### Response to Interactive comment on "A simple 3-D numerical model of thermal convection in Earth's growing inner core: on the possibility of the formation of the degree-one structure with lateral viscosity variations" by M. Yoshida

#### S. Zhang (Referee)

Although this isn't exactly my area of expertise, I do find the topic of this paper interesting. Degree-one like inner core structure has been proposed by recent seismological studies. This paper carried out a series of 3D thermal convection simulations to explore the possibility of generating this kind of structure by an "endogenic factor". While exploring the major uncertainties of the model parameters such as rheology and the thermal conductivity of the inner core, the author concludes that an "endogenic factor" is less probable. The lateral viscosity variation considered here is a good addition to previous works, and this improves our understanding of the core evolution, which is worth publishing. However, the numerical treatment borrowed directly from mantle convection simulations requires a few changes to be suitable for inner core convection. And some details of those treatments carried out in this paper are oversimplified or improper to me (listed below). I would like to suggest some corrections or justifications before the paper can be published.

[Reply] I sincerely thank the reviewer for constructive comments. I have carefully incorporated all the comments and suggestions into the revised manuscript attached below. The revised parts are highlighted by red. I provide below my response to reviewer's comments.

Some detailed comments are listed below:

1. This study uses mantle convection simulations to deal with convection within a growing inner core. In contrast to mantle convection studies, the author uses a timedependent inner core radius to get dimensionless equation Eq. (2-4) to account for a growing inner core. However, the Eq. (2-4) is build based on an Eulerian specification that is fixed on space. With a growing inner core radius, the grid is actually slowly expanding through time. So strictly speaking, mass, momentum, and energy are no longer conserved with this expending mesh. One could argue that the growth rate is small enough to make it negligible, and there are also some previous studies using a similar treatment. However, considering the significant accumulating growth of the inner core during the whole simulation, I still feel this requires some improvement or at least a more detailed justification.

[Reply] Thank you for your comment. I agree with your comment. The framework of present model is based on mantle convection simulations to deal with convection within the growing inner core. In contrast to mantle convection simulations, I used a time-dependent inner core radius to get dimensionless equations Eq. (2–4) to account for the growing inner core. However, Eq. (2-4) is build based on an Eulerian specification that is fixed on space. With increasing inner core radius, the grid is actually slowly expanding through time. Thus, strictly

speaking, mass, momentum, and energy may be no longer conserved with this expending grid, although the growth rate could be small enough to make it negligible. However, considering the significant accumulating growth of the inner core during the whole simulation, improvement of the present numerical model should be required. I openly discussed this problem in Section 4.

2. p. 3820, l. 12-14. As the small sphere is imposed, it created an additional inner boundary, what's the boundary condition here? And how is it made consistent with reality? The temperature difference across the inner core seems to be constant during the whole simulation. What is the justification for that? As the inner core radius grows significantly through time, and it cools as well, I don't see any particular reason that this will stay almost the same.

[Reply] About the boundary condition, impermeable, shear-stress-free, adiabatic conditions are imposed on the top boundary of the small virtual-sphere for the purpose of technical convenience. However, this is just for the purpose of computational convenience, and this setup does not mean the existence of the real singularity that violates the mass and heat transport near the center of the model sphere. This is explicitly explained in Section 2. And, about the temperature difference across the model domain, I ignore the secular cooling of the whole inner core, because the cooling rate and the resulting time change of the temperature difference across the inner core can not be estimated a priori. However, the absolute time change in the temperature difference across the inner core formation, and the effect of the time change on the magnitude of thermal Rayleigh number (Eq. 5) is negligibly small compared to other physical values. Therefore, I consider that this assumption would be justified at least in the framework of this numerical model. I explicitly explained this point in Section 2.

3. Ep.(8) and p.3823, I. 1-4 "The heat source associated with solidification of the inner core are ignored because these effects play a secondary role in the growth of the inner core (Buffett et al., 1992)". This isn't correct. Buffett et al., (1992) keeps the latent heat and gravitational energy terms in their equation, and most other research keep them as well. For example, in core evolution models from Gubbins et al. (2003), Nimmo et al. (2004), the latent heat plus gravitational energy is larger than the specific heat term for present day Earth. These research also show once the inner core starts to freeze, the core temperature dropping rate decreases significantly. So this isn't a secondary effect that can be ignored.

[Reply] Thank you for your comment. Following your comment, I removed this sentence and modified based on new references below:

- Gubbins, D., Alfè, D., Masters, G., Price, G.D. & Gillan, M.J., 2003. Can the Earth's dynamo run on heat alone?, *Geophys. J. Int.*, **155**, 2, 609-622, doi:10.1046/j.1365-246X.2003.02064.x.

- Nimmo, F., Price, G.D., Brodholt, J. & Gubbins, D., 2004. The influence of potassium on core and geodynamo evolution, *Geophys. J. Int.*, **156**, 2, 363-376, doi:10.1111/j.1365-246X.2003.02157.x.

Although Buffett et al. (1992) implicitly evaluated that these effects play a secondary role in the growth of the inner core, most of other studies kept these effect. For example, the modeling studies of the core evolution by Gubbins et al. (2003) and Nimmo et al. (2004) revealed that the latent heat plus gravitational energy is larger than the specific heat for the present Earth, and once the inner core starts to freeze, the core temperature decreases significantly with time, which has probably influence on the growth speed of the inner core and the generation and maintenance of geodynamo. I explicitly discussed this point in Section 2.

4. The gravity acceleration seems to be treated as a constant in this study. Different from the mantle, the gravity acceleration should be almost linearly increasing from 0 at the centre to \_4.4 m/s<sup>2</sup> at the present day ICB (e.g. PREM model). I would expect depth dependent g will have some influence on the convection that should be considered.

### [Reply] Yes, I treated it as a constant with radius in this model for the simplicity. I explained explicitly this point in Section 2.

5. Although model uncertainties of CMB heat flow and inner core age are mentioned in the discussion, the heat flow is assumed to be constant in this study. Moreover, only low CMB heat flow and a slowly growing inner core with an age of \_4.5Gry are tested in this study, which are extreme cases rather than "realistic" ones. As mentioned in the discussion of this paper, there are many studies that suggest larger CMB heat flow and younger inner core age. And the CMB heat flow may have a significant variation through the whole Earth's history. Whether the fast growing inner core leads to a different flow pattern or not needs to be explored. So, I would like to suggest an additional test model with fast growing inner core.

[Reply] According to a more recent paleogeomagnetic study, the inner core formed at ~1.5 Ga (Biggin et al. 2015). This "young" inner core age is consistent with the indirect evidences from seismology and mineral physics that the CMB heat flow is larger than previously thought (Hernlund et al. 2005; Lay et al. 2006; van der Hilst et al. 2007). In the present numerical model, the CMB heat flow is assumed to be constant, Fm' = 2.56×1012 W, based on the model constants used in Buffett et al. (1992) and the initial radius of the inner core arbitrarily set. As stated in Section 2, this value would be a lower limit value for the present Earth considering an even relationship between the total plume buoyancy flux observed at the Earth' surface and the inferred total CMB heat flow (e.g., Davies 1988; Sleep 1990; Davies & Richards 1992) and a minimal power requirement for maintenance of the geodynamo (Buffett 2002). However, the CMB heat flow in the past Earth would be lower than that in the present Earth, because the average mantle temperature may increase as the Earth older. More than that, there is a possibility that the CMB heat flow may have a significant variation throughout the Earth's history. Although the implementation of time-dependent CMB heat flow is beyond the framework of the present simple numerical model, it might be a serious problem for the growing speed of the inner core. However, if the inner core grows faster than the present model, the degree-one structure would only appeared for a further limited range of viscosity contrast of temperature dependence than the results presented in this paper. Therefore I believe that the conclusion on the less possibility of an endogenic origin for the degree-one thermal/mechanical structure of the inner core is justified. I discussed this point explicitly in Section 4.

Technical correction:

p. 3821, l. 15 "g" should be g0

[Reply] Fixed.

I hope these comments/suggestions will be found useful by the author when preparing a revised version of the article.

[Reply] I again deeply appreciate you for the careful reading and significant improvement of this paper.

Masaki Yoshida

1	A simple 3D numerical model of thermal convection in Earth's growing inner core:
2	On the possibility of the formation of the degree-one structure with lateral
3	viscosity variations
4	
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13	Submitted to <i>Solid Earth</i> on December 4, 2015.
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16	Abstract
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18	An east-west hemispherically asymmetric structure for Earth's inner core has been
19	suggested by various seismological evidence, but its origin is not clearly understood. Here,
20	to investigate the possibility of an "endogenic origin" for the degree-one
21	thermal/mechanical structure of the inner core, I performed new numerical simulations
22	of thermal convection in the growing inner core. A setup value that controls the viscosity
23	contrast between the inner core boundary and the interior of the inner core, $\Delta \eta_T$ , was taken
24	as a free parameter. Results show that the degree-one structure only appeared for a limited
25	range of $\Delta \eta_T$ ; such a scenario may be possible but is not considered probable for the real
26	Earth. The degree-one structure may have been realized by an "exogenous factor" due to
27	the planetary-scale thermal coupling among the lower mantle, the outer core, and the inner
28	core, not by an endogenic factor due to the internal meological neterogeneity.
29	Var monda, averagical simulation, ingen come the much convection, do and some construction.
30	<b>Key words:</b> numerical simulation; inner core; thermal convection; degree-one structure;
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30 36	After the segregation of the rocky mantle and molton iron core in the early stage of
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Earth's formation (e.g., Stevenson 1981), the inner core was formed by gradual 37 solidification of the molten iron core and the size increased with age (Jacobs 1953; Buffett 38 39 et al. 1992) (for example, see reviews by Buffett (2000) and Sumita and Yoshida (2003)). The solidification of the solid core could affect the vigor of outer-core convection owing 40 41 to the release of latent heat and the passage of light elements toward the liquid outer core 42(e.g., Sumita & Yoshida 2003). The resulting growth of the inner core may have changed the convection style in the outer core and affected the intensity of Earth's magnetic field 43 44 throughout Earth's history.

Although the structure of the present inner core cannot be inferred from surface
geophysical observations, various seismological evidence suggests that the inner core has
an east–west, hemispherically asymmetric structure in terms of seismic velocity,
anisotropy, and attenuation (Tanaka & Hamaguchi 1997; Creager 1999; Niu & Wen 2001;
Cao & Romanowicz 2004; Deuss *et al.* 2010; Irving & Deuss 2011; Waszek *et al.* 2011;
Lythgoe *et al.* 2014) (see also a recent review by Tkalčić (2015)).

Previous numerical simulations of Earth's mantle convection clarified that for 5152convecting rocky materials confined in a spherical shell, the spherical harmonic degreeone structure was observed for a relatively wide range of the parameter that controls the 53lateral viscosity variations due to temperature variations (McNamara & Zhong 2005; 54Yoshida & Kageyama 2006). This is because when the temperature-dependence of 55viscosity is moderate, the highly viscous lid that develops at the surface of the convecting 5657mantle has the longest-wavelength scale, and the dynamic instability at the bottom of the lid concentrates in one area. This scenario characterizes the degree-one thermal structure 58of mantle convection that lies between the "mobile-lid regime" with weakly temperature-59dependent viscosity and the "stagnant-lid regime" with strongly temperature-dependent 60 viscosity (Yoshida & Kageyama 2006). 61

62It is possible that the degree-one seismic structure in the present inner core originated from lateral temperature variations, which in turn originated from lateral viscosity 63 64 variations. Even given the uncertainties in the rheological properties and composition of the inner core materials, lateral viscosity variation offers considerable potential as an 65 "endogenic factor" that may explain the formation of the degree-one seismic structure. It 66 67 is therefore worth examining whether a degree-one thermal/mechanical structure generated from the lateral viscosity variations can be realized for solid materials confined 68 in a sphere. This topic had not been investigated in previous numerical simulation models 69 of inner core convection (Deguen & Cardin 2011; Cottaar & Buffett 2012; Deguen 2013; 70 Deguen et al. 2013). 71

In this study, to explore the time-dependent behavior of the convection regime in

Earth's growing inner core and the possibility of generating the degree-one thermal/mechanical structure from the internal rheological heterogeneity, a new simple numerical model of the growing inner core is constructed and a series of numerical simulations of thermal convection are performed, assuming that the solidification of the liquid core started at 4.5 Ga and that the radius of the inner core gradually increased with the square root of age.

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#### 80 2 Model

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82 Convection in the inner core is computed numerically using a staggered grid-based, 83 finite-volume code, ConvGS (e.g., Yoshida 2008). The material of the inner core is 84 modeled as a Boussinesq fluid with an infinite Prandtl number confined in a sphere and modeled in spherical coordinates  $(r, \theta, \phi)$ . Impermeable, shear-stress-free, isothermal 85 conditions are imposed on the inner core boundary (ICB) with a fixed dimensionless 86 87 radius of  $r_c$ ' = 1 (Fig. 1a). The driving force of convection is primordial heat alone because the radiogenic heat production is negligibly small in the inner core (e.g., Karato 2003) 88 (see also below). The number of computational grids is taken as 64 (r)  $\times$  64 ( $\theta$ )  $\times$  192 ( $\phi$ ) 89  $\times$  2 (for Yin and Yang grids) (Yoshida & Kageyama 2004). To avoid the mathematical 90 singularity at the Earth's center, an extremely small virtual-sphere with a dimensionless 91 radius of  $r_{\delta} = 10^{-6}$  is imposed at the center of the model sphere. For the purpose of 92computational convenience, impermeable, shear-stress-free, adiabatic conditions are 93 94 imposed on the top boundary of the small sphere. However, this setup does not mean the existence of the real singularity that violates the mass and heat transport near the center 9596 of the model sphere.

Following standard techniques for mantle convection simulations (e.g., Schubert *et al.* 2001), the length *L*, time *t*, velocity vector v, stress tensor (or pressure)  $\sigma$ , viscosity  $\eta$ , and temperature *T* are non-dimensionalized as follows:

100 
$$L = r_c(t)L', \ t = \frac{r_c(t)^2}{\kappa_0}t', \ \mathbf{v} = \frac{\kappa_0}{r_c(t)}\mathbf{v}', \ \mathbf{\sigma} = \frac{\kappa_0\eta_0}{r_c(t)^2}\mathbf{\sigma}', \ \eta = \eta_0\eta', \ T = \Delta T_0 \cdot T'$$
(1)

101 where  $r_c(t)$  denotes the time-dependence of the radius of the inner core, which depends 102 on time (age);  $\kappa_0$  denotes the reference thermal diffusivity;  $\eta_0$ , the reference viscosity;  $\Delta T_0$ , 103 the characteristic temperature variation; and the subscript "0" refers the reference values 104 for the inner core (Table 1). In these equations, symbols with primes represent 105 dimensionless quantities.



107 momentum, and energy, which govern inner core convection, are expressed as

$$\nabla \cdot \mathbf{v} = 0, \tag{2}$$

109 
$$-\nabla p + \nabla \cdot \boldsymbol{\tau} + Ra(t)\zeta(t)^{-3}Te_r = 0, \ \boldsymbol{\tau} = \eta \left(\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^{tr}\right)$$
(3)

110 
$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} T = \boldsymbol{\nabla}^2 T + H(t) \boldsymbol{\zeta}(t)^{-2}, \qquad (4)$$

111 respectively, where *p* represents the pressure;  $\tau$ , the deviatoric stress tensor; and  $e_r$ , the 112 unit vector in the radial direction. Primes representing dimensionless quantities are 113 omitted in Eq. (2)–(4).

In the numerical simulation for this study, instead of fixing the dimensionless radius of the ICB, the radius of the inner core in the thermal Rayleigh number, Ra, the internal heatsource number, H, and the "spherical-shell ratio",  $\zeta$ , depend on age. They are given by

$$Ra(t) = \frac{\rho_0 \alpha_0 \Delta T_0 g_0 \left[ r_c(t) - r_\delta \right]^3}{\kappa_0 \eta_0} = \frac{\rho_0^2 \alpha_0 \Delta T_0 g_0 c_{p0} \left[ r_c(t) - r_\delta \right]^3}{k_0 \eta_0},$$

$$H(t) = \frac{\Omega \left[ r_c(t) - r_\delta \right]^2}{M_m \Delta T_0 \kappa_0 c_{p0}} = \frac{\Omega \rho_0 \left[ r_c(t) - r_\delta \right]^2}{M_m \Delta T_0 k_0},$$

$$\zeta(t) = \frac{r_c(t) - r_\delta}{r_c(t)},$$
(5)

where  $\rho_0$  is the reference density;  $\alpha_0$ , the reference thermal expansion coefficient;  $\Delta T_0$ , 118119the reference temperature difference across the inner core;  $g_0$ , the reference gravitational acceleration;  $c_{p0}$ , the reference specific heat at constant pressure;  $k_0$ , the reference thermal 120conductivity; and  $M_m$ , the mass of the mantle (Table 1). Note that I ignore the secular 121122cooling of the whole inner core, because the cooling rate and the resulting time change of 123the temperature difference across the inner core can not be estimated a priori. However, 124the absolute time change in the temperature difference across the inner core should be 125small throughout the inner core formation, and the effect of the time change on the magnitude of thermal Rayleigh number (Eq. 5) is negligibly small compared to other 126physical values. Therefore, I consider that this assumption would be justified at least in 127128the framework of this numerical model (see also discussion in Section 5). Furthermore, 129although the gravity acceleration is almost linearly increasing from 0 at the Earth's centre to ~4.4 m s<sup>-2</sup> at the present ICB (Dziewonski & Anderson 1981), I treated it as a constant 130with radius for the simplicity. 131

Thermal conductivity has received a lot of attention in recent mineral physics studies, and the main finding is that it may be much larger than the value of 36 W m<sup>-1</sup> K<sup>-1</sup> from Stacey and Davis (2008), i.e., 100–200 W m<sup>-1</sup> K<sup>-1</sup> (de Koker *et al.* 2012; Pozzo *et al.*  135 2012, 2014). Therefore, two end-member models with  $k_0 = 36$  and 200 W m<sup>-1</sup> K<sup>-1</sup> are 136 investigated in this study.

Because the radioactive potassium may be major heat source in the inner and outer 137core, and there is a large possibility that the role of potassium is important in determining 138139 the history of growth of inner core and geodynamo power (e.g., Nimmo et al. 2004). 140 However, I ignore explicitly radioactive heating in the present model because the amount of radioactive potassium in the inner and outer core would be still under the debate in 141 142mineral physics, geodynamics, geomagnetism and seismological communities, and 143instead consider primordial heating in the heat source term of the conservation equation 144of energy. This is because the temperature the inner core is normalized by the 145characteristic temperature variation, although there is another way for normalizing the temperature the inner core by the amount of heat production. 146

147 The amount of approximated primordial heat that has existed since Earth's formation, 148  $\Omega$ , is taken as a free parameter estimated from the heat released by mantle cooling in the 149 present Earth (Turcotte & Schubert 2014):

150 
$$\Omega = -\frac{4}{3} \pi r_e^3 \rho_e c_{pe0} \frac{dT}{dt} \Big|_{t=t_0},$$
(6)

where  $r_e$  is the radius of Earth;  $\rho_e$ , the average density of Earth; and  $c_{pe0}$ , the average specific heat at constant pressure for Earth. Here  $\left. \frac{dT}{dt} \right|_{t=t_0}$  is the present cooling rate of the mantle (Turcotte & Schubert 2014):

154 
$$\left. \frac{dT}{dt} \right|_{t=t_0} = -3\lambda \left( \frac{RT_m^2}{E_a} \right), \tag{7}$$

where  $\lambda$  is the average radiogenic decay constant in the mantle; *R*, the gas constant, *T<sub>m</sub>*, the mean mantle temperature; and *E<sub>a</sub>*, the activation energy of dry olivine. Using the values in Table 1,  $\Omega$  is 11.3 TW, which is a reasonable value compared with the total heat release by mantle cooling estimated from a global heat-flow balance (Lay *et al.* 2008).

Following the work of Buffett *et al.* (1992), who studied an analytical model for solidification of the inner core, the global heat balance of the outer core is

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$$\frac{4\pi}{3} \left( r_b^3 - r_c^3 \right) \rho c_p \frac{dT_s(r)}{dt} = 4\pi r_c^2 f_i(t) - 4\pi r_b^2 f_m(t), \tag{8}$$

where  $T_s(r)$  is the "solidification temperature", and  $f_i(t) \equiv F_i(t)/(4\pi r_c^2)$  and  $f_m(t)$ ( $\equiv F_m(t)/(4\pi r_b^2)$ ) are the heat fluxes across the ICB and the core-mantle boundary (CMB), respectively (Fig. 1b). The potential temperature in the well-mixed liquid outer core is

assumed to be spatially uniform and slowly decreases with time, and the ICB is assumed 165166 to be in thermodynamic equilibrium with the surrounding liquid. Under these assumptions, the temperature through the outer core is uniquely defined by the solidification 167 temperature,  $T_s(r)$ , as a function of pressure or depth (see the explanation in Buffett *et al.* 168169 (1992) for details). Here, the heat sources associated with solidification of the inner core 170(i.e., the release of latent heat and gravitational energy) are ignored. Although Buffett et al. (1992) implicitly evaluated that these effects play a secondary role in the growth of 171172the inner core, most of other studies kept these effect. For example, the modeling studies 173of the core evolution by Gubbins et al. (2003) and Nimmo et al. (2004) revealed that the 174latent heat plus gravitational energy is larger than the specific heat for the present Earth, 175and once the inner core starts to freeze, the core temperature decreases significantly with time, which has probably influence on the growth speed of the inner core and the 176generation and maintenance of geodynamo. 177

178 According to Eq. (4) in Buffett *et al.* (1992), the radial dependence of  $T_s$  is expressed 179 as

180 
$$T_{s}(r) = T_{s}(0) - \frac{2\pi}{3}G\rho_{0}^{2}r^{2}\frac{\partial T_{s}}{\partial p}, \qquad (9)$$

181 where *G* is the gravitational constant, and  $\partial T_s / \partial p$  is the solidification profile (Table 1).

182 Substituting Eq. (9) into Eq. (8), an expression for the radius of the inner core is 183 obtained:

184 
$$r_{c}(t) = r_{b} \left[ \frac{1}{4\pi r_{b}^{2} N} \int_{0}^{t} F_{m}(t) dt \right]^{\frac{1}{2}},$$
 (10)

185 where the model constant *N* is expressed as

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$$N = \frac{2\pi}{9} r_b^3 c_{p0} \rho_0^2 G \frac{\partial T_s}{\partial p}.$$
 (11)

187 Assuming that the heat flux across the CMB is constant throughout Earth's history 188 (Buffett *et al.* 1992), a model constant,  $F_m$ ', is obtained:

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$$F_{m}' = \frac{4\pi N}{t} \left( r_{c} - r_{c}^{i} \right)^{2}, \qquad (12)$$

where  $r_c{}^i$  is an arbitrary value that represents the initial radius of the inner core at the beginning of the simulation. When the age of Earth's core is assumed to be 4.5 Ga (Lister & Buffett 1998), the solidification of the inner core is assumed to have begun at this age, and  $r_c{}^i$  is taken to be 21.5 km so that  $r_c-r_c{}^i$  is exactly 1200 km,  $F_m{}^i$  is  $2.56 \times 10^{12}$  W, which would be a lower limit value for the real Earth considering an even relationship between the total plume buoyancy flux observed at the Earth' surface and the inferred total CMB 196 heat flow (e.g., Davies 1988; Sleep 1990; Davies & Richards 1992). Eq. (12) indicates

that the relationship between the total CMB heat flow and the radius of the growing inner core is  $F_m \propto r_c^2$ , which means that the radius of the inner core is proportional to the square

root of  $F_m$ '. It should be noted that there is a trade-off between the choices of  $F_m$ ' and  $r_c^{i}$ ,

which are critical for identifying the age of the present-day inner core in the real Earth. In the present model, I set  $r_c^i$  to a significantly small value to see the behavior of inner core convection over the longest geological time, i.e., 4.5 Gyr (see Section 4 for discussion).

Finally, the time-dependent radius of the growing inner core used in the present simulation is expressed as

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$$r_c(t) = \left(\frac{F_m'}{4\pi N}\overline{t}\right)^{\frac{1}{2}} + r_c^i, \qquad (13)$$

where the dimensional time is scaled as  $\overline{t} = (\overline{r_c}(t)^2 / \kappa_0) \cdot t'$  using the radius of the inner core at the previous time step of the simulation,  $\overline{r_c}(t)$ . Eq. (13) means that  $r_c$  increases with the square root of age, and eventually reaches  $r_c = 1221.5$  km after 4.5 Gyr, which matches the present radius of Earth's inner core.

At the beginning of the simulation, the initial condition for dimensionless temperature with significant small-scale lateral perturbations is given as

213 
$$T'(r,\theta,\phi) = 0.5 + \omega \cdot Y_{34}^{17}(\theta,\phi) \cdot \sin\left[\pi \frac{r_1' - r'}{r_1'}\right],$$
(14)

where  $Y_{\ell}^{m}(\theta, \phi)$  is the fully normalized spherical harmonic function of degree  $\ell$  and order *m* and  $\omega$  (=0.1) is the amplitude of perturbation.

The viscosity of the inner core materials,  $\eta_T$ , in this model depends on temperature according to a dimensionless formulation (Yoshida 2014):

218 
$$\eta_{T}' = \eta_{0}' \exp\left[2T_{ave}'(t)\left(\frac{E'}{T' + T_{ave}'(t)} - \frac{E'}{2T_{ave}'(t)}\right)\right],$$
 (15)

where  $T_{ave}'(t)$  is the dimensionless average temperature of the entire sphere at each time step. A model parameter, *E*', controls the viscosity contrast between the ICB with T' = 0and the interior of the inner core. In the present model,  $E' = \ln(\Delta \eta_T)$  varies from  $\ln(10^0)$ = 1 (i.e., no laterally variable viscosity) to  $\ln(10^5) = 11.51$ . The essence of the viscosity equation in Eq. (15) is just a simple relationship between the temperature and viscosity under the assumption that non-Newtonian rheology does not work in the iron-nickel alloy unlike in the rocky mantle (e.g., Karato 2003, 2008).

227 **3 Results** 

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229Fig. 2 shows the time evolution of the convection pattern in the inner core for models with  $\eta_0 = 10^{17}$  Pa s,  $k_0 = 36$  W m<sup>-1</sup> K<sup>-1</sup>, and with  $\Delta \eta_T = 10^0$  (Fig. 2a–d),  $\Delta \eta_T = 10^3$  (Fig. 2302e-h),  $\Delta \eta_T = 10^{3.5}$  (Fig. 2i-l), and  $\Delta \eta_T = 10^4$  (Fig. 2m-p). When  $\Delta \eta_T = 10^0$  (i.e., the 231viscosity of the inner core is homogeneous) the convection pattern kept a short-232233wavelength structure over almost all of the simulation time and the "present" inner core 234at 0.0 Ga has numerous downwellings uniformly distributed in the sphere (Fig. 2d). On 235the other hand, when  $\Delta \eta_T$  is large enough to make the surface thermal boundary layer stagnant ( $\Delta \eta_T \ge 10^4$ ), the convection pattern tends towards a short-wavelength structure 236237with age, and secondary cold plumes from the bottom of the highly viscous lid are evenly distributed in the sphere (Fig. 2p). A remarkable change in the convection pattern is found 238for moderate values of  $\Delta \eta_T$ : when  $\Delta \eta_T = 10^3$  and  $10^{3.5}$ , the convection patterns shift 239towards a long-wavelength structure with increasing age, and eventually the longest-240241wavelength thermal structures develop for the inner core at 0.0 Ga (Fig. 2h, 2l)

To quantitatively assess the variations in thermal and mechanical heterogeneities in the 242243inner core with time, Fig. 3 shows the power spectra of the temperature and root-meansquare velocity fields throughout the modelled inner core at each time step. On the other 244hand, when  $\Delta \eta_T$  is  $\leq 10^{2.5}$ , it is found that the scales of the thermal and mechanical 245heterogeneities generally tend to shorten with increasing age, although long-wavelength 246structures develop just after the beginning of the simulation ("A" in Fig. 3a, c) in spite of 247an initial temperature condition with a short-wavelength structure (Eq. (14)). When  $\Delta \eta_T$ 248is moderate (i.e.,  $\Delta \eta_T = 10^3$  and  $10^{3.5}$ ) it appears that the scales of the thermal and 249mechanical heterogeneities generally tend to shorten with increasing age before c. 2.0 Ga, 250but the degree-one structure begins to develop after 1.0 Ga (see "B" in Fig. 3e-h). 251

Even when  $\eta_0 = 10^{16}$  Pa s and  $k_0 = 36-200$  W m<sup>-1</sup> K<sup>-1</sup>, these conclusions remain 252essentially unchanged: the degree-one thermal/mechanical structure only appeared for a 253limited range of parameter values for lateral viscosity variations, i.e.,  $\Delta \eta_T = 10^4$  and  $10^{4.5}$ 254for the model with  $k_0 = 36$  W m<sup>-1</sup> K<sup>-1</sup> ("A" in Fig. 4) and  $\Delta \eta_T = 10^3$  and  $10^{3.5}$  for the 255model with  $k_0 = 200 \text{ W m}^{-1} \text{ K}^{-1}$  ("A" in Fig. 5). These results imply that the degree-one 256structure is found in the models with a wide range for the Rayleigh number, as also shown 257in mantle convection simulations (McNamara & Zhong 2005; Yoshida & Kageyama 2582006). 259

260 This degree-one convection pattern is similar to the familiar "sluggish-lid regime" or

261the "transitional regime" in thermal convection of the mantle that has already been found 262in numerical simulations of mantle convection (e.g., Solomatov 1995) (Fig. 6b). In this 263regime, the flow velocities of downwelling plumes from the sluggish-lid are large 264compared with the interior of the inner core, and the global flow pattern in the sphere 265corresponds to the temperature distribution. This is because temporal changes in thermal 266heterogeneity roughly correlate with those in mechanical heterogeneity, as shown in Fig. 3, 4 and 5. Also, once the degree-one structure is formed, the sluggish-lid regime 267268generates the largest magnitude of flow velocity and the most laterally heterogeneous velocity field in the inner convecting region (Fig. 6b and 7a) when compared with other 269270regimes such as the mobile-lid regime (Fig. 6a) and the stagnant-lid regime (Fig. 7b).

When  $\Delta \eta_T$  is 10<sup>4</sup> or larger, the scales of the thermal and mechanical heterogeneities under the stagnant-lid, whose thickness is approximately 100 km, are quite small (the dominant degrees are below 16) even after 4.5 Gyr (Fig. 3i–l). Although the relatively long-wavelength mode is intermittently dominant during the simulation even when  $\Delta \eta_T$ is quite large ("C" in Fig. 3k–l), it is immediately damped over a short time-scale of  $\leq$  c. 0.5 Gyr. As a result, the stagnant-lid regime is maintained for almost the entire simulation time.

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#### 279 4 Discussion

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281The degree-one structure of the inner core suggested by various seismological evidence may have been realized by an "exogenous factor" from outside the ICB, rather than by an 282"endogenic factor" due to the internal rheological heterogeneity. The exogenous origin 283284for the hemispherically asymmetric structure of the inner core and the related 285hemispherical difference in the degree of crystallization, i.e., freezing and melting, of the 286inner core materials is consistent with a previous suggestion that planetary-scale thermal 287coupling among the lower mantle, the outer core, and the inner core plays a primary role in the growth and evolution of the inner core (Aubert et al. 2008; Gubbins et al. 2011; 288289Tkalčić 2015). More recently, a new isotopic geochemical analysis study (Iwamori & 290 Nakamura 2015; Iwamori et al. 2015) revealed that such planetary-scale thermal coupling 291operates in the whole-Earth system through so-called "top-down hemispheric dynamics", 292in which the hemispheric supercontinent-ocean distribution at Earth's surface controls the 293thermal convection system in the whole Earth via the mantle, outer core, and inner core over the course of Earth's history (see Fig. 13 of Iwamori and Nakamura (2015)). 294

It should be noted that the style of convection envisioned in this study is only one form of degree-one convection. Another form involves melting and solidification processes at 297the ICB, as suggested by Alboussière et al. (2010) and Monnereau et al. (2010), who 298concluded that the inner core translates laterally as a rigid body and the return flow 299effectively occurs in the fluid outer core. Because the present study adopts impermeable boundary conditions at the ICB, this second form of degree-one convection is not 300 301 permitted. This scenario provides a strong alternate candidate for the form of degree-one 302convection through top-down hemispheric dynamics, if degree-one convection due to internal rheological heterogeneity is not possible. As another new idea, some authors 303 304 suggested that the hemispheric seismic heterogeneity in the inner core would be caused 305 by the magnetic field (Takehiro 2011; Lasbleis et al. 2015; Takehiro 2015). The effects 306 of magnetic field on the 3D numerical model of thermal convection in the growing inner 307 core should be investigated in future. As a start, we are developing a new 3-D numerical simulation code for multiple-layered thermal convection with a large viscosity contrast 308 309 among each layer, considering the methodology used in the present study and several 310 issues discussed in this Section. The relationship between the degree-one thermal 311convection and characteristic of geomagnetic field may be an interesting topic not only 312in earth science but also in computational fluid dynamics.

313 The age of the inner core is one of the most controversial issues in Earth Science. As mentioned in Section 2, the value of the total heat flow at the CMB is a key parameter 314 that controls the speed of growth of the inner core. A previous study (Cottaar & Buffett 315316 2012) has shown that the minimum heat flow for inner core convection is 4.1 TW, giving 317 a maximum inner core age of 1.93 Ga. Heat flow greater than 6.3 TW leads to the present convection regime, but the inner core age is then less than 1.26 Ga. Following their 318 analysis, if the total CMB heat flow is c. 5–15 TW (Lay et al., 2008 and references therein), 319 320 the age of inner core should be younger (i.e., less than 1 Ga). If the value of the total CMB 321heat flow is significantly larger than that used in this study, the possibility of the formation 322of degree-one structure by an endogenic factor due to rheological heterogeneity becomes less likely, because it takes c. 3.0 Gyr to form the degree-one structure when  $\eta_0 = 10^{17}$  Pa 323 s,  $k_0 = 36$  W m<sup>-1</sup> K<sup>-1</sup>, and  $\Delta \eta_T$  is 3.5 (Fig. 3g). At the very least, the conclusion that the 324325convective motion in the inner core is maintained even for the present Earth can be drawn 326 from the models studied here.

Almost all of recent paleomagnetic and geodynamics studies have suggested that the age of inner core would be less than 1.3 Ga to 2.0 Ga (Labrosse *et al.* 2001; Nimmo *et al.* 2004; Davies 2015; Nimmo 2015). According to a more recent paleogeomagnetic study, the inner core formed at ~1.5 Ga (Biggin *et al.* 2015). This "young" inner core age is consistent with the indirect evidences from seismology and mineral physics that the CMB heat flow is larger than previously thought (Hernlund *et al.* 2005; Lay *et al.* 2006; van der

333 Hilst et al. 2007). In the present numerical model, the CMB heat flow is assumed to be constant,  $F_m' = 2.56 \times 10^{12}$  W, based on the model constants used in Buffett *et al.* (1992) 334 and the initial radius of the inner core arbitrarily set. As stated in Section 2, this value 335would be a lower limit value for the present Earth considering an even relationship 336 337 between the total plume buoyancy flux observed at the Earth' surface and the inferred total CMB heat flow (e.g., Davies 1988; Sleep 1990; Davies & Richards 1992) and a 338 minimal power requirement for maintenance of the geodynamo (Buffett 2002). However, 339 340 the CMB heat flow in the past Earth would be lower than that in the present Earth, because 341the average mantle temperature may increase as the Earth older. More than that, there is 342a possibility that the CMB heat flow may have a significant variation throughout the 343 Earth's history. Although the implementation of time-dependent CMB heat flow is beyond the framework of the present simple numerical model, it might be a serious 344problem for the growing speed of the inner core. However, if the inner core grows faster 345 than the present model, the degree-one structure would only appeared for a further limited 346 347 range of  $\Delta \eta_T$  than the results presented in this paper. Therefore I believe that the 348 conclusion on the less possibility of an endogenic origin for the degree-one 349 thermal/mechanical structure of the inner core is justified.

350Seismic observations provide evidence for a seismic velocity discontinuity about 200 km below the ICB separating an isotropic layer in the uppermost inner core from an 351underlying anisotropic inner core (e.g., Song & Helmberger 1998). The results presented 352353 here may imply that this "inner core transition zone" represents the boundary between a sluggish, highly viscous, cold layer and an underlying hot convection region (Fig. 6b and 3547a). Sumita and Yoshida (2003) predicted that there may be a characteristic structure in 355 356 the topmost section of the inner core that is similar to the plate tectonic mechanism at 357 Earth's surface, which is explained by the existence of a crust-like, thin, low-degree 358partial-melting layer and an underlying asthenosphere-like, high-degree partial-melting 359 layer. The ICB is, by definition, at melting temperature, which means that it is possible 360 that the viscosity could be lowest at the top and gradually increase with depth into the 361 interior of the inner core. However, because the temperature across the inner core is never 362 very far from the melting temperature (Stacey & Davis 2008), depending in detail on the 363 contribution from light element impurities, viscosity variations in the underlying hot 364 convection region should be small.

Here I need to mention the lack and limitation of the present simple numerical model. First, the framework of present model is based on mantle convection simulations to deal with convection within the growing inner core. In contrast to mantle convection simulations, I used a time-dependent inner core radius to get dimensionless equations Eq. 369 (2–4) to account for the growing inner core. However, Eq. (2-4) is build based on an 370 Eulerian specification that is fixed on space. With increasing inner core radius, the grid 371 is actually slowly expanding through time. Thus, strictly speaking, mass, momentum, and 372 energy may be no longer conserved with this expending grid, although the growth rate 373 could be small enough to make it negligible. However, considering the significant 374 accumulating growth of the inner core during the whole simulation, improvement of the 375 present numerical model should be required.

- 376 Next, on the issue of secular cooling raised in Section 2, the dimensionless internal 377 heating production shown in Eq. (5) is incomplete because the inner core got cooling 378 down rapidly once the inner core gets started growing in the present model. The exact 379 way is for the secular cooling to be addressed as the boundary condition problem, not the internal heating production, because the initial temperature of inner core should be 380 381determined by the solidus temperature of iron-alloy, adiabatic heat flux across the ICB, the latent heat release and gravitational energy caused by light element release. 382383 Furthermore, the molten core might have initial accretion energy before the inner core 384started growing, but not in the inner core. Although the integration of these two serious 385problem into the present numerical model is numerically difficult and is beyond the 386 present study on the possibility of the formation of the degree-one structure with lateral viscosity variations, they should be considered in the future model. 387
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#### 389 **5 Conclusions**

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The systematic numerical simulations conducted in this study investigated the 391 392 possibility of the formation of a degree-one structure with lateral viscosity variations in 393 thermal convection within the age of Earth for a simulated inner core confined in a sphere. 394 The degree-one thermal/mechanical structure, however, only appeared for a limited range 395 of parameter values for lateral viscosity variations. Considering the uncertainties in the exact magnitude of lateral temperature variations, the rheology, and the composition of 396 397 Earth's inner core materials, the formation of a degree-one structure with lateral viscosity 398 heterogeneity under the limited geophysical conditions confirmed here would be 399 considered possible but not probable for the real Earth. If the value of the total CMB heat 400 flow is significantly larger than that used in this study, the possibility of the formation of 401 degree-one structure by an endogenic factor due to rheological heterogeneity becomes less likely, because it takes c. 3.0 Gyr to form the degree-one structure. Namely, the 402403 degree-one structure of the inner core may have been realized by an "exogenous factor" from outside the ICB, rather than by an "endogenic factor" due to the internal rheological 404

405 heterogeneity.

In future, the evolution of the inner core should be resolved by numerical simulations 406 407 of the whole-Earth thermal convection system because it is highly possible that the growth rate of the inner core is determined by heat flow at the CMB, which largely depends on 408 409 the behavior and style of mantle convection (Buffett et al. 1992; Sumita & Yoshida 2003). 410 The combined effects of the thermal and compositional buoyancies (Lythgoe et al. 2015) and the effects of non-uniform heat flux boundary condition at the ICB, rather than a fixed 411 412temperature condition on the style and regime of inner core convection, should also be 413 studied numerically in future.

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Symbol	Definition	Value	Unit	Refs.
$\alpha_0$	Thermal expansion	9.74×10 <sup>-6</sup>	K-1	a, b
	coefficient*			
$c_{p0}$	Specific heat at constant	$7.03 \times 10^2$	$J kg^{-1} K^{-1}$	a
	pressure*			
<b>g</b> 0	Gravitational acceleration*	4.4002	m s <sup>-2</sup>	a
$ ho_0$	Density*	$1.27636 \times 10^4$	kg m <sup>-3</sup>	a
$k_0$	Thermal conductivity*	36 or 200	$W m^{-1} K^{-1}$	a/c, d, e
<b>K</b> 0	Thermal diffusivity*	4.01×10 <sup>-6</sup>	$m^2 s^{-1}$	а
$\eta_0$	Viscosity*	$10^{17} \text{ or } 10^{16}$	Pa s	f
-	Temperatures at the inner	5000	K	a
	core boundary			
$\Delta T_0$	Characteristic temperature	30	K	a
	variation*			
$M_m$	Mass of the mantle	4.043×10 <sup>24</sup>	kg	g
$ au_c$	Formation age of the core	4.5	Ga	h
C <sub>pe0</sub>	Average specific heat at	$9.2 \times 10^2$	$J kg^{-1} K^{-1}$	g
	constant pressure for Earth			
$ ho_{e0}$	Average density of Earth	5.520×10 <sup>3</sup>	kg m <sup>-3</sup>	g
$T_m$	Mean mantle temperature	2250	K	g
$\overline{E_a}$	Activation energy for dry	540×10 <sup>3</sup>	J mol <sup>-1</sup>	g, i
	olivine			
λ	Average decay constant for	$2.77 \times 10^{-10}$	yr <sup>-1</sup>	g
	the mixture of radioactive			
	isotopes in the mantle			
	Present cooling rate of the	64.7	K Gyr <sup>-1</sup>	Eq. (7)
$\left. u \right _{t=t_0}$	mantle			
G	Gravitational constant	6.67384×10 <sup>-11</sup>	$m^3 kg^{-1} s^{-2}$	g
R	Gas constant	8.3144621	J mol <sup>-1</sup> K <sup>-1</sup>	g
$\partial T_s / \partial p$	Solidification profile	7×10 <sup>-9</sup>	K Pa <sup>-1</sup>	 j, k
$r_e$	Radius of Earth	6.371×10 <sup>6</sup>	m	g
r <sub>b</sub>	Radius of the outer core	3.48×10 <sup>6</sup>	m	1
$r_c(t)$	Radius of the inner core	Time-dep.	m	_
$r_c$	Radius of the present inner	1.2215×10 <sup>6</sup>	m	1
· .		1.2212/110		-

**Table 1.** Physical parameters for the simulation used in this study.

	core						
$r_c^i$	Initial radius of the inner core	$2.15 \times 10^4$	m	Х			
Ω	Model constant (see text)	1.13×10 <sup>13</sup>	W	Eq. (6)			
N	Model constant (see text)	$2.01 \times 10^{16}$	$J m^{-2}$	Eq. (11)			
$F_m$ '	Model constant (see text)	2.56×10 <sup>12</sup>	W	Eq. (12)			
Dimensionless parameters							
Ra(t)	Thermal Rayleigh number	Time-dep.	-	Eq. (5)			
H(t)	Internal heat-source number	Time-dep.	-	Eq. (5)			
$\zeta(t)$	Spherical-shell ratio number	Time-dep.	-	Eq. (5)			

Definition: "\*" indicates reference values for the inner core. References: a, Stacey and
Davis (2008); b, Vočadlo *et al.* (2003); c, de Koker *et al.* (2012); d, Pozzo *et al.* (2012);
e, Pozzo *et al.* (2014); f, Karato (2003); g, Turcotte and Schubert (2014); h, Lister and
Buffett (1998); i, Karato and Wu (1993); j, Verhoogen (1980); k, Buffett *et al.* (1992); l,

596 Dziewonski and Anderson (1981). "x" indicates arbitrary values (see text).

#### 600 **Figure captions**

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Figure 1. (a) Illustration for the numerical model of inner core convection. (b) Schematic 602 profile of the potential temperature in the cooling core of the Earth used in an analytical 603 604 model for solidification of the inner core (after Buffett et al., 1992). The solid line 605 represents the temperature in the upper part of the thick solid inner core, the liquid outer core, and the lower mantle at some instant. The thick dashed line represents the 606 607 subsequent evolution of temperature as heat is continuously extracted from the core. Heat fluxes  $f_m$  and  $f_i$  pertain to the core-mantle and inner-core boundaries (CMB and ICB), 608 respectively. The solidification temperature  $T_s(r_c)$  is defined at the ICB, and  $T_s(r, t)$  is the 609 610 temperature within the inner core.

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Figure 2. Time evolution of the convection pattern in the inner core for models with  $\eta_0$   $= 10^{17}$  Pa s,  $k_0 = 36$  W m<sup>-1</sup> K<sup>-1</sup>, and (a–d)  $\Delta \eta_T = 10^0$ , (e–h)  $\Delta \eta_T = 10^3$ , (i–l)  $\Delta \eta_T = 10^{3.5}$ , and (m–p)  $\Delta \eta_T = 10^4$ . Blue and copper isosurfaces indicate regions with lower and higher than average temperatures at each depth. (a–d and m–p) blue: –0.3 K; copper: +0.3 K. (e– f and i–k) blue: –0.6 K; copper: +0.6 K. (g, h, and l) blue: –1.5 K; copper: +1.5 K.

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Figure 3. Temporal changes in the heterogeneity of temperature and root-mean-square velocity fields in the inner core for the models with  $\eta_0 = 10^{17}$  Pa s,  $k_0 = 36$  W m<sup>-1</sup> K<sup>-1</sup>, and (a–b)  $\Delta \eta_T = 10^0$ , (c–d)  $\Delta \eta_T = 10^2$ , (e–f)  $\Delta \eta_T = 10^3$ , (g–h)  $\Delta \eta_T = 10^{3.5}$ , (i–j),  $\Delta \eta_T = 10^4$ , and (k–l)  $\Delta \eta_T = 10^5$ . The logarithmic power spectra are normalized by the maximum values at each elapsed time. The four black wedges on the panels (a), (e), (g), and (i) represent the ages that correspond to Fig. 2.

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Figure 4. Temporal changes in the heterogeneity of temperature and root-mean-square velocity fields in the inner core for the models with  $\eta_0 = 10^{16}$  Pa s,  $k_0 = 36$  W m<sup>-1</sup> K<sup>-1</sup>, and (a–b)  $\Delta \eta_T = 10^0$ , (c–d)  $\Delta \eta_T = 10^2$ , (e–f)  $\Delta \eta_T = 10^3$ , (g–h)  $\Delta \eta_T = 10^{3.5}$ , (i–j),  $\Delta \eta_T = 10^4$ , and (k–l)  $\Delta \eta_T = 10^5$ . The logarithmic power spectra are normalized by the maximum values at each elapsed time.

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Figure 5. Temporal changes in the heterogeneity of temperature and root-mean-square velocity fields in the inner core for the models with  $\eta_0 = 10^{16}$  Pa s,  $k_0 = 200$  W m<sup>-1</sup> K<sup>-1</sup>, and (a–b)  $\Delta \eta_T = 10^0$ , (c–d)  $\Delta \eta_T = 10^2$ , (e–f)  $\Delta \eta_T = 10^3$ , (g–h)  $\Delta \eta_T = 10^{3.5}$ , (i–j),  $\Delta \eta_T = 10^4$ , and (k–l)  $\Delta \eta_T = 10^5$ . The logarithmic power spectra are normalized by the maximum values at each elapsed time. 636

Figure 6. Cross sections of temperature and velocity fields for the models with  $\eta_0 = 10^{17}$ Pa s,  $k_0 = 36$  W m<sup>-1</sup> K<sup>-1</sup>, and (a)  $\Delta \eta_T = 10^0$  and (b)  $\Delta \eta_T = 10^3$  at 0.0 Ga. The top panels show the cross sections at the central depth of the inner core (i.e., radius of 610.8 km) and bottom panels show the cross sections cut along the great circles shown by dashed lines in the top panels. The contour interval in the temperature plots is 3 K. These figures correspond to (a) Fig. 2d and Fig. 3a, and (b) Fig. 2h and Fig. 3e.

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**Figure 7.** Cross sections of temperature and velocity fields for the models with  $\eta_0 = 10^{17}$ Pa s,  $k_0 = 36$  W m<sup>-1</sup> K<sup>-1</sup>, and (a)  $\Delta \eta_T = 10^{3.5}$  and (b)  $\Delta \eta_T = 10^4$  at 0.0 Ga. The top panels show the cross sections at the central depth of the inner core (i.e., radius of 610.8 km) and bottom panels show the cross sections cut along the great circles shown by dashed lines in the top panels. The contour interval in the temperature plots is 3 K. These figures correspond to (a) Fig. 21 and Fig. 3g, and (b) Fig. 2p and Fig. 3k.

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