

Modelling of the wave fields by the modification of the matrix method

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Modelling of the wave fields by the modification of the matrix method in anisotropic media

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for anisotropic layered medium the matrix propagator can be represented by a “wave propagator” in each layer. The displacement field on the free surface of an anisotropic medium is obtained from the received system of equations considering the radiation condition and that the free surface is stressless.

5 In recent years one of the most important methods is the development of approaches for constructing the theoretical seismograms, which allow the study of the structure of the medium and determination of the earthquake source parameters. The effects on the wave field and seismic waves’ propagation in the earth’s interior should be considered when calculating these seismograms. Thus, the displacement field, which
 10 is registered on the free surface of an inhomogeneous medium, depends on the model of the geological structure and the physical processes in the source.

In the 1950s, Thomson and Haskell first proposed a method for constructing interference fields by simulation of elastic waves in layered isotropic half-space with planar boundaries (Haskell, 1953). The matrix method was developed in previous works
 15 (Behrens, 1967; Buchen, 1996; Cerveny, 2001; Chapman, 1974).

The stable algorithms of seismogram calculation for all angles of seismic wave propagation are obtained. The matrix method is generalized for low-frequency waves in inhomogeneous elastic concentric cylindrical and spherical layers surrounded by an elastic medium. The concept of the characteristic matrix determined by physical parameters of the environment is developed. The matrix method is used for wave propagation in elastic, liquid and thermoelastic media. In addition, it has been generalized for the study of other processes described by linear equations. The advantage of the matrix method is the ability to write matrix expressions compactly that are useful both
 20 in analytical studies and numerical calculations.

25 The matrix method and its modifications are used to simulate the seismic wave propagation in isotropic and anisotropic media. This method is quite comfortable and has several advantages over other approaches. Both advantages and disadvantages of the matrix method are well described in Helbig et al. (2001), Stephen (1981), and Thomson (1950).



Today in seismology much attention is given to mathematical modelling as one of the main tools for the analysis and interpretation of the wave fields.

2 Theory

The problem of wave fields modelling, when the source is presented by seismic moment, has practical applications in seismology. Therefore, the development of methods for determining the displacement field on the free surface of an anisotropic inhomogeneous medium for sources of this type is an actual task and needs to be resolved.

In this paper the propagation of seismic waves in anisotropic inhomogeneous medium is modelled by a system of homogeneous anisotropic layers, as shown in Fig. 1. Each layer is characterized by the propagation velocity of P and S wave and density. At the boundaries between layers, hard contact condition is met, except for the border, where the source of seismic waves is located.

The earthquake source is modelled by nine pairs of forces, which represented a seismic moment tensor. This description of the point source is sufficiently known and effective for simulation of seismic waves in layered half-space (Haskell, 1953). In general, the source is also assumed to be distributed over time (i.e. seismic moment $M_0(t)$ is a function of time). This means that the physical process in the source does not occur instantaneously, but within a certain time frame. It is known for our seismic events ($M_w \sim 2-3$) that the time during which the event occurred may be 0.1–0.7 s. The determination of the source time function is an important seismic problem. In this chapter the direct problem solution is shown, when a point source is located on an arbitrary boundary of layered anisotropic media.

We assume the usual linear relationship between stress τ_{ij} and strain e_{kl} :

$$\tau_{ij} = c_{ijkl} \cdot e_{kl} = c_{ijkl} \frac{\partial u_l}{\partial x_k}, \quad (1)$$

where $\mathbf{u} = (u_x, u_y, u_z)^T$ is displacement vector.

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where $\tilde{F} = G_{s,1}^{-1} \cdot F$, $G = G^{n+1,s+1} \cdot G_{s,1}$. Using Eq. (7) and the radiation condition (with a halfspace $(n+1)$ the waves are not returned), and also the fact that the tension on the free surface equals to zero, we obtain a system of equations:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ v_D^P \\ v_D^{S_1} \\ v_D^{S_2} \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} & G_{13} & G_{14} & G_{15} & G_{16} \\ G_{21} & G_{22} & G_{23} & G_{24} & G_{25} & G_{26} \\ G_{31} & G_{32} & G_{33} & G_{34} & G_{35} & G_{36} \\ G_{41} & G_{42} & G_{43} & G_{44} & G_{45} & G_{46} \\ G_{51} & G_{52} & G_{53} & G_{54} & G_{55} & G_{56} \\ G_{61} & G_{62} & G_{63} & G_{64} & G_{65} & G_{66} \end{pmatrix} \begin{pmatrix} u_x^{(0)} + \tilde{F}_1 \\ u_y^{(0)} + \tilde{F}_2 \\ u_z^{(0)} + \tilde{F}_3 \\ \tilde{F}_4 \\ \tilde{F}_5 \\ \tilde{F}_6 \end{pmatrix}.$$

Using only the homogeneous equations is sufficient to get the displacement field on a free surface:

$$\begin{cases} G_{11}u_x^{(0)} + G_{12}u_y^{(0)} + G_{13}u_z^{(0)} = -(G_{11}\tilde{F}_1 + G_{12}\tilde{F}_2 + G_{13}\tilde{F}_3 + G_{14}\tilde{F}_4 + G_{15}\tilde{F}_5 + G_{16}\tilde{F}_6) \\ G_{21}u_x^{(0)} + G_{22}u_y^{(0)} + G_{23}u_z^{(0)} = -(G_{21}\tilde{F}_1 + G_{22}\tilde{F}_2 + G_{23}\tilde{F}_3 + G_{24}\tilde{F}_4 + G_{25}\tilde{F}_5 + G_{26}\tilde{F}_6) \\ G_{31}u_x^{(0)} + G_{32}u_y^{(0)} + G_{33}u_z^{(0)} = -(G_{31}\tilde{F}_1 + G_{32}\tilde{F}_2 + G_{33}\tilde{F}_3 + G_{34}\tilde{F}_4 + G_{35}\tilde{F}_5 + G_{36}\tilde{F}_6) \end{cases}.$$

The stress-displacement discontinuity is determined through the components of the seismic tensor in matrix form (Fryer et al., 1984):

$$F = \begin{pmatrix} -c_{55}^{-1}M_{xz} \\ -c_{44}^{-1}M_{yz} \\ -c_{33}^{-1}M_{zz} \\ \rho_x (M_{xx} - c_{13}c_{33}^{-1}M_{zz}) + \rho_y M_{xy} \\ \rho_x M_{yx} + \rho_y (M_{yy} - c_{23}c_{33}^{-1}M_{zz}) \\ \rho_x (M_{zx} - M_{xz}) + \rho_y (M_{zy} - M_{yz}) \end{pmatrix} \delta(z - z_z),$$

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components, by conducting computational experiments with reasonable accuracy, controlled by means of the theoretical relations for variations of physical parameters of studied media and wave forms on the surface of a layered half-space, and is also confirmed by the coincidence with analytical solutions and with results obtained by other methods.

The approbation of the proposed method is done via comparative analysis of waveforms, obtained by different methods. Comparative analysis of waveforms confirms the possibility of using the matrix method for problems of seismology in the case of distributed in time earthquake sources for isotropic and anisotropic media. This paper is the first step in determining the earthquake source parameters. The algorithm to determine the source parameters is based on the expressions for displacement field on free surface of an anisotropic medium in Eq. (8) and spectra of real records from stations that recorded these events. The results of determining of the earthquake source parameters will be published in the next papers.

Thus, the methods, approaches, algorithms, software for the propagation of seismic waves and results of direct and inverse dynamic problems of seismology proposed and developed by the author and highlighted in the paper, can be successfully used in the study of the seismic regions and effective implementation in the construction of the earthquake source mechanism which is crucial for seismic regions of the country.

Probability and reliability of basic scientific terms and results is provided by well posed problems, rigidity of mathematical methods and transformations in obtaining basic analytical relations for the displacement field and the seismic moment tensor components, by conducting computational experiments with reasonable accuracy, controlled by means of the theoretical relations for variations of physical parameters of studied media and wave forms on the surface of a layered half-space, and is also confirmed by the coincidence with analytical solutions and with results obtained by other methods.

Thomson, W. T.: Transmission of elastic waves through a stratified solid medium, J. Appl. Phys., 21, 89–93, 1950.

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Table 1. The parameters of medium.

No.	layer thickness, m	c_{11} , GPa	c_{12} , GPa	c_{44} , GPa	density, kg m^{-3}
1	2000	52 992	21 896	15 548	2300
2	2800	75 625	27 575	24 025	2500
3	13 200	103 788	33 804	34 992	2700
4	6000	129 472	48 608	40 432	2800
5	21 000	204 800	68 196	68 302	3200

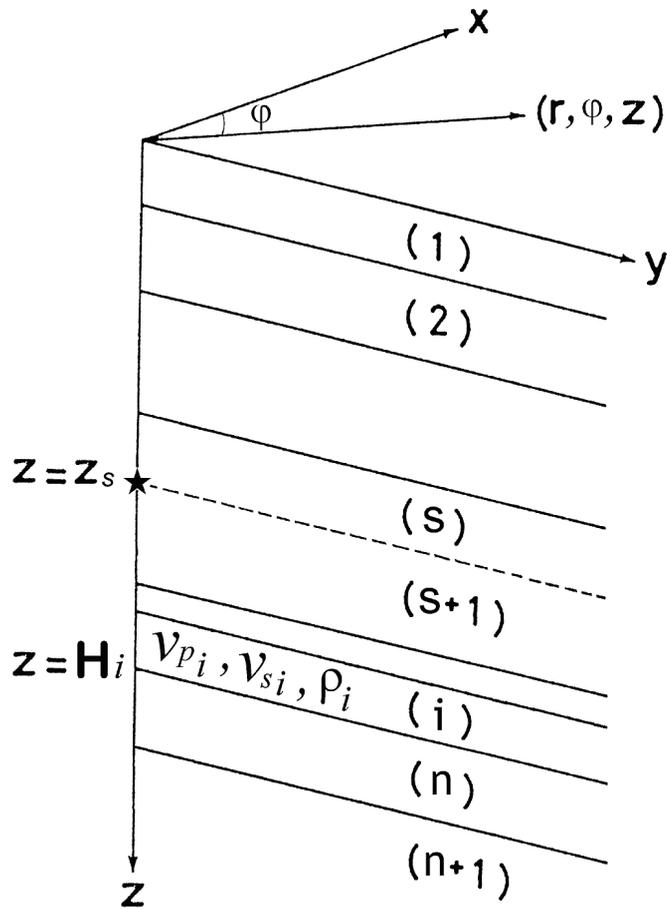


Fig. 1. Model vertically inhomogeneous medium.

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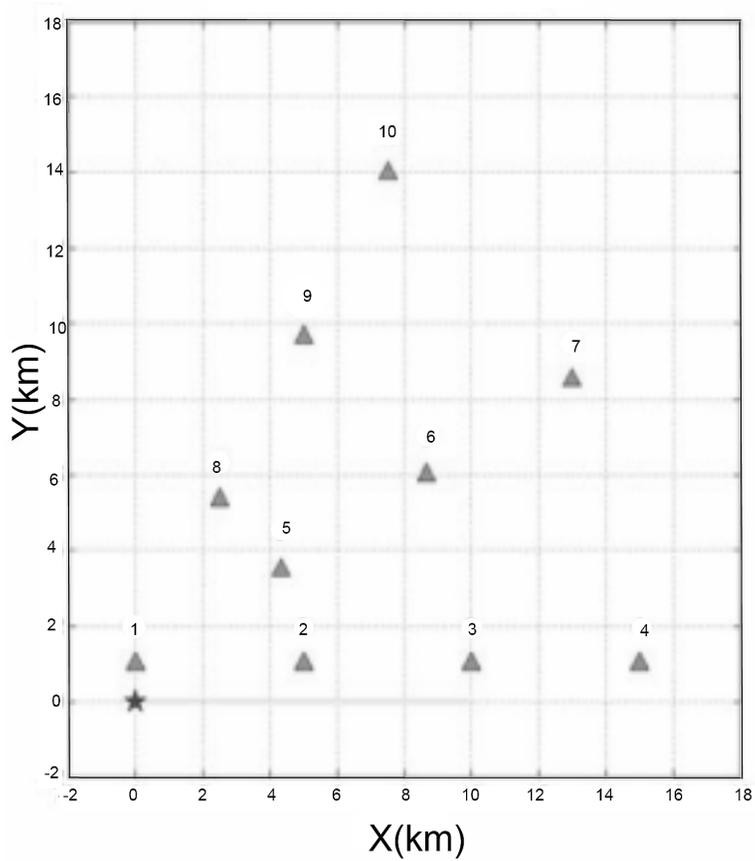


Fig. 2. Source–receiver geometry for the strike-slip point source. The star shows the epicentre in the chosen right-handed coordinate system with positive X pointing east, positive Y pointing north, and positive Z pointing up.

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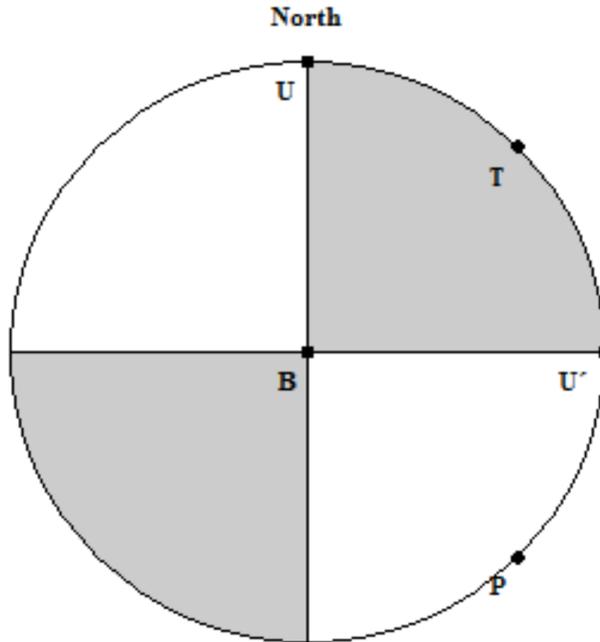
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P axis (ϕ / δ) in degrees: 135.00 / 0.00
T axis (ϕ / δ) in degrees: 45.00 / 0.00



stereographic lower
 fault plane ($\phi / \delta / \lambda$) in degrees: 0.00 / 90.00 / 0.00
 auxiliary plane ($\phi / \delta / \lambda$) in degrees: 270.00 / 90.00 / 180.00

Fig. 3. The source focal mechanism (strike-slip).

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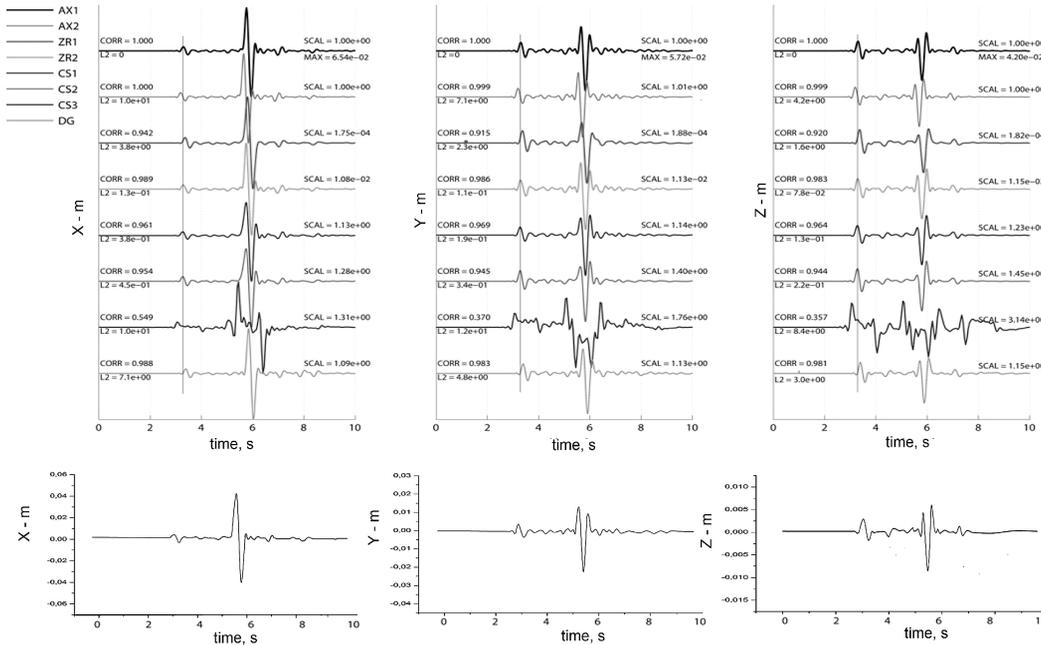


Fig. 4. Components of the displacement field on the free surface of the medium (Table 1), calculated by different methods according to the project SIV and by proposed modification of the matrix method for the receiver 10 ($X = 13990$ m, $Y = 7500$ m).

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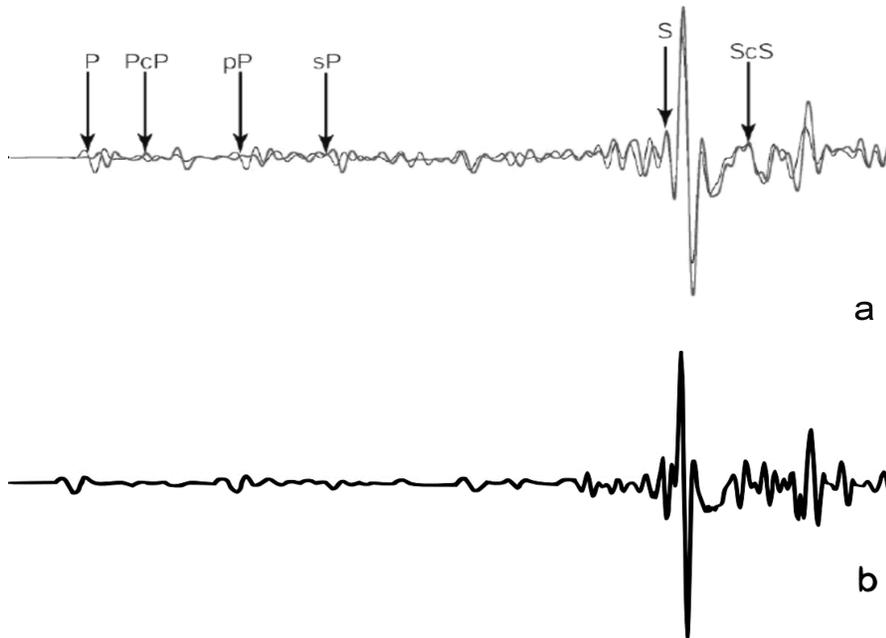


Fig. 5. (a) Synthetic seismograms for PREM and an anisotropic model (isotropic model with 5% share of weakly anisotropic perturbations). (b) Synthetic seismogram calculated by the proposed modification of the matrix method.