Supplement of Solid Earth, 12, 1563–1579, 2021
https://doi.org/10.5194/se-12-1563-2021-supplement

Solid Earth Open Access

Supplement of

Near-surface structure of the Sodankylä area in Finland, obtained by passive seismic interferometry

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The "Signal-to-noise ratio stacking" method for evaluating empirical Green's functions (SNRS) (Afonin et. al., 2019)

The general purpose of this method is to select for stacking only those cross-correlation functions that are not only coherent to each other, but also correspond to the stationary phase area.

Let us assume that ambient noise in some frequency band is recorded simultaneously at two different points with Cartesian coordinates $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$. For each frequency, the stationary phase area for the receiver located in the point $\boldsymbol{r_i}$, *i=1,2* corresponds to Fresnel zone of the wave propagating from the source to the receiver with some apparent velocity. In this case, the maximum of the cross-correlation function at some time lag would correspond to the minimum of apparent velocity and hence, the cross-correlation function would be close to the "true" empirical Green's function (EGF). We assume that noise sources are partly located in a stationary phase area while other noise sources are distributed outside it. For selection of cross-correlation functions corresponding to the stationary phase area, it is possible to use criteria of minimum apparent velocity and of the signal-to-noise ratio increasing after stacking. We consider the SNR of EGF after stacking as some generally non-linear function of apparent velocity and backazimuth of noise sources and an initial time window used to start selection of cross-correlation functions to the stack. In this case, the global optimisation of this objective function would allow to retrieve EGFs of high quality.

We assume again that the ambient seismic noise is recorded simultaneously at two different points with Cartesian coordinates $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$, $\boldsymbol{r} = [\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}]$ and continuous recordings are split into $\boldsymbol{n}$ time windows with the same durations. Let $\boldsymbol{a_i}(\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{t})$ is the cross-correlation function of these seismic records for the time window $\boldsymbol{i}$, $\boldsymbol{i} = \boldsymbol{1} \dots \boldsymbol{n}$, where $\boldsymbol{t}$ is a time lag of the seismic records. Let $\boldsymbol{t_m}$ is the maximum time lag in a cross-correlation function (length of cross-correlation); $\boldsymbol{t_{ds}}$ is a maximum time of wave propagation between the two points; $|\boldsymbol{t_m}| \gg |\boldsymbol{t_{ds}}|$ and $-\boldsymbol{t_m} \le \boldsymbol{t} \le \boldsymbol{t_m}$. Let $-\boldsymbol{t_{ds}} \le \boldsymbol{t_e} \le \boldsymbol{t_{ds}}$ is the time lag on the cross-correlation function corresponding to the expected seismic phase (body or surface wave) and $\triangle \boldsymbol{t_e} = \boldsymbol{t_e} \pm \boldsymbol{T}$, where $\boldsymbol{T}$ is the period of expected signal. Negative values of the time lags correspond to the anti-causal part of the evaluated EGF. In this case, selection of $\boldsymbol{t_{ds}}$ and $\triangle \boldsymbol{t_e}$ is based upon *a priori* information about seismic velocities in the studied area. The value of $\triangle \boldsymbol{t_e}$ is at least two periods of the expected signal dominant frequency. In the case of evaluation of surface wave parts of EGFs, this frequency usually corresponds to the frequency of noise with the largest amplitude that can be estimated by time-frequency analysis of the seismic noise records.

Let $\boldsymbol{a_i^{max}}(\boldsymbol{r_1}, \boldsymbol{r_2}, \triangle \boldsymbol{t_e})$ is the maximum value of cross-correlation function in the time interval $\triangle \boldsymbol{t_e}$. Then, the signal-to-noise ratio of the cross-correlation function calculated for the $\boldsymbol{i\ th} -$ time window ($\boldsymbol{SNR}(\boldsymbol{a_i}(\boldsymbol{r_1}, \boldsymbol{r_2}, \boldsymbol{t}))$) is:

$$SNR(a_i(\boldsymbol{r_1}, \boldsymbol{r_2}, t)) = \frac{a_i^{max}(\boldsymbol{r_1}, \boldsymbol{r_2}, \triangle t_e)}{\frac{1}{2|t_m - t_{ds}|}\left(\int_{t_{ds}}^{t_m} a_i^2(\boldsymbol{r_1}, \boldsymbol{r_2}, t)dt + \int_{-t_m}^{-t_{ds}} a_i^2(\boldsymbol{r_1}, \boldsymbol{r_2}, t)dt\right)} \quad (1)$$

Let $a_i(\boldsymbol{r_1}, \boldsymbol{r_2}, t)$ and $a_j(\boldsymbol{r_1}, \boldsymbol{r_2}, t)$ are cross-correlation functions calculated for two different time windows $i \in (1..n)$ and $j \in (1..n)$ and $c(\boldsymbol{r_1}, \boldsymbol{r_2}, t) = a_i(\boldsymbol{r_1}, \boldsymbol{r_2}, t) + a_j(\boldsymbol{r_1}, \boldsymbol{r_2}, t)$ is an EGF retrieved from these two cross-correlation functions. If $a_i(\boldsymbol{r_1}, \boldsymbol{r_2}, t)$ and $a_j(\boldsymbol{r_1}, \boldsymbol{r_2}, t)$ are coherent to each other and $i \ne j$, then expressions $SNR(a_i(\boldsymbol{r_1}, \boldsymbol{r_2}, t)) < SNR(c(\boldsymbol{r_1}, \boldsymbol{r_2}, t))$ and $SNR(a_j(\boldsymbol{r_1}, \boldsymbol{r_2}, t)) < SNR(c(\boldsymbol{r_1}, \boldsymbol{r_2}, t))$ have to be true, according to the principle of interference. Condition $i \ne j$ is necessary in order to avoid stacking of functions with itself. Therefore, increasing SNR of the retrieved EGF after stacking with some cross-correlation function can be used as a criterion for selection of this function to the stack, excluding incoherent functions from the stack and building up the EGF with high signal-to-noise ratio.

Based on the criteria described above, an expression for calculation of EGF for k-th iteration can be written as

$$G^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t) = \sum_{\substack{i=1 \\ i \ne k}}^{n} (G_i^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t) + a_i(\boldsymbol{r_1}, \boldsymbol{r_2}, t) * \delta(G_i^k, a_i)), \quad (2)$$

where $k = 1 \dots n$ is the number of initial function; n is the number of time windows; $i = 1, \dots, n$; $G_i^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t)$ is EGF corresponding to k-th – initial function and evaluated in previous iterations:

$$G_i^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t) = \begin{cases} a_k(\boldsymbol{r_1}, \boldsymbol{r_2}, t), \ i = 1 \\ G_{i-1}^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t), \ i \neq 1 \end{cases}. \quad (3)$$

The operator of selection can be written as

$$\delta(G_i^k, a_i) = \begin{cases} 0, SNR\left(G_i^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t) + a_i(\boldsymbol{r_1}, \boldsymbol{r_2}, t)\right) < SNR\left(G_i^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t)\right); \\ 1, SNR\left(G_i^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t) + a_i(\boldsymbol{r_1}, \boldsymbol{r_2}, t)\right) \geq SNR\left(G_i^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t)\right); \end{cases} \quad (4)$$

As a result of this algorithm we obtain n candidates for EGF that can be considered as solutions to the optimization problem in some parameter space. Let us consider the signal-to-noise ratio as some function *f(k)*, where *k* is the index of initial functions: $SNR(G^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t)) = f(k), k = 1, \dots, n$. Then the condition for the final EGF selection can be written as $m = argmax(f(k))$, where *m* denotes the index of EGF selected to the stack. Following this condition, the EGF with maximum signal-to-noise ratio will be selected as the final one. As the function $f(k)$ may have several local maxima in the parameter space k, *k=1,..,n,* the condition for the final EGF selection ensures that the global maximum of this function is obtained in the parameter space considered.

In the proposed algorithm, maximizing the signal-to-noise ratio of the retrieved EGF is ensured by stacking of only cross-correlation functions coherent to each other and selection of EGF with the maximum signal-to-noise ratio from all calculated candidate EGFs. In other words, the proposed algorithm is analogous to the direct search methods of global optimization. It is necessary to remember, however, that EGF with the maximum signal-to-noise ratio does not correspond to a true EGF, if the dominant noise sources are located outside the stationary phase area. Therefore, it is important to use the system of observations that allows estimating azimuthal distribution of noise sources. Moreover, the method is based on assumption that sources of the ambient seismic noise produce a signal with relatively broad bandwidth and cannot produce an ideal harmonic signal of single frequency.

The method also makes it possible to keep control over a-priory unknown azimuthal distribution of noise sources. For this, a 2-D array of seismic recording stations is necessary. In this case, the time lags, corresponding to expected signal $\Delta\ t_e$ in Eq. 1 have to be a function of apparent seismic velocity and backazimuth: $\Delta\ t_e = f(v, \varphi)$. Then signal-to-noise ratio for each pair of stations of the array is the function of initial function index, velocity and backazimuth: $SNR(G^k(\boldsymbol{r_1}, \boldsymbol{r_2}, t)) = f(k, v, \varphi), k = 1, \dots, n,\ v_{min} \leq v \leq v_{max},\ 0 \leq\ \varphi\ \leq 360$. Limits of apparent velocities have to be calculated according to *a-priory* information about seismic velocities in the studied area. A global maximum of the function corresponds to the strongest or the most coherent wavefield. Therefore, the method allows estimating azimuth to the strongest source of noise wavefield.

Reference: Afonin, N., Kozlovskaya, E., Nevalainen, J., & Narkilahti, J. (2019). Improving the quality of empirical Green's functions, obtained by cross-correlation of high-frequency ambient seismic noise. Solid Earth, 10(5), 1621-1634.