

PAPER REVIEW

Title : "Regional wave propagation using the discontinuous Galerkin method"

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GENERAL COMMENTS

This paper presents a discontinuous Galerkin (DG) finite element method for 3D seismic wave modeling in regional scale models. Such method is suitable when models with complex geometries are considered. In the framework of seismic modelling, finite elements allows to deal with topographies and/or sharp geological structures. In this respect, the work-flow presented by the authors, allows to build tetrahedral meshes that are adequate for regional models. This is related to the flexibility offered by the combination of the DG method with tetrahedral elements. As stated by the authors, this method is an interesting alternative to the more popular spectral element (SE) method, which relies on hexahedral elements.

The paper is structured as follows: after an introduction that explains clearly the purpose of the paper, the authors briefly present the principles of the discontinuous Galerkin method with the ADER time integration and its implementation within the code SeisSol. The work-flow for the generation of tetrahedral meshes with the tool Cubit is also presented with a focus on the European model. This original model has been created from the assembling of the EPcrust and ak135 models. Section 3 is devoted to benchmark tests, where the simulations of seismic waves with DG in the PREM model are compared with the SE method. Finally, the authors present the result of a simulation of the l'Aquila earthquake (Italy) with the DG method in the European model. Some conclusions and perspectives conclude the study.

To my knowledge, this is the first publication concerning the application of the DG method to the simulation of seismic wave in 3D regional models. Hence, this study presents interesting and new results. To my opinion, taking into account the potentials of the method, I believe that the contribution of the authors fully deserves a publication in an international journal such as Solid Earth.

Nevertheless, the paper suffers of the following points :

1 - The text needs a correction from an English reviewer. Some sentences sound awkward, sometimes they are misleading and they finally disturb the reading. Also, the punctuation needs a severe checking since lot of commas are missing in the text.

2 - Interesting results are presented but their analysis should be improved.

3 - I have found a general lack of technical information.

All sections are balanced but they can be improved by adding some important information and explanations. Hence, my comments will mainly focus on some weaknesses regarding important information that are missing for a comprehensive reading. In this objective, I suggest the authors to add :

1 - A new figure showing the mesh for the PREM model.

2 - A table with the statistics concerning the mesh of the European model.

3 - A table that compares the meshes used by DG and SE for the benchmark.

4 - A table with some metrics concerning the computation with DG and SE for the benchmark.

In the following, you will find my detailed comments in order to improve the quality of the paper. The list of items is quite long but since these corrections do not represent a significant additional work, I request a moderate revision.

Once again, it was a pleasure to review this paper and I hope the authors will find my comments useful.

Dr. Vincent ETIENNE

DETAILED COMMENTS FOR EACH SECTION

* ABSTRACT

No comment

* 1 - INTRODUCTION

Page 1130, lines 18-19 (definition) : "...on regional seismic wave propagation...". Since this is the main concern of your article, please define at the beginning what you mean by "regional". For instance, you can give some indications on the model dimensions, in terms of kms, degrees, etc...

Page 1131, line 5 (terminology): "Besides semi-analytical algorithms, numerical direct solvers evolved such as the Finite-Difference...". The term "direct solvers" is somehow misleading since in the applied mathematical community, this term is related to an algorithm that are solving linear system with numerical techniques such as LU factorization. Therefore, I would mention instead "explicit time marching schemes".

Page 1131, line 23 (explain): "...and explicit symplectic time extrapolation schemes". You are referring to the time integration used in SE methods. Concerning the standard SE method (Komatitsch 1997), I believe that the time integration is performed via a second-order operator based on a predictor / corrector scheme. Maybe, other modelers using SE are using different schemes. Hence, I do not understand your point. Please clarify why you specify that SE is linked to a symplectic time interpolation ? Is it really specific to SE methods ?

Page 1132, line 3 (be more precise): "...the elements can locally be adapted (h-adaptivity) without overhead". Please explain why there is no overhead ?

Page 1132, line 18 (sounds awkward): "...technical properties of the implementation..." Are you referring to "computing strategies for the implementation of the DG method" or "numerical properties of the DG scheme" ?

* 2 - WAVE PROPAGATION IN 3-D MEDIA

** 2.1 - THE ADER-DG APPROACH

Page 1133, line 9 (correct english): "Like every Finite-Element method" instead of "As every Finite-Element method" ?

Page 1133, lines 11-12 (sounds awkward): "In the scheme (add a comma) the complete 3-D computational domain..." Do you mean: "In our implementation of the DG scheme, the complete 3-D computational domain..." ?

Page 1133, lines 17-18 (add information): "We use an orthogonal basis suggested by Dubiner (1991)". I would recommend to indicate that you are using modal basis

functions. This is an important characteristic of your approach.

Page 1133, lines 24-25 (wrong statement): "Note, that differently from FV only adjacent elements communicate, and a reconstruction process is not required". First : in FV methods, the communication do actually concerns only adjacent elements. At least, this is the case for the P0 FV method where the solution is approximated as piece-wise constant per element. Second : what do you mean with a reconstruction process ? Do you refer to the assembling usually encountered in standard FE methods ? Please clarify.

Page 1134, line 3 (add information and explain): "...many occurring integrals can be precomputed in the reference space". You should give more information concerning the integrals. For the reader not familiar with the DG method, it is useful to mention the name of whose matrices : mass, stiffness and flux matrices. Then, when you are saying that many matrices can be precomputed, is it related to the fact that you are using homogeneous properties within the elements ? I think this is a really important assumption in your formalism and since you mentioned in the introduction that you will focus on the model discretization, I am surprised that this assumption is not even mentioned here. Please, add a sentence concerning the approximation of the physical properties within the elements (it is piece-wise constant ?) and explain why you have chosen this specific approximation.

** 2.2 - LOCAL ADAPTIVITY AND LOAD BALANCING

Page 1134, line 26 (english): "Until now, this is handled by zoning...". Replace "Until now" with a less abrupt expression (like "This issue can be handled by zoning...") or remove these words which are not really informative.

** 2.3 - MESH GENERATION IN GEOMETRICALLY COMPLEX 3-D MEDIA

Page 1135, line 10 (english): "...which have tremendous influence on the propagation...". Here, "tremendous" seems too excessive, use instead a more neutral adjective.

*** 2.3.1 - STRUCTURED MESHING

Page 1135, lines 24-26 (move statement or explain): "In contrast, Komatitsch (1997) assign the material properties directly to single integration nodes in the SE scheme, at least, if interfaces cannot be respected within the computational mesh." Actually, this corresponds to the simplest strategy you mentioned at the beginning of section 2.3.1. (line 15 "material properties are simply assigned to nodes...") Please explain why you mention here the SE as a contrast of the approach of Kristek et al. ? This statement should be moved around the line 15.

*** 2.3.2 - UNSTRUCTURED MESHING

Page 1136, lines 25-26 (english): "Furthermore, assembling a scheme that is..." The term "assembling" may be associated to finite-element methods and therefore I would recommend to replace it with another word.

Page 1137, lines 9-10 (english): "...but are usually only in low-order formulations efficiently implementable that are very dispersive." Please reformulate this sentence which sounds awkward.

Page 1137, line 14 (english): "...tetrahedral grids are much more flexible to align". The term "align" is misleading. Please replace this word with another.

Page 1137, line 14-15 (explain): "This tremendously reduce the meshing effort potentially at the expense of longer simulation time". Please explain your point concerning the simulation time. If the mesh allows for a better discretization of the model, one may expect a more precise solution with a reduced numerical cost. Here, you should explain how and why the simulation time can be affected.

** 2.4 - WORKFLOW OF TETRAHEDRAL MESH GENERATION USING CUBIT

*** 2.4.1 - GEOMETRY GENERATION - EUROPEAN MODEL

Page 1138, line 1 (punctuation): "Recent studies investigated, whether internal material...". Remove the comma after "investigated".

Page 1138, lines 1-4 (add information) : you are referring to studies (Komatitsch, Lee, Casarotti, Stupazzini and Cupillard) but you are not saying anything about the conclusions of these studies. Please add some information. What are the conclusions of the authors ? Do we need absolutely to respect the interfaces within the mesh ?

Page 1138, lines 5-6 (explain): "Not respecting material interfaces of strong contrasts requires a high resolution for ADER-DG schemes". Do you mean here that you need a very fine discretization ? I guess that this is linked to the constant properties per element. If so, you should indicate it clearly since at the beginning of the section 2.4.1 you refer to the SE method which is not based on this assumption.

Page 1138, lines 17-18 (useless statement): "A Matlab parser can read the file to a Matlab structure which contains the material information and locations of the 3 D interfaces". The fact that you are using a Matlab program is not informative for the reader. Remove this statement.

Page 1138, lines 18-20 (explain): "The upper-mantle discontinuities of the ak135 model are manually projected on spherical shells using the same lateral sampling points". What do you mean by "manually projected" ? Does this task can be done automatically which some kind of interpolation ?

Page 1138, lines 24-25 (explain): "In Cubit a surface reconstruction directly from these pointsets failed". Include a comma after "Cubit". If you want to enter into such details when using Cubit, you should indicate why the process failed. Does Cubit give information about the failure ?

Page 1138, lines 25-26 (explain): "Therefore, parallel spline curves along a row or column of the structured pointsets had to be created using the vertices as spatial support". Does it that after this procedure, the surface has been smoothed ?

Page 1138, lines 26-27 (punctuation): Add a comma after "From the generated lineset".

Page 1139, lines 4-6 (explain): "Furthermore, so-called imprinting surfaces have to be defined to generate conforming meshes at each layer boundary". What are the imprinting surfaces ? Does the mesh built in several pieces ? Why the need to define surface in order to generate conforming mesh. Please explain better.

*** 2.4.2 - TETRAHEDRAL MESH GENERATION - EUROPEAN MODEL

Page 1139, line 20 (add information): "Cubit provides geometry adaptive sizing functions to control the element size...". What is the parameter for measuring the element size ? Edge length, insphere radius, volume ? Be more precise.

Page 1140, line 6 (add information): "In case of distorted elements in the mesh..." Again, indicate how you detect a distorted element ? What is the criterion you adopt with Cubit ? Also can you provide more information about the algorithm implemented in Cubit ? How the mesh is actually built ? Does the process rely on a Delaunay triangulation ?

Page 1140, line 13 (punctuation): Add a comma after "despite the mentioned difficulties".

Page 1140, lines 13-14 "...the generation of a high-quality tetrahedral mesh took one day...". An important part of your article is devoted to the construction of tetrahedral meshes. Here, you should give some statistics about the mesh you built for the European model. I would suggest to add a table with at least the total nb of elements, min/max quality factors, min/max edge lengths and some other useful information. This table will help the reader to understand the complexity of the mesh while illustrating the flexibility offered by tetrahedral elements.

* 3 - BENCHMARKS

Page 1140, line 24 (punctuation): Add a comma after "In the first experiment".

Page 1141, line 5 (punctuation): Add a comma after "With respect to the results of the SE method".

** 3.1 - EXPLOSIVE SOURCE

*** 3.1.1 - SETUP

Page 1141, line 12 (punctuation): Add a comma after "In this test".

Page 1141, line 18 (be more precise): "with a main period of $T_{peak} = 20s$ ". Here, I guess you refer to the dominant period of the Ricker wavelet ? In this case, you may indicate that the minimum period of the source function is 8 s (i.e. $20 / 2.5$).

Page 1141, lines 19-20 (add figure and explain): "The physical domain of the SeisSol simulation, is a cuboid of $\Omega = [-1000 \text{ km}, 1000\text{km}] \times [-500 \text{ km}, 3500\text{km}] \times [2400 \text{ km}, 6400\text{km}]$...". I would suggest to add a figure that shows the geometry of the mesh and possibly a view of its interior. It is particularly illustrative to show how the mesh honors the Earth discontinuities and how you adapt the size of the element with depth.

Page 1141, lines 25-26 (explain): "This keeps a constant number of $n_{min} = 3$ tetrahedral elements per shortest wavelength in each subdomain to model a shortest wave period of $T_{min} = 20s$ ". First point : Please justify why you adopt an average spatial discretization of 3 elements per shortest wavelength. I guess that with $O=5$, the recommended discretization with the SE method is only one element per shortest wavelength. Explain, why your discretization rules with DG-ADER are more severe than with SE. This is an important point, taken into account that for the same discretization length, the nb of tetrahedra is more important than with hexahedra. Second point: here you mention a shortest period of 20 s. This is not consistent with my precedent comment (see above).

Page 1142, lines 1-2 (add more information): "This high spatial discretization of the mesh is also suited to model wavefields over long propagation distances". Can you be more precise and indicate a typical range of propagation distances in terms of nb of wavelength ?

Page 1142, lines 3-8 (add a table): to conclude the descriptions of the meshes used by the SeisSol and SpecFEM codes, I would suggest to add a comparison table with some basic information : the volume of the modelling domain, the total nb of elements, min/max quality factors, min/max edge lengths and some other useful information for both SeisSol and SpecFEM.

Page 1142, lines 9-10 (add information): Can you indicate the range of propagation distances in terms on nb wavelength between the source and the receivers ?

*** 3.1.2 - RESULTS

Page 1142, line 14 (punctuation): Add a comma after "For each station".

** 3.2 - SHEAR DISLOCATION SOURCE

*** 3.2.1 - SETUP

Page 1143, lines 12 (explain): "For the SeisSol simulation (add a comma) we reduced the block model of...". Please explain why the model has been reduced in this case.

At the end of this paragraph, you should indicate the position of the receivers since they probably are not located at the same positions than in the precedent test. Again, specify the range of propagation distances in terms of nb of wavelength.

*** 3.2.2 - RESULTS

Page 1143, line 20 (punctuation): Add a comma after "As expected".

** 3.2 - DISCUSSION

Page 1144, lines 8-9 (add information): "In SeisCol (add a comma) a constant value for each parameter is interpolated in one single tetrahedral element..." I believe that this assumption should have been introduced in section 2.1 (see my precedent comment). Can you also indicate how the properties are interpolated per element ? Do you perform a kind of averaging ? Is it based on the barycenter of the element ?

Page 1144, lines 13-15 (english and explain): "But, (remove the comma) tests have shown that the spatial discretization already is determined by an accurate approximation of the wavelength due to the CFL-condition". I do not understand your point, please reformulate this sentence.

Page 1144, lines 15-16 (shortcoming): "Therefore, the sampling of the relatively low material gradient in PREM is sufficient". Can you justify your statement ? For instance you can rely on the theoretical variation of properties within the elements, i.e. what is the typical percentage of velocity variation at the scale of the elements ?

Page 1144, line 18 (punctuation): Add a comma after "At near offset stations".

Page 1144, lines 21-22 (punctuation): Add comma after "Käser and Igel (2001) claimed that" and after "an isotropic elastic medium".

Page 1144, lines 26-27 (comment): "the effect is purely numerical and can be diminished by refining the mesh around the source". I guess you implement the source on a single element. What happens if the source coincide with one corner of the element ? Also, have you tried to spread the source over several elements ? You may discuss a little bit about the implementation of the source since it seems that it is mesh-dependent.

** 3.4 - COMPARISON OF CODE PERFORMANCE

Page 1145, lines 8-10 (add information): "Since the simulations of this study provides a different level of accuracy on different meshes and physical domains (add a comma) a quantitative comparison is not possible (, remove the comma) in fair terms". I believe you have a valuable information to provide here. It is true that the modeling with DG-ADER and SE have been performed on completely different meshes but there are some interesting parameters to look at. Therefore and again, I would suggest to add a comparison table. In this table you should indicate, for both SE and DG-ADER: the nb of unknowns, the dimension of the modelling domain, the nb of time steps and also the measured computation time with the nb of MPI process. Then, from this values, you can estimate the average computation time per unknown, per MPI process and per time step. In Etienne et al. 2010, we found out at this computation time is comparable between SE and DG.

Is it also the case with your specific DG-ADER formulation ? Let me say that this is just an interesting parameter and the objective is not to indicate that a code is better than another and vice-versa. But since you decided to include a paragraph entitled "Comparison of code performance", I think you should give some statistics for the reader.

Page 1145, line 11 (punctuation): Add a comma after "...efficiency of numerical codes".

* 4 - APPLICATION OF THE ADER-DG METHOD TO REAL DATA: THE 2009 L'AQUILA EARTHQUAKE

Page 1145, line 20 (punctuation): Add a comma after "In the previous test". Remove comma after "...it could be demonstrated".

Page 1145, line 21 (punctuation): Add a comma after "In this section".

** 4.1 - MODEL SETUP

Page 1146, line 11 (punctuation): Add a comma after "of the Earth (and remove 's) crust"

** 4.2 - GEOMETRICAL REPRESENTATION

Page 1147, line 14 (punctuation): Add a comma after "To generate a mesh inside the volume"

Page 1147, lines 19-20 (justify): "a spatial sampling of 2 tetrahedral elements per smallest wavelength, if an $O = 5$ scheme is applied". Here you adopt a different discretization from the validation test of section 3 where you decided to use 3 elements per smallest wavelength (with also $O = 5$). Please, explain why you are changing your discretization criteria. Actually, one would have expected that the validation tests allow to estimate the required discretization to be used later in the real application.

Page 1147, line 21 (punctuation and shortcoming): "As already mentioned, in SeisSol (add comma ,) the materials values of tetrahedral elements are averaged over all vertex values". Well, it has not been "already mentioned". Please refer to my precedent comments.

Page 1147, line 23 (punctuation): Add a comma after "...mesh generation".

Page 1147, line 24-26 (shortcoming): "Assuming a seismic source signal at a peak frequency of 0.03 Hz (add a comma ,) the smallest wavelength of 36 km can be sampled correctly". If I understand well, the average size of the element is 18 km (line 22). Then you have indicated that in the EPcrust model the S-wave velocity varies from 0.4 to 4.1 km/s. Then this leads to a wavelength of $0.4 / 0.03 = 13.3$ km. You can see here that the elements are larger than the wavelength and therefore the near subsurface (where the surface waves propagate) is not well sampled. But due to the interpolation of the physical properties, the minimum V_s is 1.1 km and this changes a lot the physical model. Please, comment the effect of the interpolation that produced higher V_s at the surface than in the real Earth.

Page 1148, line 1 (punctuation): Add a comma after "In the mantle".

Page 1148, line 3 (typo): "3.7Melements" should be written "3.7 million elements". Do not use the point for the thousand separator in "1.164 M degrees" since it has been used before as the decimal separator. This makes around 315 degrees of freedom per element. Is is correct ?

** 4.3 - DATA PROCESSING

Page 1148, line 8 (punctuation): Add a comma after "For the l'Aquila

earthquake".

Page 1148, line 10 (punctuation): Add a comma after "From these networks".

Page 1148, line 15 (punctuation): Add a comma after "For the simulation".

Page 1148, line 17 (punctuation and acronym) : Define "STF". Add a comma after "To obtain the STF".

Page 1148, line 19 (punctuation): Add a comma after "Subsequently".

** 4.4 - RESULTS AND DISCUSSIONS

Page 1148, line 22 (punctuation): Add a comma after "For real and synthetic data".

Page 1148, line 23 (punctuation): Add a comma after "Here".

Page 1149, lines 1-2 (explain): "The misfit between data and synthetics can be attributed mainly to the approximation of the material values inside the Earth by the applied velocity models". You could probably improve your analysis. Is seems that the S and the surface waves arrive earlier in the numerical simulation with SeisSol than in the observed data. This can be due to higher velocities in the near sub-surface of the modelling mesh. Is it related to the approximation that changes for instances S-wave velocity from 0.4 to 1.1 km / s at the surface ? Also, you did not indicate if an attenuation law was applied in your modelling. Please clarify.

Page 1149, line 3 (punctuation): Add a comma after "Concluding".

Page 1149, line 4 (english): Use "For instance" instead of "Exemplary"

Page 1149, lines 4-6 (be more precise): "...this can be seen, at the near offset station MATE where boundary reflections only occur after the surface wave has passed". To help the reader, indicate at what time these reflections can be observed in the seismograms.

* 5 - CONCLUSIONS

Page 1149, line 17 (punctuation): Add a comma after "Due to the use of unstructured tetrahedral meshes".

Page 1149, line 19 (punctuation): Add a comma after "In the second part".

Page 1149, line 25 (punctuation): Add a comma after "As described in Sec 2.2".

page 1149, line 27 (english): "...this study can focus the computational effort using the ADER-DG method". Instead of "focus" do you mean "justify" ?

Page 1150, line 3 (punctuation): Add a comma after "In a future study".

* TABLES

No comment

* FIGURES

Figure 2 : Could you explain why the seismograms are not complete for each receiver ?

Figure 3 : Same comment than above.

Figure 4 : This is an interesting figure but it is really too small. I suggest to enlarge it (over two columns). Indicate the scale for the dimension of the mesh (left). In the zoom (bottom right), we have the impression that the properties are represented with gradient within the elements while it should be piece-wise constant per element. Please clarify.

* REFERENCES

No comment

*** END OF REVIEW ***