Response to Referee 2's comments concerning manuscript se-2014-82

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We have carefully revised our manuscript 'Wave-equation based traveltime seismic tomography - Part 1: Method' [MS No. se-2014-82], taking into account ALL of the comments and suggestions by anonymous Referee 2. We are grateful for the comments and suggestions which have been very helpful in improving the paper. Below, we have provided detailed point-by-point replies to the comments, highlighted in blue colour. Meanwhile, our revisions are also highlighted in blue in the updated manuscript.

We look forward to hearing from you (Referee 2) on the revised manuscript. Thank you very much.

Responses to Referee 2

Review report

The paper by Tong et al. presents a computationally economic numerical method to accurately calculate finite-frequency sensitivity kernels of seismic traveltimes for tomographic inversion. The method basically consists of two parts: the 2-D forward modelling and 3-D traveltime inversion. A finite-difference solver is first introduced to simulate 2-D acoustic wave propagation and compute the forward and adjoint wavefields used in the construction of Frechet traveltime kernels. The adjoint approach for finite-frequency full waveform tomography is not novel and has been developed for years led by Tromps group based on the 3D SPECFEM. The paper makes a thorough re-derivation of the formulation of the Frechet traveltime kernel starting from 2-D acoustic wave equation. In addition, two traveltime picking techniques are presented to automatically determine the arrival times of specific phases in computed synthetic seismograms. The second part of the paper describes the choice of model parameterization and regularization invoked in tomographic inversion and reviews two common strategies, LSQR and conjugate gradient method, to solve for the parameterized velocity model. Overall, the paper is well written, providing clear and comprehensive descriptions of the theoretical foundation how to obtain the numerical solutions of wave-based 2-D traveltime kernels and use them to invert for the 3-D velocity model. I think the paper deserves being published, but there are a few questions and comments on the paper needed to be addressed and elaborated more clearly in the revision before being accepted.

Reply: Thank you very much for your kind comments and suggestions. Yes, as the reviewer said that, the main purpose of this study is to present a computationally efficient wave-equation based seismic tomography method. We have carefully considered the comments and suggestions and revised the manuscript accordingly.

Comment 1: *In the derivation of the traveltime kernel in eq. (11), a test function $q(t, x)$ is introduced to multiply the wave equation for the perturbed wavefield in eq. (6). It is shown that this function is not arbitrary and has to satisfy the conditions listed in eq. (9) and seems to be similar to the adjoint source used to generate the traveltime kernel in Tromp et al. (GJI, 2005). It would be more insightful to understand the resulting kernel in eq. (11) if the paper could add more specific descriptions of the test function following eq. (9).*

Reply: Thanks for this good suggestion. The $q(t, x)$ in the kernel expression Eq. (11) is no longer an arbitrary test function. Instead it has to satisfy Eq. (9), i.e., the adjoint seismic wave equation. We have added some descriptions of the test function $q(t, x)$ following Eq. (9) in the revised manuscript. $q(t, x)$ is actually a wavefield generated reversely in time by backpropagating the windowed and normalized velocity signal recorded at the receiver in the velocity model $c(x)$ from the receiver to the source, also known as the the adjoint wavefield.

***Comment 2**: The sensitivity kernels for seismic observables depend largely on how the observed data are measured. The derivation of the Frechet traveltime kernels is essentially founded on the cross correlation measurements as defined in eq. (2) which leads to the final formulation of the kernel in eq. (11). In section 3, the paper, however, places emphasis on the determination of onset times of phase arrivals using manual or automatic picking methods, such as STA/LTA and proposed envelop energy ratio methods. These onset picking methods tend to determine the arrival times at the highest possible frequencies so that the infinite-frequency approximation assumed in traditional ray-based tomography is valid. It is an apparently contradictory concept from the finite-frequency theory based on the cross-correlation traveltime measurement. Moreover, unlike the cross correlation method, all these means are difficult to obtain the mathematical expressions of the corresponding Frechet traveltime kernels for tomographic applications. On the other hand, the manually or automatically picked traveltime residuals obtained by onset picking would differ from those by cross correlation, because the former results in the higher-frequency phase arrivals that experience less severe wavefront healing effects (referred to the study by Hung et al., 2001, GJI). Therefore, I don't think the onset picking method highlighted in the paper is an appropriate approach to measuring traveltime residuals for the proposed finite-frequency tomographic method, unless the authors can demonstrate the difference is small for their case.*

Reply: Thanks for the critical comments. Cross-correlation is a measure of similarity of two waveforms as a function of a time-lag applied to one of them. Ideally, if two waveforms $s(t)$ and $d(t)$ differ only by an unknown shift along the time axis, one can use the cross-correlation technique to exactly find how much $s(t)$ must be shifted along the time axis to make it identical to $d(t)$. In this and other finite-frequency traveltime seismic tomography studies (e.g. Dahlen et al., 2000; Zhao et al., 2000; Tromp et al., 2005; Fichtner et al., 2006), the derivations of the Frechet traveltime kernels are mainly based on the Born approximation, which requires that the reference velocity model $c(\mathbf{x})$ for $s(t)$ is very close to the real model $c(\mathbf{x}) + \delta c(\mathbf{x})$ for $d(t)$, i.e., $|\delta c(\mathbf{x})| << c(\mathbf{x})$. Since $|\delta c(\mathbf{x})| << c(\mathbf{x})$, it is straightforward to get that $|s(t) - d(t)| << |s(t)|$ if both $s(t)$ and $d(t)$ satisfy the same wave equation. Providing that $|s(t) - d(t)| << |s(t)|$, we can say that the data and synthetic broad pulses $d(t)$ and $s(t)$ have very similar waveforms, then the time differences of onset, peak and end times of the two pulses should be almost the same. The time difference of onsets on synthetic and data pulses should be identical or very close to the one calculated with the cross-correlation traveltime measurement, since cross-correlation traveltime difference is also a kind of traveltime difference. But in this study, we take a detour to get the onset time of synthetic seismogram $s(t)$. It is computationally expensive to simulate the propagation of seismic waves in 3D elastic or even anelastic models. To reduce the computation cost, we alternatively use 2D simulation to get synthetic seismogram $u(t)$, which may only match the 3D synthetic seismogram $s(t)$ against the onset time. Since we only need to know the exact onset time of the synthetic pulse, this 2D approximation is accurate enough.

Regarding to the finite-frequency effects, it is true that seismic waves of different frequencies may have different sensitivities to various regions and arrive at different times. Since the sensitivity kernel is a finite-frequency one, we need to use finite-frequency traveltime residuals in our tomography study. Actually, the traveltime of the first arrival on synthetic seismogram, which is generated by a source time function with dominant frequency f_0 (please see Eq. 4), is a finite-frequency traveltime. To be consistent, the raw data seismograms should be first filtered with a bandpass filter that is consistent with the frequency spectrum of the synthetic seismograms. After that, we manually pick the onset times of the filtered seismic phase of our interest. The crital matter left is how we can accurately pick the onset times.

In the application manuscript se-2014-83, we use *P*- and *S*-wave arrival times of local earthquakes recorded by the Southern California Seismic Network and compiled by the Southern California Earthquake Data Center (SCEDC), since we found that the differences between the onset times archived by SCEDC and the corresponding ones picked on filtered seismograms are very small and can be viewed as negligible noise for such a regional seismic tomography study. The reason for these small differences is probably that the frequency spectrum of the computed synthetics actually covers the frequency band of the dominant energy in the unfiltered observed data.

In the revised manuscript, we have added a detailed discussion on the validity of traveltime residuals obtained with the energy envelop method or the combined ray and cross-correlation method in the last paragraph of Section 2 and the second paragraph of Section 7.

Comment 3: *In the first part of forward modelling, the paper gives nice illustrations in Figs. 5 and 6 to show how the 2-D traveltime kernels of different phase arrivals look likes and provide complimentary information to constrain the implemented synthetic structures. In the second part, the paper describes somewhat detailed, step-by-step procedures in section 5 how to invert for the 3D velocity model using LSQR or conjugate gradient method. To make this part more comprehensible and the entire paper more complete, I suggest using the same synthetic model, phase arrival-time data and corresponding kernels shown in Figs. 5 and 6 as the illustrative example and adding one or two figures in this section which show the 3-D tomographic results based on these two inversion methods.*

Reply: Thank you for the kind comments and the suggestion for adding the results of a synthetic inversion test. This manuscript se-2014-82 (Paper I) is the first paper of our two-paper submission and focuses on presenting the theoretical and computational fundation, as well as step-by-step procedures of the wave-equation based traveltime seismic tomography (WETST). The examples of 2-D sensitivity kernels for different seismic phases shown in Figs. 5 and 6 demonstrate that complimentary information provided by different seismic phases can be used by the proposed 2-D–3-D seismic tomography method in exploring the subsurface structures, as also mentioned by the reviewer. The kernels in Figs 5. and 6 are computed for velocity anomalies in the 2-D vertical plane as our forward solver is based on the 2D acoustic wave equation. But in the second manuscript se-2014-83 (Paper II), we have applied WETST to explore the 3-D velocity and Poison's ratio structures of the 1992 Landers earthquake area. We also conducted synthetic inversion examples such as checkerboard resolution tests in Paper II and therefore do not include them in Paper I to reduce repetition. Admittedly, in Paper II, mainly direct arrivals and head waves refracted from different discontinuities in the crust are used and no Moho-reflected phases are included in the studies. To further constrain the subsurface structures with additional information from reflected seismic phases, we are working with Xueyuan Huang, a PhD student from Tsinghua University, and manually picked several thousands of PmP and SmS arrival times recorded by the southern California seismic network. In that case, we use direct arrivals, head waves and Moho reflected phases to image the crustal velocity structure of southern California. This is still work in progress, and once it is finished, we may have a more comprehensive understanding of the performances of WETST in imaging subsurface structure with different seismic reflected/refracted phases in real applications.