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Multi-quadric collocation model of horizontal crustal movement

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Abstract

To establish the horizontal crustal movement velocity field of the Chinese mainland, a Hardy multi-quadric fitting model and collocation are usually used, but the kernel function, nodes, and smoothing factor are difficult to determine in the Hardy function interpolation, and in the collocation model the covariance function of the stochastic signal must be carefully constructed. In this paper, a new combined estimation method for establishing the velocity field, based on collocation and multi-quadric equation interpolation, is presented. The crustal movement estimation simultaneously takes into consideration an Euler vector as the crustal movement trend and the local distortions as the stochastic signals, and a kernel function of the multi-quadric fitting model substitutes for the covariance function of collocation. The velocities of a set of 1070 reference stations were obtained from the Crustal Movement Observation Network of China (CMONOC), and the corresponding velocity field established using the new combined estimation method. A total of 85 reference stations were used as check points, and the precision in the north and east directions was 1.25 and 0.80 $\mathrm{mm\,yr^{-1}}$, respectively. The result obtained by the new method corresponds with the collocation method and multi-quadric interpolation without requiring the covariance equation for the signals.

1 Introduction

Horizontal movement velocity fields provide the main basic data for Earth science research. Because measuring a station's velocity by repeated observation is always limited, with many points that cannot be directly measured, a mathematical velocity field model is always used to obtain the velocity field. How to obtain reliable horizontal velocity fields from measured points has been the focus of much research at home and abroad, and it has produced a lot of research literature (Argus and Gordon, 1991; Huang et al., 1993; Liu et al., 2001, 2002; Chai et al., 2009; Jiang and Liu, 2010; Hu and Wang, 2012; Zeng et al., 2012, 2013). As we know, the horizontal velocity of a ground point mainly consists in two aspects: the overall movement and the local deformation reflected ground motion. The motion model for horizontal velocity is often used in geophysical models and the statistical fitting method. The geophysical model is often implemented using the Euler vector method (Argus and Gordon, 1991). Generally, the condition for using the Euler vector method is to divide the block reliably and treat each division as a rigid body, but many areas do not satisfy the requirements of a rigid body, which limits the usefulness of the method. The statistical fitting method mainly includes multi-quadric functions (Huang et al., 1993; Liu et al., 2001; Zeng et al., 2013) and collocation (Liu et al., 2002; Chai et al., 2009; Jiang and Liu, 2010; Zeng et al., 2012). For simply fitting the parameters of measured points and estimating the parameters of unmeasured points, you can use multi-quadric functions (Hardy, 1978), which are pure mathematical methods and their physical meaning is not clear; the key issues and difficult problems in their application is the choice of kernel function, smoothing factor, and node. Some scholars have systematically researched their application in horizontal velocity field models in the Chinese mainland (Huang et al., 1993; Nie et al., 2007; Zeng et al., 2013). In order to consider the changing information of velocity fields in different local regions, the least squares collocation method, whose key problem is determining the covariance function of signal vectors, can be adopted (Yang, 1992). To balance the contribution of the estimation results from the signal covariance matrix and observation noise, we can use the variance components to estimate the collocation solutions (Yang and Liu, 2002; Yang et al., 2008) or adaptive collocation solutions (Yang et al., 2009; Yang and Zeng, 2009; Yang et al., 2011), but both of them require iterative calculations. Because the collocation covariance function is very difficult to build, some scholars have analyzed and compared the relationship between the multi-quadric function and covariance collocation (Tao et al., 2002); some scholars, using the compensation principle of least squares, have pointed out that choosing the appropriate regularization parameters after the semi-parametric model may include the collocation model (Tao and Yao, 2003); and some scholars have established a parameter estimation model combining the multi-quadric function and configuration models

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and used the distance function as the signal covariance when estimating the gravity field to obtain results close to the multi-quadric function model (Wang and Ou, 2004).

In view of the above-mentioned facts, in this paper, we take Euler vectors as the long-term overall movement trend of the function model and regard the local variations of horizontal movements as stochastic parameters. At the same time, the stochastic characters are taken into account. We utilize a multi-quadric kernel function to obtain its normalized matrix and combine the characters of the collocation and multi-quadric functions, which not only avoids having to establish the covariance function but also achieves an integrated effect using both collocation and multi-quadric functions (Tao and Yao, 2003). On the basis of the method proposed above, we established a Chinese mainland horizontal movement velocity field by using the velocities of a set of 1041 reference stations, obtained from the Crustal Movement Observation Network of China.

2 Collocation model of horizontal movement

It is well known that the horizontal velocity of a ground point can be described in two parts. One is the holistic long-term trend of horizontal movement expressed by the Euler vector, which is related to the block where the sites are. The other part is the local movement change, modelled as stochastic signals, which is mainly affected by local surface displacement (Freymueller et al., 2013). Therefore, the function model of point horizontal velocity is:

$$V = \mathbf{A}\hat{\boldsymbol{\omega}} + \mathbf{B}\hat{S} - L, \tag{1}$$

where $\mathbf{A}$ represents the designed matrix of size $n \times 3$, $\mathbf{B}$ denotes the coefficient matrix of size $n \times u$, where n and u are the station quantity and estimated parameter numbers; $\hat{\boldsymbol{\omega}}$ represents the estimation of the Euler vector because of plate movement, and $\boldsymbol{L}$ is the observation vector of horizontal velocity on a station; the covariance matrix is $\boldsymbol{\Sigma}_\mathrm{e} = \sigma_0^2 \mathbf{P}_\mathrm{e}^{-1}$ where $\mathbf{P}_\mathrm{e}$ denotes its weight matrix. $\hat{\boldsymbol{S}}$ is the signal estimation of observed points, and its covariance matrix is $\boldsymbol{\Sigma}_S = \sigma_0^2 \mathbf{P}_S^{-1}$ where $\mathbf{P}_S$ denotes its weight matrix.

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The relation between the long-term movement trend of horizontal velocity on a ground station and the Euler vector of plate movement can be expressed as follows (Jin et al., 2006; Yang and Zeng, 2009):

$$\begin{bmatrix} v_n \\ v_e \end{bmatrix} = \begin{bmatrix} R\sin\lambda & -R\cos\lambda & 0 \\ -R\sin\phi\cos\lambda & -R\sin\phi\sin\lambda & R\cos\phi \end{bmatrix} \times \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \tag{2}$$

where v_e denotes the easterly component of the ground station's horizontal velocity in a topocentric coordinate system, and v_n represents the northerly component. R represents the radius of earth, and λ, ϕ are the latitude and longitude of the ground point. Thus, the matrix **A** in Eq. (1) is:

$$A = \begin{bmatrix} R\sin\lambda & -R\cos\lambda & 0 \\ -R\sin\phi\cos\lambda & -R\sin\phi\sin\lambda & R\cos\phi \end{bmatrix}. \tag{3}$$

Generally, the measured point signal is the local motion variation without the plate rigid motion, which is the horizontal residual velocity in measured points. So **B** = **I** (unit matrix) in Eq. (1).

If we take the local movement variation signal $\hat{S}'$ of a non-observed point into account and assume that there exists a covariance matrix $\mathbf{\Sigma}_{SS'} = \mathbf{\Sigma}_{SS'}^T \neq 0$ between the non-observed point's signal $\hat{S}'$ and the observed point's signal $\hat{S}$ then, if the covariance matrix is known, the collocation solution considering the non-observed point's signal simultaneously (Yang and Zeng, 2009; Zeng et al., 2012) is:

$$\begin{cases} \hat{\boldsymbol{\omega}} = \left(\mathbf{A}^T\mathbf{P}_L\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{P}_L L \\ \hat{S} = \mathbf{\Sigma}_S\mathbf{B}^T\mathbf{P}_L\left(L - \mathbf{A}\hat{\boldsymbol{\omega}}\right) \\ \hat{S}' = \mathbf{\Sigma}_{S'S}\mathbf{\Sigma}_S^{-1}\hat{S} \end{cases}, \tag{4}$$

where $\mathbf{P}_L = \left(\mathbf{B}\mathbf{\Sigma}_S\mathbf{B}^T + \mathbf{\Sigma}_e\right)^{-1}$.

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The Euler vector estimation $\hat{\boldsymbol{\omega}}$ of plate movement and velocity estimation $\hat{S}$ of local movement in the collocation model above depend not only on the covariance matrix $\boldsymbol{\Sigma}_e$ of observed horizontal velocity L but also on the covariance matrix $\boldsymbol{\Sigma}_S$ of the local velocity signal $\hat{S}$. Before data processing, the covariance matrices $\boldsymbol{\Sigma}_S$ and $\boldsymbol{\Sigma}_{SS'}$ must be known, which is the key point of the problem. This is the most difficult aspect of applying the collocation method. In order to overcome the difficulty, the elements in the covariance matrix are calculated according to the covariance function. There are a wide variety of stochastic signal covariance functions, such as the Gauss exponent function, the Hirvonen function, etc. The principles for determining the covariance function are also diverse. Different covariance functions can be based on different data and principles (Zeng et al., 2012). Moreover, different researchers can obtain different covariance functions even where the data and principles are the same. That is to say, different physical applications spend a lot of energy in determining the covariance function, which greatly limits the collocation model applications.

3 Multi-quadric function of horizontal movement

In practice, the most difficult aspect of adopting the collocation method is that the signal's a priori variance cannot be determined accurately. The signal covariance functions are generally built by adopting observed data based on certain principles (Zeng et al., 2012). Thus, the priori covariance matrix determined in this way has a strong correlation with the current measured data. It not only affects the optimality of collocation, but also reduces its range of application. Allowing for that, the requirements of precision and density of observed data are relatively high when estimating covariance functions. At the same time, the user should be well informed about the physical field that it is used in. Thus, it is difficult to obtain a general application and influence the optimality of the collocation model (Tao and Yao, 2003). We noticed that the signal covariance function of collocation is a kind of function about distance in general (Zeng et al., 2012). Therefore, in this paper, we do not look upon the local deformation parameters as ran-

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dom signals, like the collocation, but as non-random variables, taking into account their random nature (Tao and Yao, 2003), reflected by the distance function, to construct the covariance matrix of the local signals. The covariance matrix of the local deformation is

$$\mathbf{G}_S = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1m} \\ g_{21} & g_{22} & \cdots & g_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ g_{m1} & g_{m2} & \cdots & g_{mm} \end{bmatrix}, \tag{5}$$

where g is the value obtained by the distance function.

We still use the Eq. (1) as the error equation. Because of this, the distance function does not explicitly consider the physical nature of the local deformation. Using the compensation least squares method estimation criterion:

$$V^T \boldsymbol{\Sigma}_e^{-1} V + \hat{S}^T G_S^{-1} \hat{S} = \min. \tag{6}$$

Then, according to Tao and Yao (2003) and Wang and Ou (2004),

$$\begin{cases} \hat{\boldsymbol{\omega}} = \left(\mathbf{A}^T \left(\mathbf{B}\mathbf{G}_S\mathbf{B}^T + \boldsymbol{\Sigma}_e\right)^{-1} \mathbf{A}\right)^{-1} \mathbf{A}^T \left(\mathbf{B}\mathbf{G}_S\mathbf{B}^T + \boldsymbol{\Sigma}_e\right)^{-1} \dot{X} \\ \hat{S} = \mathbf{G}_S\mathbf{B}^T \left(\mathbf{B}\mathbf{G}_S\mathbf{B}^T + \boldsymbol{\Sigma}_e\right)^{-1} (\dot{X} - \mathbf{A}\boldsymbol{\omega}). \end{cases}$$

Considering the collocation problem of the non-observed point's signal parameter $\boldsymbol{S}'$, we use the distance function to contact the measured point's signal parameter $\boldsymbol{S}$ and the unmeasured point's signal parameter $\boldsymbol{S}'$. So the $\boldsymbol{\Sigma}_{SS'}$ in Eq. (4) is transformed to

$$\mathbf{G}_{S'S} = \begin{bmatrix} g_{S'_1 S_1} & \cdots & g_{S'_1 S_m} \\ \cdots & \cdots & \cdots \\ g_{S'_p S_1} & \cdots & g_{S'_p S_m} \end{bmatrix}. \tag{7}$$

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Then the estimation of the unmeasured point's signal parameter $\boldsymbol{S}'$ is

$$\hat{S}' = \mathbf{G}_{S'S}\mathbf{B}^T\left(\mathbf{B}M_S\mathbf{B}^T + D_\Delta\right)^{-1}\left(\dot{X} - A\varpi\right). \tag{8}$$

Thus, the key to the above problem is still to determine the covariance matrix of the local deformation parameters; in this paper we will transform the local deformation parameter covariance matrix to the covariance function to determine it.

Multi-quadric functions are first proposed in 1978 by Hardy (Hardy, 1978). The basic idea is that any smooth surface can be approached at any precision by using a series of finite and regular mathematical functions, and the non-observed points can be estimated by making use of the observed points. Because it can be designed flexibly and the controllability is strong, the method has been widely used in the interpolation problems involved in geoscience since the approach was proposed. Tao and Yao (2003) has discussed the relationship between multi-quadric and collocation in detail and thought that the multi-quadric function is a kind of special covariance function. Based on this idea, we introduce a multi-quadric kernel function to determine the covariance matrix of local deformation. Thus, the covariance matrix Eqs. (5) and (7) in Eq. (4) are completely determined by the multi-quadric kernel function. Currently, the most commonly used multi-quadric kernel functions in addition to the conical surface are: positive double curved surface, inverted double curved surface, positive three surface, and inverted three surface.

1. Positive double curved surface:

$$g\left(x, y, x_{0i}, y_{0i}\right) = \left[(x - x_{0i})^2 + (y - y_{0i})^2 + \delta^2\right]^{\frac{1}{2}} \tag{9}$$

   The symbol δ represents the smoothing factor. Obviously, if $\delta = 0$, the positive double curved surface is a conical surface.

2. Inverted double curved surface:

$$g\left(x, y, x_{0i}, y_{0i}\right) = \left[(x - x_{0i})^2 + (y - y_{0i})^2 + \delta^2\right]^{-\frac{1}{2}} \tag{10}$$

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Again, δ represents the smoothing factor, and if $\delta = 0$, the inverted double curved surface is a conical surface.

3. Positive three surface:

$$g(x, y, x_{0i}, y_{0i}) = \left[(x - x_{0i})^2 + (y - y_{0i})^2 + \delta^2\right]^{\frac{3}{2}} \tag{11}$$

4. Inverted three surface:

$$g(x, y, x_{0i}, y_{0i}) = \left[(x - x_{0i})^2 + (y - y_{0i})^2 + \delta^2\right]^{-\frac{3}{2}} \tag{12}$$

As can be seen, the actual covariance matrix $\mathbf{\Sigma}_S$ of the signal is replaced by the matrix G_S determined in the multi-quadric function, which is exactly the same as the standard configuration model in the form; namely, Eqs. (4) and (6) are exactly identical. Although the two methods are similar in form, their theoretical basis is different. According to the rule of maximum posterior estimation, the standard configuration model obtains the optimal estimate under the condition of the signal a priori variance being known. However, the latter considers the randomness of the parameters, gives a certain compensation and balance, and still considers it as a parameter estimation of non-random parameters, and the optimization is better than the previous method (Wang and Ou, 2004). In this way, we not only overcome the problem that the covariance function of the classical fit model is difficult to determine but also keep the formula invariance of the fitting model. Because of its simplicity and rationality, the application scope of the model can be greatly expanded (Tao et al., 2002; Tao and Yao, 2003).

4 Calculation and analysis

We used the same observation data as in (Yang and Zeng, 2009), that is, 1041 repeat observation stations of the Crustal Movement Observation Network Engineering,

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whose precision of horizontal velocity is superior to fitting point coordinates velocity by 3 mm yr^{-1}. This data includes 85 high precision points (29 consecutive observation stations and 56 irregular observation basis points) as the external inspection points to assess the accuracy of the constructed velocity field model, and a net of the remaining 985 regular observation points after subtracted 56 irregular observation basis points from the 1041 repeat observation stations to establish the velocity field model.

In order to evaluate the internal precision of the model, the calculation formula for the root-mean-square error (RMS) of the horizontal residual velocity of the 985 observation stations is calculated:

$$\mathrm{RMS} = \sqrt{\frac{V^T \cdot V}{985}}, \tag{13}$$

where V is residuum calculated by different solutions, that is, station horizontal residual velocity.

The RMS of the external checkpoint horizontal velocity can be defined by:

$$\mathrm{RMS} = \sqrt{\frac{(\dot{X}_\mathrm{m} - \dot{X}_\mathrm{c})^T \cdot (\dot{X}_\mathrm{m} - \dot{X}_\mathrm{c})}{85}}, \tag{14}$$

where $\dot{X}_\mathrm{m}$ is the measured horizontal velocity of the 85 checkpoints, and $\dot{X}_\mathrm{c}$ is the horizontal velocity of the 85 stations, calculated by different functions.

4.1 Calculation results for different kernel functions

Since the multi-quadric function is used to determine the signal priori random information in the multi-quadric collocation model, different kernel functions have different results. We use inverted and positive, double curved, two, and three surfaces to obtain results. Table 1 shows statistics for velocity residuals for the different kernel functions

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and Table 2 shows statistics for the 85 points of external inspection accuracy of the different kernel functions. It can be seen that the different kernel functions have different effects on the accuracy of the results.

1. From the 85 external checkpoint precision statistics, the RMS for east and north of the positive three surface function is 1.25 and 0.89 $mm\,yr^{-1}$, respectively, but from the velocity residuals statistics, it is 1.29 and 1.14 $mm\,yr^{-1}$, respectively. The accuracy of the external statistics is basically the same as that of the internal statistics, which shows that the fitting and prediction results are stable. The external precision of the inverted double curved surface can also reach 1.38 and 1.18 $mm\,yr^{-1}$, but its residual velocity is only 0.56 and 0.51 $mm\,yr^{-1}$, and the RMS is obviously better than that of the positive three surface.

2. From the internal and external statistical accuracy, the positive surface function and the inverse surface function of the statistical results have obvious differences. The 85-point accuracy of the inverted double curved surface and inverted two and three surfaces in the north direction are, respectively, 1.18, 1.45 and 1.77 $mm\,yr^{-1}$; in the east direction they are 1.38, 1.67 and 1.93 $mm\,yr^{-1}$. All of them are superior to 2 $mm\,yr^{-1}$. The variation of the different inverse curve functions is small, and this may be due to the fact that inverse curve performs as the reciprocal of distance. The closer the distance, the greater the correlation between points, and the physical characteristics of the velocity field are basically identical. However, the 85-point accuracy of the positive double curved surface and positive two and three surface in the north and east directions are, respectively, 1.27, 5.02 and 0.89 $mm\,yr^{-1}$, and 2.61, 5.02, and 1.25 $mm\,yr^{-1}$. The difference between different positive surface functions is obvious which may be due to the positive surface function and the distance of the positive correlation. The greater the distance between the points, the larger the correlation, and this is contrary to the physical characteristics of the velocity field, which therefore has not been well described by the physical properties of the velocity field.

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4.2 Comparison with other common methods

In order to compare the effect of the multi-quadric collocation model in calculating the horizontal velocity field, we compared it with other common methods using four calculated solutions as follows:

Scheme 1: with the very clear physical significance of the plate tectonics Euler vector model, we use the least square principle to solve the unified Euler vector for Chinese mainland then solving point speed.

Scheme 2: we use the collocation method to estimate the long-term overall movement trend (Euler vector) and the local point change velocity. In the process of calculation, we use the Hirvonen function as a covariance function of random signal, and use 1041 station velocities, in which the Euler vector parts had been subtracted, to estimate the covariance function. The covariance function (Yang et al., 2011) of the north ($C_N(d)$) and east ($C_E(d)$) velocity signals can be defined by

$$C_N(d) = \frac{38.33170}{1 + 0.651987 \cdot d^2} \quad (15)$$

$$C_E(d) = \frac{21.47733}{1 + 1.063019 \cdot d^2}. \quad (16)$$

Scheme 3: the multi-quadric function method, being a kind of pure mathematics method and being often used in the establishment of the velocity field, has been systematically researched in the literature (Zeng et al., 2013). It has no clear physical meaning, cannot solve the Euler vector, and can only estimate the coefficient of the node. Because the key issue for establishing a horizontal velocity field model using the multi-quadric function method is how to determine the kernel function, smooth factor, and node number. This paper will use a hyperbolic function as the kernel function, a smooth factor of 10, and 56 basis points for the node.

Scheme 4: using a multi-quadric collocation model that has clear physical significance and can estimate the long-term overall movement trend (Euler vector) and the local point change velocity to solve the point change horizontal velocity, we will use Eqs. (7), (8) and the inverted two surface in the calculation.

Table 3 lists the Euler vectors of the Chinese mainland calculated by different schemes. Table 4 lists the external precision of the Euler vectors of the Chinese mainland calculated by different schemes. Table 5 lists the velocity residuals statistics for different schemes. Table 6 shows the precision statistics of 85 external check points. Figure 1 shows the velocity residual statistical results of 85 external check points.

A deep analysis of the results shows the following.

1. As can be seen from Table 3, the Euler vector, calculated by Euler vector estimation method (Scheme 1), collocation (Scheme 2), and multi-quadric collocation models (Scheme 4) are equivalent in magnitude and trends, but as a result of the difference of the function model and the signal covariance function between collocation and the multi-quadric collocation model, they are slightly different in values and in the estimated remaining horizontal velocity.

2. The least squares model based on the Euler vector only takes into account the entire horizontal movement of the Chinese mainland, so its precision is poor. From the perspective of internal precision (Table 4), horizontal residuals for the east and north directions are up to 30.17 and 27.69 mm yr^{-1} maximum, respectively, and statistical precision for east and north is 4.63 and 6.19 mm yr^{-1}, respectively. Viewed from the external checkpoint, the external precision (Table 6) of the Euler vector least squares solutions for east and north are 5.89 and 7.06 mm yr^{-1}, respectively. This is because the Chinese mainland is not a block; the Euler vector least squares solution can determine only the overall orientation of the Chinese mainland, and does not take the systemic change of the local area into account.

3. Solving using the collocation method not only takes into account the overall orientation, but also determines the local area random effects. It makes a more reasonable distribution of the residuals (Table 5). The internal precision is 1.60 mm yr^{-1}

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for east and 1.43 $mm\,yr^{-1}$ for north. The external check precision also improved, to 2.14 $mm\,yr^{-1}$ for east and 1.47 $mm\,yr^{-1}$ for north.

4. Using a multi-quadric function method can obviously improve the precision of the velocity model. The internal residual statistic is 2.79 $mm\,yr^{-1}$ for east and 2.25 $mm\,yr^{-1}$ for north; the external check precision is 1.85 $mm\,yr^{-1}$ for east and 1.40 $mm\,yr^{-1}$ for north. Theoretically, multi-quadric functions can approximate the measured surface with arbitrary accuracy, but in practice it is impossible to select all the observation points for the node in order to improving the fitting internal precision without considering the estimate precision of the non-observation points (i.e. the precision of the external check points). However, extrapolation precision is what we need to focus on.

5. Synthesizing collocation method and multi-quadric function method not only obviously improved internal precision, with east and north precision reaching 0.78 and 0.73 $mm\,yr^{-1}$, respectively, which is obviously superior to the collocation method and the multi-quadric function method; but external precision has also obviously improved, with east and north direction precision reaching 1.67 and 1.45 $mm\,yr^{-1}$, respectively, which is slightly superior to the collocation method and the multi-quadric function method. This approach integrates the characteristics of the two methods. It avoids the establishment of the covariance function, and achieves a combined effect.

6. The Euler vector, collocation and multi-quadric collocation models have physical meaning, and can work out the Euler vectors of Chinese mainland blocks. However, the Euler vector method has larger differences from the other two methods, which, in particular, have smaller differences in latitude and longitude components and rotating angle speed aspects. The Euler vectors determined by the collocation method and the multi-quadric function configuration model are very close.

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5 Conclusion

Multi-quadric functions and the collocation method are the most commonly used methods of fitting the observed points and estimating the non-observed points. However, it is difficult to select the kernel function, smoothing factor, and node when we use the multi-quadric function method. The key to applying the collocation model is to establish reliable covariance function. Usually, it tends to adopt an empirical formula in a gravitational field, where the majority of applications are established, by using a measured data covariance model, which not only affects the optimality of the collocation method but also reduces its usable area.

Taking into account the establishment of the covariance function is a relatively high requirement for data, and generally, it is difficult to achieve. Bearing in mind that the covariance function is a function of the distance, the local deformations covariance matrix is often chosen as a simple function of distance in multi-quadric functions. The estimating effects of this integrated approach are similar to the collocation method. Because the solution is simple and reasonable, it can greatly expand the scope of application of the collocation model.

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Table 1. Velocity residuals statistics for different kernel functions (mm yr^{-1}).

Plan	East				North			
	Max	Min	Average	RMS	Max	Min	Average	RMS
Inverted double curved surface	4.17	−7.65	0.01	0.56	3.03	−6.49	0.01	0.51
Inverted two surface	5.75	−11.27	0.02	0.78	3.76	−9.633	0.02	0.73
Inverted three surface	11.00	−22.61	0.02	1.46	9.05	−19.52	0.02	1.41
Positive double curved surface	–	–	–	–	–	–	–	–
Positive two surface	21.50	−30.13	0.00	3.79	27.68	−22.94	0.00	4.54
Positive three surface	13.25	−22.59	0.00	1.29	5.02	−19.30	0.00	1.14

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Table 2. External inspection accuracy of different kernel functions ($mm\,yr^{-1}$).

Plan	East				North			
	Max	Min	Average	RMS	Max	Min	Average	RMS
Inverted double curved surface	5.79	−4.59	0.08	1.38	6.20	−3.32	0.08	1.18
Inverted two surface	6.21	−4.75	0.00	1.67	7.69	−2.79	0.12	1.45
Inverted three surface	7.06	−5.79	0.15	1.93	8.49	−4.22	0.03	1.77
Positive double curved surface	4.76	−20.83	0.09	2.61	2.59	−3.68	0.49	1.27
Positive two surface	21.24	−14.73	0.78	5.02	21.71	−12.48	0.34	5.02
Positive three surface	3.73	−4.79	0.13	1.2	3.01	−2.61	20.24	0.89

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Table 3. Euler vectors of Chinese mainland.

Scheme	ω_x (rad Myr^{-1})	ω_y (rad Myr^{-1})	ω_z (rad Myr^{-1})	Lon. λ (°)	Lat. ϕ (°)	ω (° Myr^{-1})
1	−0.0004415	−0.0037950	0.0035848	−96.6	43.2	0.300
2	0.0001159	−0.0042049	0.0031925	−88.4	37.2	0.302
4	0.0000206	−0.004275	0.0032031	−89.7	36.8	0.306

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Table 4. Internal precision of the Euler vectors (mm yr^{-1}).

Scheme	East				North			
	Max	Min	Average	RMS	Max	Min	Average	RMS
1	30.17	−23.30	0.16	4.63	23.48	−27.69	−0.20	6.19
2	30.29	−23.89	0.04	4.89	25.37	24.08	0.24	6.59
4	30.46	−23.53	0.04	4.89	24.69	24.59	−0.17	6.35

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Table 5. Velocity residuals statistics ($mm\,yr^{-1}$).

Scheme	East				North			
	Max	Min	Average	RMS	Max	Min	Average	RMS
1	30.17	−23.30	0.16	4.63	23.48	−27.69	−0.20	6.19
2	23.12	−13.30	0.03	1.60	19.68	−7.49	−0.03	1.43
3	29.13	−15.20	0.00	2.79	24.54	−9.12	0.00	2.25
4	5.75	−11.27	0.00	0.78	3.76	−9.633	0.00	0.73

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Table 6. External examination precision ($mm\,yr^{-1}$).

Scheme	East				North			
	Max	Min	Average	RMS	Max	Min	Average	RMS
1	13.34	−20.29	−0.05	5.89	16.47	−23.29	1.315	7.06
2	9.10	−8.54	0.00	2.14	3.12	−5.34	0.00	1.47
3	12.48	−3.75	0.00	1.85	4.50	−6.94	0.00	1.40
4	6.21	−4.75	0.00	1.67	7.69	−2.79	0.12	1.45

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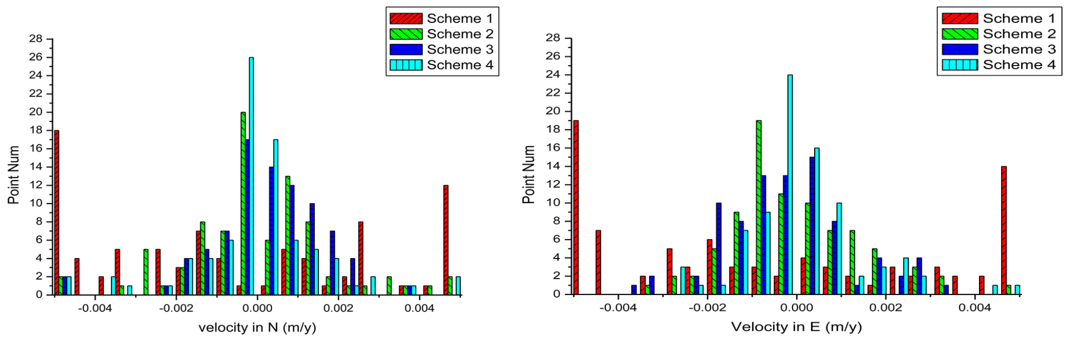

Figure 1. The velocity residuals statistics of 85 external check points.