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Anisotropic P-wave travel-time tomography implementing Thomsen's weak approximation in TOMO3D

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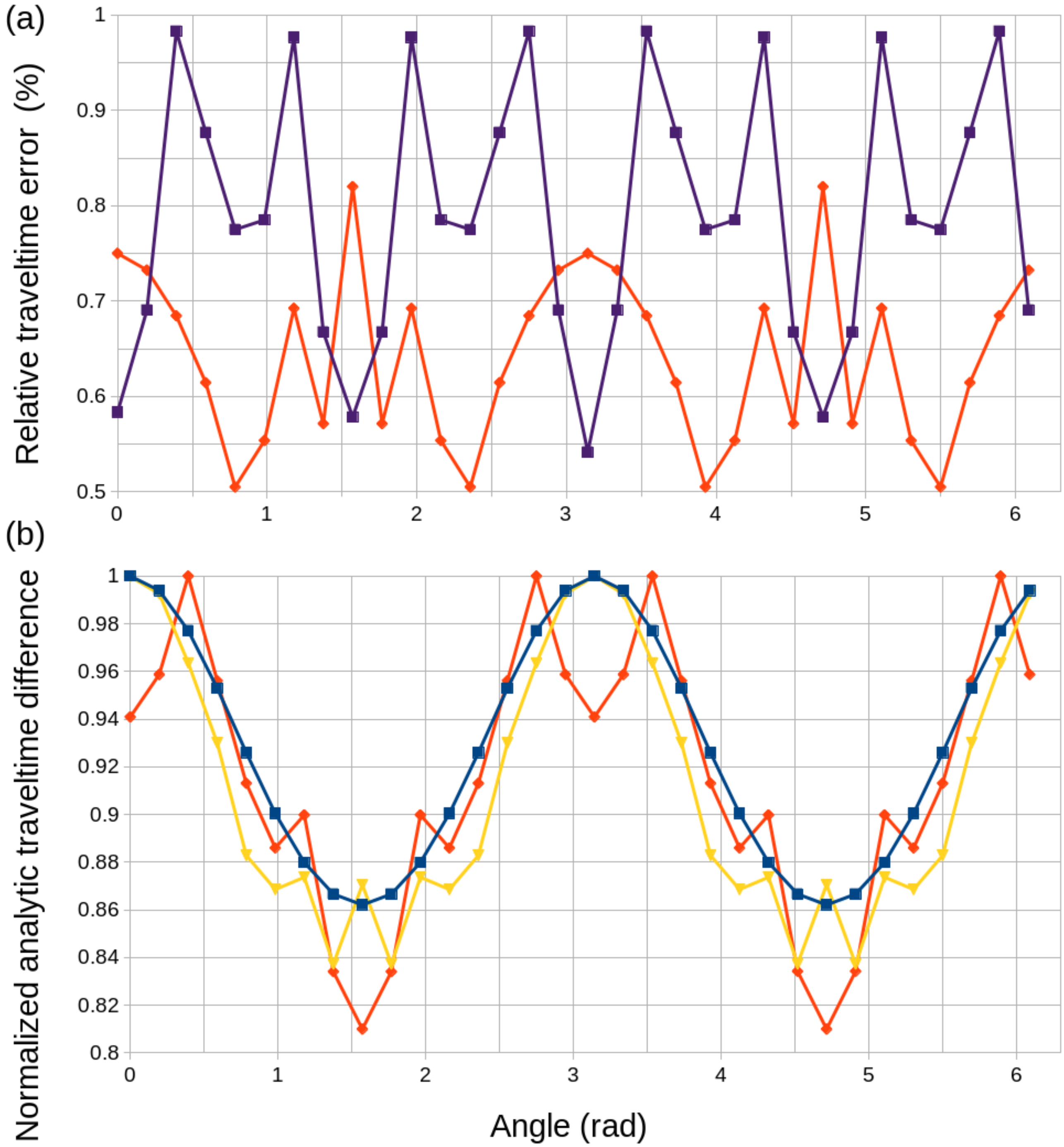

Figure S1: Improvement of (a) accuracy and (b) sensitivity pattern of v in P[ε] for the 0 rad meridian with refinement of the model grid by a factor of 2. In (a) the purple line marks the accuracy for the grid used in the synthetic tests in this paper as a reference for comparison with the orange line indicating the accuracy achieved using a refined grid. The refined average relative traveltime error is 0.64% ± 0.08% compared to the 0.8% ± 0.1%. In (b) the comparison is established among the blue line marking the analytic sensitivity as a reference, and the sensitivities for the grid that we used (orange line) and the refined grid (yellow line). The refined grid reduces the relative traveltime error and produces a better fit of the analytic sensitivity.

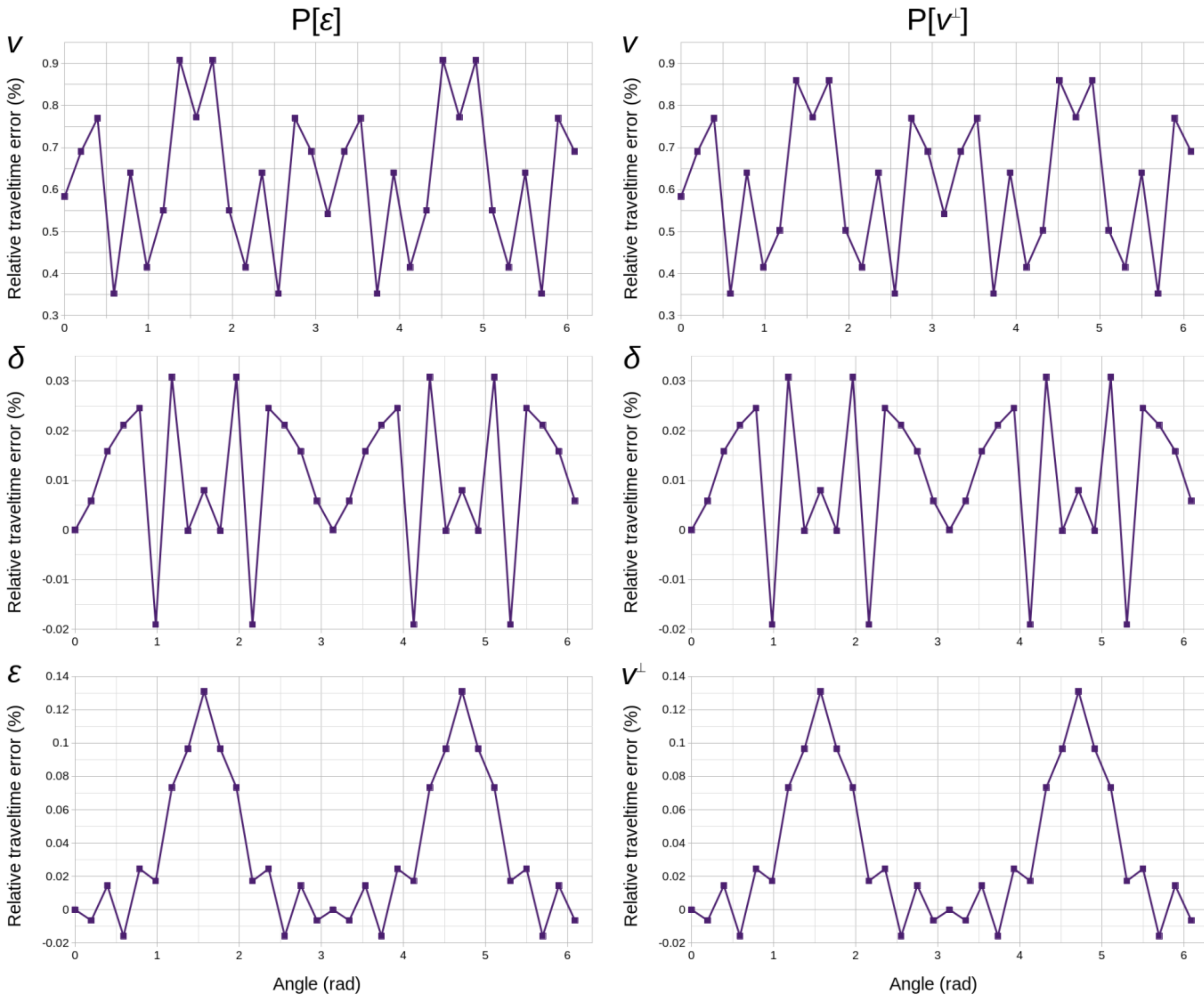

Figure S2: For the π/4 rad meridian, relative traveltime errors in percentage with respect to the analytic value for three pairs of equivalent simulations used in the sensitivity tests. Polar angle origin is in the downward vertical axis. Both parametrizations produce almost identical accuracies.

Mathematical proof for the shapes of v sensitivity in both parametrizations

Regarding the shapes of relative and normalized sensitivities for v, in the following we provide mathematical proof showing that the former is constant whereas the latter is sinusoidal.

We use Δt to refer to the difference between the travel times measured with and without the 25% anomaly. x_A is the thickness of the anomaly whereas x_B is the total ray path length from source to receiver. v_A and v_B are the anisotropic velocities in the anomaly and in the background respectively.

$$\Delta t = \Delta\left(\frac{x}{v}\right) = \frac{x_B}{v_B} - \left(\frac{x_A}{v_A} + \frac{x_B - x_A}{v_B}\right) = \frac{x_A(v_A - v_B)}{v_A v_B}$$

According to the definition we have just given, the sensitivity expressed as relative difference is

$$S_R = \frac{\frac{x_A(v_A - v_B)}{v_A v_B}}{t_R}$$

where t_R is the travel time measured without the anomaly

$$t_R = x_B / v_B$$

With that S_R becomes

$$S_R = \frac{x_A}{x_B}(1 - \frac{v_B}{v_A})$$

In the case of a 25% anomaly in the v parameter in P[ε], v_A and v_B are as follows

$$v_A = v_{PA}(1 + \delta\, sin^2\theta\, cos^2\theta + \varepsilon\, sin^4\theta)$$

$$v_B = v_{PB}(1 + \delta\, sin^2\theta\, cos^2\theta + \varepsilon\, sin^4\theta)$$

We use v_{PA} and v_{PB} to refer to the axis-parallel velocity parameter v. For a 25% anomaly in v their proportion is $v_{PA} = 1.25\, v_{PB}$. The values for x_A and x_B are $x_A = 1\, km$ and $x_B = \ km$. Thus, the final expression for S_R for a 25% anomaly in v in P[ε] is a constant 0.05 (or 5%).

$$S_R = \frac{1}{5}\left(1 - \frac{1}{1.25}\right) = 0.05$$

Regarding the sensitivity expressed as normalized difference, the general expression for an anomaly in any of the four parameters is

$$S_N = \frac{\frac{x_A(v_A - v_B)}{v_A v_B}}{\Delta t_{MAX}} = \frac{x_A}{\Delta t_{MAX}}\left(\frac{1}{v_B} - \frac{1}{v_A}\right)$$

where Δt_{MAX} is the maximum travel time difference among all four parameters. This expression will contain some combination of sinusoidal functions for all four possible anomalies. The sensitivity (normalized and relative) pattern for v in P[$v^{\perp}$] is different than for P[ε] although it follows the same sinusoidal pattern and it has equal maxima. We can see that in the case of P[$v^{\perp}$] both S_R and S_N will display a sinusoidal shape. Now v_A and v_B are

$$v_A = v_{PA}(1 + \delta\, sin^2\theta\, cos^2\theta + \left(\frac{v^{\perp}}{v_{PA}} - 1\right) sin^4\theta)$$

$$v_B = v_{PB}(1 + \delta\, sin^2\theta\, cos^2\theta + \left(\frac{v^{\perp}}{v_{PB}} - 1\right) sin^4\theta)$$

and the sine and cosine functions do not cancel out in S_R, as they did in the case of an anomaly in v in P[ε], nor in S_N. For the latter it is trivial to see that the sinusoidal dependencies are identical to the case in P[ε].