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Comment

# ***Interactive comment on “Modelling complex geological angular data with the Projected Normal distribution and mixtures of von Mises distributions” by R. M. Lark et al.***

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Again, we are grateful for this review of our paper. Our responses and proposals for revision are as follows.

## **1. Results**

As noted in response (4) to the Anonymous reviewer 1, we could add a table of the MVM parameter estimates. However, as Wang and Gelfand (2013) note, the parame-

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ters of the PN model are not directly very informative, and the density plots on Figures 2 and 3 are much more useful.

## 2. Resolution of data

It is not uncommon in the geological literature to find that angular data are presented as counts in bins, and for the analysis to proceed from the binned data. It seems that the reviewer thinks that is what we have done here. However, the binning of our data is for the rose diagrams only, the maximum likelihood estimates, for either the MVM or the PN distributions, are done directly from the original angular (or doubled orientation) data. There is no binning of the data to some resolution which, as the reviewer correctly states, would reduce the precision of our models. To clarify this we could add the following sentence to the end of section 3.1.1.:

‘Note that this binning is done only for the presentation of the data as rose diagrams; all statistical analysis is done on the raw angular observations.’

## 3. Sensitivity of the selection of the number of terms in the MVM to sample size

The referee is correct that the sample size may affect our decision on the number of terms to include in the MVM model. However, our bootstrapping procedure deals with this directly. The decision whether to select a model with  $g + 1$  terms over against a model with  $g$  terms is based on the log-likelihood ratio. This is tested against an empirical distribution of the ratio generated from simulated instances in which the underlying number of terms is  $g$  and *for the sample size of our data set*. Our decision to reject, for example, the hypothesis of  $g$  terms in favour of  $g + 1$  is therefore based on the empirical distribution of the test statistic in the null case for a sample size equal to the one we



have. We could add a sentence to the end of section 2.1 to state:

‘Note that, by using this bootstrapping procedure we also avoid testing the log-likelihood ratio against an asymptotic distribution by using an empirical distribution for the same sample size as our data.’

#### 4. Accessibility

We think that the referee’s concerns about accessibility could largely be met by giving a more intuitive account of the PN distribution where it first appears (page 2183, line 27 of the discussion paper). We propose to edit this section to read:

‘An alternative model for more complex distributions of angular data is the projected normal (PN) distribution. This distribution, and its flexibility, can easily be understood intuitively. Consider a distribution of points on a scatter plot formed by observations drawn from a pair of correlated normal random variables. One may draw a line from each point to the origin, forming a vector with some angle,  $\theta$ . If the correlation between the two variables was zero, and they each had mean, zero, then the angles will have a uniform distribution around the circle. If the mean was zero but the correlation was quite large, then an ‘antipodal’ angular variable with two peaks in its distribution, separated from each other by  $\pi$  radians (180 degrees) would be generated. By allowing the means of the variables to vary the distribution of angles can be made unimodal, or bimodal but asymmetric and non-antipodal. The model has considerable flexibility despite its simple conception. In mathematical terms, if  $\mathbf{y}$  is a realization of a bivariate random variable,  $\mathbf{y}$ , on the plane  $\mathbb{R}^2$ , and  $\Pr\{\mathbf{y} = \mathbf{0}\} = 0$ , then its radial projection  $\|\mathbf{y}\|^{-1}\mathbf{y}$  is a random vector on the unit circle which can be converted to a vector of random angles relative to some direction treated as 0. In the PN distribution  $\mathbf{y}$  is a realization of a bivariate normal variable  $\mathcal{N}_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$  on the plane...’

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