Detailed comments

<u>Pag. 1346 line 5:</u> "where the scaling factor has a specified variance". I would eliminate this sentence because, reading the abstract, the reader has no idea about scaling factor and about its statistical behavior used in the paper.

<u>Pag. 1350 line 4:</u> "Models of the core field……". In my opinion, the first describing sentences appear not totally clear and it is difficult to understand the following parts. I think that the authors should explain more exhaustively what is the noise in magnetic field, maybe using simple equations. For example, the topic can be introduced saying that the measured magnetic field contains these contributes:

$$T = T^{Nuc} + T^{ext} + T^{lit} + T^{res}$$

Where

T^{Nuc} is the contribute of the earth nucleus.

T^{Ext} is the external field

T^{Lit} is the field generated by lithosphere

T^{res} is the residual field where only short wavelengths should remain.

The first three contributes are removed from measured field by means of mathematical models. The removing producers can be erroneous because mathematical models are not perfect especially for external field. Due to this, sometimes the field T^{res} is the superimposition of:

$$T^{res} = T^{noise} + T^{short\_wav\_res}$$

Where T^{noise} is the field not correctly removed and shows long wavelengths and . $T^{short\_wav\_res}$ is the real filtered short wavelength field. The paper try to model how T^{noise} affects the lithospheric model.

I would like to say that I'm not sure about this explanation but it was what I could understand with difficulty from the text. If it is right, in my opinion these equations have to be inserted. If not, similar equations are necessary for a clear explanation

<u>Pag. 1350 line 16:</u> "We will also often use the index l for the degree and m for the order". I would delete this sentence because, in the paper, the indexes n and k are always used for Brp, while the indexes l and m are always used for Bri. This sentence at this point can make the reader think that the indexes l and m can be used also for Brp and create some confusion in equations (6) and (7) where appear the indexes l' and m' that comes from the derivation of

$$\Phi = \sum_{i=1}^{\vartheta\_num} \sum_{j=1}^{\phi\_num} w_i \left| \sum_{l=1}^{L} \sum_{m=-l}^{-1} \left(\frac{a}{r}\right)^{l+2} (l+1)\widetilde{g_l^m}\sin(|m|\phi_j)P_l^m(\theta_i) \right.$$

$$+ \sum_{l=1}^{L} \sum_{m=0}^{l} \left(\frac{a}{r}\right)^{l+2} (l+1)\widetilde{g_l^m}\cos(|m|\phi_j)P_l^m(\theta_i)$$

$$\left. - \sum_{n=1}^{N} \sum_{k=-n}^{-1} \eta_i \left(\frac{r}{a}\right)^{n-1} n\epsilon_n^k \sin(|k|\phi_j)P_n^k(\theta_i) - \sum_{n=1}^{N} \sum_{k=0}^{n} \eta_i \left(\frac{r}{a}\right)^{n-1} n\epsilon_n^k \cos(|k|\phi_j)P_n^k(\theta_i) \right|^2$$

This formula is not showed in the text, so the reader, without all the mathematical steps, can think that the indexes l' and m' can come from the equation (1) and not from the product of the two sums.

Pag. 1351 line 2: "We simply need to minimize". Why there is the need to minimize? I think that authors want to describe the external noise field with a lithospheric internal field but, if it is the case, it is not so obvious for readers. So, in my opinion, the authors should clarify this point. For example, if I'm right, at page 1349 line 18 they can replace "We consider also a model of….." with "the noise field can be well approximated also by a magnetic field on internal origin….". Again, I'm not sure that my interpretation of the text is right but, in my opinion, the text has to be written without the possibility of wrong interpretations.

Pag. 1351 line 5: "…and wi are weights that are defined below". The weights are never clearly defined in the paper. The next reference to weights in at line 10 of page 1351 where they are posed $\sin(\theta_i)$ without any explanation. In my opinion the authors should clarify the meaning of the weights and explain why they use some values.

Pag. 1353 line 9: "However an adaptive Gaussian quadrature….". In order to reproduce the experiments, more information about this method are necessary. In my opinion, the authors can explain the method or add a reference where readers can find the details about the computation.

Pag. 1354 line 26: "We do not aim at a precise description of the disturbance….". In order to reproduce the experiment, the authors should explain how to evaluate coefficients l_n^k or insert a reference where the reader can find how they can be computed.

Pag. 1355 lines 19-25: "The process we applied….". I could not understand exactly the steps. Until now, no real data have been used and the models have been obtained by imposing some values for gauss coefficients on a noise field data. Here the authors say that a least-square approach has to be used on real satellite data. How? Is it possible to get the noise field? Maybe the coefficients g_m^l have to be calculated by means of an inversion on real data? The three fields indicated from line 3 to line 8 of pag. 1356 are all noise fields or output fields. I would like to know how can apply their method starting from magnetic field data Even if in the next sections, the authors describe better the three steps; in my opinion, the authors should add some equations here in order to better explain what they estimate in the "first straightforward least square minimization", what are the results used as data in the second step, and what is the relation to perform the second step. In other words they have to explain the general algorithm A flow chart should be a good way to explain the three-steps algorithm.

Pag. 1357 lines 9-10: "the associated weight…..looks acceptable". What does it mean "acceptable"? In order to reproduce this experiment I would know how I have to adjust the weights and above all when I

have to stop in adjusting. In my opinion, the authors should explain more quantitatively when the model is acceptable.

Pag. 1357 lines 16-16: "the data weights used depend only on the data density". Can the authors explain better the dependency of the weights by the data density?

Pag. 1360 line 14: "…an acceptable power spectrum". . See comment about page 1357 lines 9-10.