Response to **Interactive comment on "A simple 3-D numerical model of thermal convection in Earth's growing inner core: on the possibility of the formation of the degree-one structure with lateral viscosity variations" by M. Yoshida**
Anonymous Referee #2

This manuscript is aimed to reveal the style of thermal convection in Earth's inner core with its thermal history that might have degree-one structure. There have been several studies on the style of inner core convection mainly done by R. Deguen and his colleagues that was called as 'Translational Regime'. The author discussed formation of degree-one convection caused by the rheological heterogeneity because a series of studies by Deguen and his colleagues could not address the lateral viscosity variation due to their numerical limitation. The lateral viscosity variation could be caused by the generation of hemispherical feature observed by various seismological analyses but the time-scale is too much long compared to the current understanding on the age of the inner core, which would be over 3.0 Gyrs. This means that the 'translational regime' found by Deguen and colleague would be still a great candidate for understanding the large-scale seismic heterogeneity and anisotropy observed in the inner core but this study would be somewhat interesting and important step for the community. However, unfortunately, I CANNOT recommend publishing this manuscript without huge amount of revisions experienced because I found a bunch of technical errors in the model assumptions and treatment of boundary conditions as well as missed citations on various important literatures and important discussion on inner core dynamics and its implication for seismic observations. If the author fix all of his mistakes in his model assumption and numerical procudure, cite various important literatures and add some important discussion on the age of the inner core, I would re-consider my recommendation. Again, currently, this manuscript SHOULD NOT be published in anywhere. Before discussing contexts of science in the original manuscript, author should address significant issues listed-up (also required with re-run for all cases as well as several additional runs will be required. In addition to that, both thermal and mechanical boundary conditions used in this study MUST be changed.) below because entire discussions in this manuscript would NOT be quite understandable and convinced for readers.

[Reply] I sincerely thank the reviewer for critical comments. I have carefully incorporated all the comments and suggestions into the revised manuscript attached below. The revised parts are highlighted by red. Although I regret that there are some misleading descriptions for readers in the original manuscript, this paper did not violate any of previous studies published so far. I have total respect for all of the previous studies on the core modeling. However, I would acknowledge sincerely the lack of explanation of our model and discussion with the previous studies. Thanks to the reviewer's thoughtful comments, I could find new literatures, expand discussion largely, and largely improve the manuscript, separating the previous Section 4 to new Sections 4 (Discussion) and 5 (Conclusions). However, the main conclusions was not changed from that in the original manuscript. Of course, I knew that there are many theoretical and numerical studies on core dynamics, but am only trying to present a simple numerical model and open the discussion. This is because I submitted the manuscript to Solid Earth, not to other journals. However, again, I would acknowledge the lack of required discussion in the original manuscript and I deeply regret this point and deeply acknowledge all of reviewer's comments.

I provide below my response to reviewer's comments.

1. I demand author to check effects of radioactive heating because the possibility of radioactive elements in the outer and inner core would be still under the debate in mineral physics, geodynamics, geomagnetism and seismological communities.

[Reply] Thank you very much for your comment. Because the radioactive potassium may be major heat source in the inner and outer core, and there is a large possibility that the role of potassium is important in determining the history of growth of inner core and geodynamo power (e.g., Nimmo et al. 2004). However, I ignore explicitly radioactive heating in the present model because the amount of radioactive potassium in the inner and outer core would be still under the debate in mineral physics, geodynamics, geomagnetism and seismological communities, and instead consider primordial heating in the heat source term of the conservation equation of energy. I have explained explicitly this point in Section 2.

2. Regarding the treatment of convective vigor and secular cooling (eq. (5) and eq. (6)), I did NOT quite understand why the author used such formulations. For example, the heat capacity was appeared in the Rayleigh number, which was quite odd to me. Why was the heat capacity appeared in the Ra? Please justify how to formulate the Rayleigh number.

[Reply] I opened eq. 5 to enable readers to understand a relationship of (thermal diffusivity) = (thermal conductivity) / (density x heat capacity)

3. I did not get it on the non-dimesionalization done by author's formulation. I did not understand the temperature difference without bottom boundary as the convection was occurred in the inner core. Since the temperature at the center of inner core could not be determined uniquely due to its singularity, using a temperature difference across the convective region would not be correct. The only correct way for determining the temperature difference across the inner core should be scaled by the amount of heat source. The author MUST correct it.

[Reply] Thank you for your comment. Of course, there is another way for normalizing the temperature the inner core by the amount of heat production. However, as stated in reply to reviewers comment #1, I ignore explicitly radioactive heating in the present model. This is because the temperature the inner core is normalized by the characteristic temperature variation, although there is another way for normalizing the temperature the inner core by the amount of heat production. I have explicitly mentioned this point in Section 2. I agree that the word “reference temperature difference” misleads the readers, so, it is replaced by “characteristic temperature variation”.

4. On addressing the secular cooling, the non-dimensional heating shown in Eq. (5) was somewhat odd to me as well. Yes, it should be zero without the inner core BUT the inner core got cooling down rapidly (11.3 TW!) once the inner core gets started growing. I am not quite sure if the inner core might have the primordial heating or not. I guess that it would be probably NOT. The correct way is for secular cooling to be addressed as the boundary condition NOT the internal heat source in the convection system. The initial temperature of inner core should be determined by the solidus temperature of iron-alloy and adiabatic heat flux across the ICB plus the latent heat release and gravitational energy caused by light element release. I would understand that the molten core might have initial accretion energy before inner core started growing but, again, NOT in the inner core. Therefore, this assumption is completely WRONG. The author MUST fix it then re-run all cases.

[Reply] Thank you very much for your thoughtful comment. I absolutely agree your comment. In section 4, I have explicitly mentioned the lack and limitation of the present simple numerical model. I absolutely agree your comment and expanded the discussion. As reviewer pointed out, the dimensionless internal heating production shown in Eq. (5) is incomplete because the inner core got cooling down rapidly once the inner core gets started growing in the present model. The exact way is for the secular cooling to be addressed as the boundary condition problem, not the internal heating production, because the initial temperature of inner core should be determined by the solidus temperature of iron-alloy, adiabatic heat flux across the ICB, the latent heat release and gravitational energy caused by light element release. Furthermore, the molten core might have initial accretion energy before the inner core started growing, but not in the inner core. Although the integration of these two serious problem into the present numerical model is numerically difficult and is beyond the present study on the possibility of the formation of the degree-one structure with lateral viscosity variations, they should be considered in the future model. In any case, I have explicitly discussed these point in Section 4.

5. Moreover, related with comment #3 listed here, I demand author to check if singularity of center of the inner core (Earth) could avoid correctly because the spectral method approach done by Deguen and his colleague and Takehiro [2011; 2015] could avoid the singularity at the center of inner core with special technique. In other words, I demand author to check the validity of model comparing with results of Deguen and his colleague if the author’s way to avoid the singularity at the center of inner core would be robust or not. I do not really think that the initial state of Earth’s core should not have such a small particle as the author stated before inner core got started growing. As far as I understood, a numerical method used by the author (finite volume/difference scheme) that could avoid the singularity at the center of sphere should be found in various

literatures [e.g. a series of papers on core formation by Taras Gerya's group]. Please check them out and ask author to avoid the singularity completely. Otherwise, numerical results shown in this manuscript would NOT be quite RELIABLE for the community.

[Reply] About the extremely small virtual-sphere, I regret that the model description in the original manuscript misleads the readers. This is for the purpose of the computational convenience characteristic of the Yin-Yang grid, and this setup does not mean the existence of the real singularity that violates the mass and heat transport near the center of the model. I have explicitly mentioned this point in Section 2. Yes, I know that there are other theoretical models that avoid the mathematical singularity in the Earth's center. Just in case, I read the papers below by Gerya's and Takehiro's groups that reviewer kindly listed. A largest difference in the methodologies used in our 3D spherical code and Geryas' 2-D spherical code is that they used finite difference method based on a fully staggered rectangular Eulerian grid with a Lagrangian marker-in-cell technique for solving the momentum, continuity and temperature equations, whereas we used finite volume method based on a fully staggered spherical Eulerian gird for solving these basic equations by a primary variable method (Yoshida and Kageyama, 2004 in GRL; 2006 in JGR, Yoshida, 2008 in G-cubed).

- Gerya, T.V. & Yuen, D.A., 2007. Robust characteristics method for modelling multiphase visco-elasto-plastic thermo-mechanical problems, Phys. Earth Planet. Int., 163, 1-4, 83-105.
- Golabek, G.J., Gerya, T.V., Kaus, B.J.P., Ziethe, R. & Tackley, P.J., 2011. Rheological controls on the terrestrial core formation mechanism, Geochem. Geophys. Geosyst., 10, 11, Q1107.
- Deguen, R., 2013. Thermal convection in a spherical shell with melting/freezing at either or both of its boundaries, J. Earth Sci., 24, 5, 669-682.
- Deguen, R., Alboussière, T. & Cardin, P., 2013. Thermal convection in Earth's inner core with phase change at its boundary, Geophys. J. Int., 194, 3, 1310-1334.
- Deguen, R. & Cardin, P., 2011. Thermochemical convection in Earth's inner core, Geophys. J. Int., 187, 3, 1101-1118.
- Lin, J.-R., Gerya, T.V., Tackley, P.J., Yuen, D.A. & Golabek, G.J., 2009. Numerical modeling of protocore destabilization during planetary accretion: Methodology and results, Icarus, 204, 2, 732-748.
- Takehiro, S., 2011. Fluid motions induced by horizontally heterogeneous Joule heating in the Earth's inner core, Phys. Earth Planet. Int., 184, 3-4, 131-142.
- Takehiro, S., 2015. Influence of surface displacement on solid state flow induced by horizontally heterogeneous Joule heating in the inner core of the Earth, Phys. Earth Planet. Int., 241, 15-20.

6. Besides of comments on technical issues mentioned above, the first issue was how to address rheological properties of inner core material, i.e, iron-nickel alloy. Author used the simple temperature-dependent viscosity based on numerical mantle convection simulation but not quite sure if this type of rheological properties could be applicable for the inner core material or not. Please give

appropriate references in the manuscript with some justification why the inner core rheology could be similar to that of the mantle one.

[Reply] I could not find appropriate references for rheology of the inner core material, i.e, iron-nickel alloy, except Karato's textbooks [2003; 2008], but The essence of the viscosity equation in Eq. (15) is just a simple relationship between the temperature and viscosity under the assumption that non-Newtonian rheology does not work in the iron-nickel alloy unlike in the rocky mantle I have explicitly mentioned this point in Section 2.

7. The second issue in this manuscript was a treatment of inner core growth model – Author assumed that the energy releases caused by latent heat and gravitational energy were IGNORED because they were secondary effects (see eq. (8)). To understand inner core growth itself, the energy balance should be SATISFIED even if they were secondary effects. Why did author ignore those two important energy resources in the core evolution system? I could not find it in the manuscript. Thus, I demand author to give sufficient justifications (explanations) on reason that those could be ignored because I could not get explanations or justifications in the manuscript.

[Reply] Yes, this issue is raised by another reviewer, who gave me appropriate references (below) required for discussion. First, "the heat source associated with solidification of the inner core are ignored because these effects play a secondary role in the growth of the inner core" in the original manuscript have been removed because there is a possibility to mislead the readers. Next, I have modified the relevant sentences in Section 2 to clarify that most of other studies kept these effect. For example, the modeling studies of the core evolution by Gubbins et al. (2003) and Nimmo et al. (2004) revealed that the latent heat plus gravitational energy is larger than the specific heat for the present Earth, and once the inner core starts to freeze, the core temperature decreases significantly with time, which has probably influence on the growth speed of the inner core and the generation and maintenance of geodynamo. The added references for discussion are as follows:

-Gubbins, D., Alfè, D., Masters, G., Price, G.D. & Gillan, M.J., 2003. Can the Earth's dynamo run on heat alone?, *Geophys. J. Int.,* **155**, 2, 609-622, doi:10.1046/j.1365-246X.2003.02064.x.
- Nimmo, F., Price, G.D., Brodholt, J. & Gubbins, D., 2004. The influence of potassium on core and geodynamo evolution, *Geophys. J. Int.,* **156**, 2, 363-376, doi:10.1111/j.1365-246X.2003.02157.x.

8. Third issue is about the comparison between previous studies finding the 'translational regime' and author's model result. L.5 to L.13 of page 3228 was absolutely unclear to me. As far as I understood, the degree one convection generated from lateral viscosity variation such like 'Sluggish Lid Regime' would be alternative idea compared to the 'translational regime' (=this second form). As discussed the following paragraph (pointed out the next comments), the

formation time-scale of degree one convection would be _3.0 Gyr from author's model simulations, which would be much longer timescale than the age of the inner core, thus translational regime would be STILL one great candidate for understanding to find out the large-scale seismic heterogeneity (anisotropy) observed in the inner core. Such a logical flow seems to be an ethic of self-denial, which means that this type of simulation of convection in the inner core by the author would NOT be worth investigating. To get more worth doing, author should change his boundary conditions on both mechanical and thermal (similar boundary condition to other groups' studies instead of impermeable boundary condition) as well as laterally heterogeneous heat flux condition at ICB should be applied if mantle heat flux pattern could be transparent to the ICB suggested from 'top-down hemispherical dynamics'. If author could do these stuffs, author's results might be more comparable with other investigations to check if the 'translational regime' would be a strong candidate or not. Again, the author should avoid the singularity in his model at the center of the inner core.

[Reply] Again, thank you for your comment. On the lack and limitation of the present simple numerical model, in particular, boundary condition problem, I have further expanded the discussion in Section 4. Again, on the singularity problem, no worries as stated above. Next, about the sluggish lid regime and translational regime, the degree-one thermal structure of mantle convection that lies between the mobile-lid regime and the stagnant-lid regime. In 3-D spherical convection, the sluggish lid regime are replaced by degree-two or degree-one convection with increasing degree of the temperature-dependence of viscosity (e.g., Yoshida and Kageyama, 2004 in GRL, 2006 in JGR; McNamara and Zhong, 2005 in GRL). In this study, I show that the degree-one thermal/mechanical structure is only appeared for a limited range of parameter values for lateral viscosity variations.
If the value of the total CMB heat flow is significantly larger than that used in this study, the possibility of the formation of degree-one structure by an endogenic factor due to rheological heterogeneity becomes less likely, because it takes c. 3.0 Gyr to form the degree-one structure. I have emphasized this point in new Section 5.

9. Forth issue is about the age of the inner core. Looking at Figure 3, the inner core has already grown 714.3 km at t = 3.0 Ga. It seems for author to integrate 0 to 4.6 Gyr in his model. This is very odd because, as quoted by the author, the age of inner core would be less than 1.3 Ga to 2.0 Ga with referring some literatures. Why did the author decide to integrate over 4.6 Gyrs? To be more comparable for other studies, again, author should improve his own boundary condition. In addition to that comment, author ignored a bunch of literatures by S. Labrosse and F. Nimmo as well as by C. Davis. I do think that their accomplishments would be quite important for both evolution and dynamics of the inner core. To behave a fairplay in the scientific research, the author should not ignore those important literatures to cite in his manuscript. They are really IMPORTANT for discussing the thermal history of Earth's core and its influence to the inner core dynamics. Please cite them and add appropriate discussion.

[Reply] I totally agree your comment and regret the missing references on the young inner core age estimated from paleomagnetic and geodynamics studies. In Section4, I have expanded the discussion about this point and added new references below:

- Biggin, A.J., Piispa, E.J., Pesonen, L.J., Holme, R., Paterson, G.A., Veikkolainen, T. & Tauxe, L., 2015. Palaeomagnetic field intensity variations suggest Mesoproterozoic inner-core nucleation, Nature, 526, 245-248.
- Davies, C.J., 2015. Cooling history of Earth's core with high thermal conductivity, Phys. Earth Planet. Int., 247, 65-79.
- Labrosse, S., Poirier, J.-P. & Mouël, J.-L.L., 2001. The age of the inner core, Earth Planet. Sci. Lett., 190, 3-4, 111–123.
- Nimmo, F., Price, G.D., Brodholt, J. & Gubbins, D., 2004. The influence of potassium on core and geodynamo evolution, Geophys. J. Int., 156, 2, 363-376.

10. Additional idea for understanding the seismic heterogeneity in the inner core would be caused by the magnetic field – Please check literatures by Takehiro [2011; 2015 both in PEPI] and Lasbreis et al. [2015 in GJI] and author should discuss effects of magnetic field with his simulation results without the magnetic field effects. Here I listed 10 significant and critical points that author should address getting the revised manuscript. Since I think that a bunch of additional works and model validations should be done by author to get the revised version of manuscript and it would take a long time, I would not give any detailed comments here (line-by-line comments). Nevertheless, those comments would be useful for getting revised manuscript done by the author.

[Reply] Thank you very much for your comment. As pointed out by the reviewer, some authors suggested that the hemispheric seismic heterogeneity in the inner core would be caused by the magnetic field (Takehiro 2011; Lasbleis et al. 2015; Takehiro 2015). The effects of magnetic field on the 3D numerical model of thermal convection in the growing inner core should be investigated in future. As a start, we are developing a new 3-D numerical simulation code for multiple-layered thermal convection with a large viscosity contrast among each layer, considering the methodology used in the present study and several issues discussed in this Section. The relationship between the degree-one thermal convection and characteristic of geomagnetic field may be an interesting topic not only in earth science but also in computational fluid dynamics. I have expanded the discussion in Section 4 and, following your suggestion, I have cited the following papers:

- Lasbleis, M., Deguen, R., Cardin, P. & Labrosse, S., 2015. Earth's inner core dynamics induced by the Lorentz force, Geophys. J. Int., 202, 1, 548-563.
- Takehiro, S., 2011. Fluid motions induced by horizontally heterogeneous Joule heating in the Earth's inner core, Phys. Earth Planet. Int., 184, 3-4, 131-142.
- Takehiro, S., 2015. Influence of surface displacement on solid state flow induced by horizontally heterogeneous Joule heating in the inner core of the Earth, Phys. Earth Planet. Int., 241, 15-20.

[Reply] I again deeply appreciate you for the careful reading and significant improvement of this paper. I will accept all of the valuable reviewer's comments with sincerely and would like to use these comments to improve the numerical model in future.

Masaki Yoshida

A simple 3D numerical model of thermal convection in Earth's growing inner core: On the possibility of the formation of the degree-one structure with lateral viscosity variations

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Abstract

An east–west hemispherically asymmetric structure for Earth's inner core has been suggested by various seismological evidence, but its origin is not clearly understood. Here, to investigate the possibility of an "endogenic origin" for the degree-one thermal/mechanical structure of the inner core, I performed new numerical simulations of thermal convection in the growing inner core. A setup value that controls the viscosity contrast between the inner core boundary and the interior of the inner core, $\Delta\eta_T$, was taken as a free parameter. Results show that the degree-one structure only appeared for a limited range of $\Delta\eta_T$; such a scenario may be possible but is not considered probable for the real Earth. The degree-one structure may have been realized by an "exogenous factor" due to the planetary-scale thermal coupling among the lower mantle, the outer core, and the inner core, not by an endogenic factor due to the internal rheological heterogeneity.

Key words: numerical simulation; inner core; thermal convection; degree-one structure; east–west hemispherical structure; viscosity variation

1 Introduction

After the segregation of the rocky mantle and molten iron core in the early stage of

Earth's formation (e.g., Stevenson 1981), the inner core was formed by gradual solidification of the molten iron core and the size increased with age (Jacobs 1953; Buffett *et al.* 1992) (for example, see reviews by Buffett (2000) and Sumita and Yoshida (2003)). The solidification of the solid core could affect the vigor of outer-core convection owing to the release of latent heat and the passage of light elements toward the liquid outer core (e.g., Sumita & Yoshida 2003). The resulting growth of the inner core may have changed the convection style in the outer core and affected the intensity of Earth's magnetic field throughout Earth's history.

Although the structure of the present inner core cannot be inferred from surface geophysical observations, various seismological evidence suggests that the inner core has an east–west, hemispherically asymmetric structure in terms of seismic velocity, anisotropy, and attenuation (Tanaka & Hamaguchi 1997; Creager 1999; Niu & Wen 2001; Cao & Romanowicz 2004; Deuss *et al.* 2010; Irving & Deuss 2011; Waszek *et al.* 2011; Lythgoe *et al.* 2014) (see also a recent review by Tkalčić (2015)).

Previous numerical simulations of Earth's mantle convection clarified that for convecting rocky materials confined in a spherical shell, the spherical harmonic degree-one structure was observed for a relatively wide range of the parameter that controls the lateral viscosity variations due to temperature variations (McNamara & Zhong 2005; Yoshida & Kageyama 2006). This is because when the temperature-dependence of viscosity is moderate, the highly viscous lid that develops at the surface of the convecting mantle has the longest-wavelength scale, and the dynamic instability at the bottom of the lid concentrates in one area. This scenario characterizes the degree-one thermal structure of mantle convection that lies between the "mobile-lid regime" with weakly temperature-dependent viscosity and the "stagnant-lid regime" with strongly temperature-dependent viscosity (Yoshida & Kageyama 2006).

It is possible that the degree-one seismic structure in the present inner core originated from lateral temperature variations, which in turn originated from lateral viscosity variations. Even given the uncertainties in the rheological properties and composition of the inner core materials, lateral viscosity variation offers considerable potential as an "endogenic factor" that may explain the formation of the degree-one seismic structure. It is therefore worth examining whether a degree-one thermal/mechanical structure generated from the lateral viscosity variations can be realized for solid materials confined in a sphere. This topic had not been investigated in previous numerical simulation models of inner core convection (Deguen & Cardin 2011; Cottaar & Buffett 2012; Deguen 2013; Deguen *et al.* 2013).

In this study, to explore the time-dependent behavior of the convection regime in

Earth's growing inner core and the possibility of generating the degree-one thermal/mechanical structure from the internal rheological heterogeneity, a new simple numerical model of the growing inner core is constructed and a series of numerical simulations of thermal convection are performed, assuming that the solidification of the liquid core started at 4.5 Ga and that the radius of the inner core gradually increased with the square root of age.

2 Model

Convection in the inner core is computed numerically using a staggered grid-based, finite-volume code, ConvGS (e.g., Yoshida 2008). The material of the inner core is modeled as a Boussinesq fluid with an infinite Prandtl number confined in a sphere and modeled in spherical coordinates (r, θ, φ). Impermeable, shear-stress-free, isothermal conditions are imposed on the inner core boundary (ICB) with a fixed dimensionless radius of $r_c' = 1$ (Fig. 1a). The driving force of convection is primordial heat alone because the radiogenic heat production is negligibly small in the inner core (e.g., Karato 2003) (see also below). The number of computational grids is taken as 64 (r) × 64 (θ) × 192 (ϕ) × 2 (for Yin and Yang grids) (Yoshida & Kageyama 2004). To avoid the mathematical singularity at the Earth's center, an extremely small virtual-sphere with a dimensionless radius of $r_\delta' = 10^{-6}$ is imposed at the center of the model sphere. For the purpose of computational convenience, impermeable, shear-stress-free, adiabatic conditions are imposed on the top boundary of the small sphere. However, this setup does not mean the existence of the real singularity that violates the mass and heat transport near the center of the model sphere.

Following standard techniques for mantle convection simulations (e.g., Schubert *et al.* 2001), the length L, time t, velocity vector $\boldsymbol{v}$, stress tensor (or pressure) $\boldsymbol{\sigma}$, viscosity η, and temperature T are non-dimensionalized as follows:

$$L = r_c(t)L', \; t = \frac{r_c(t)^2}{\kappa_0}t', \; \boldsymbol{v} = \frac{\kappa_0}{r_c(t)}\boldsymbol{v}', \; \boldsymbol{\sigma} = \frac{\kappa_0\eta_0}{r_c(t)^2}\boldsymbol{\sigma}', \; \eta = \eta_0\eta', \; T = \Delta T_0 \cdot T' \tag{1}$$

where $r_c(t)$ denotes the time-dependence of the radius of the inner core, which depends on time (age); κ_0 denotes the reference thermal diffusivity; η_0, the reference viscosity; ΔT_0, the characteristic temperature variation; and the subscript "0" refers the reference values for the inner core (Table 1). In these equations, symbols with primes represent dimensionless quantities.

Using these dimensionless factors, the dimensionless conservation equations for mass, momentum, and energy, which govern inner core convection, are expressed as

$$\nabla \cdot \boldsymbol{v} = 0, \tag{2}$$

$$-\nabla p + \nabla \cdot \boldsymbol{\tau} + Ra(t)\zeta(t)^{-3} T \boldsymbol{e}_r = 0, \quad \boldsymbol{\tau} = \eta\left(\nabla \boldsymbol{v} + \nabla \boldsymbol{v}^{tr}\right) \tag{3}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T = \nabla^2 T + H(t)\zeta(t)^{-2}, \tag{4}$$

respectively, where p represents the pressure; $\boldsymbol{\tau}$, the deviatoric stress tensor; and $\boldsymbol{e}_r$, the unit vector in the radial direction. Primes representing dimensionless quantities are omitted in Eq. (2)–(4).

In the numerical simulation for this study, instead of fixing the dimensionless radius of the ICB, the radius of the inner core in the thermal Rayleigh number, Ra, the internal heat-source number, H, and the “spherical-shell ratio”, ζ, depend on age. They are given by

$$\begin{aligned} Ra(t) &\equiv \frac{\rho_0 \alpha_0 \Delta T_0 g_0 \left[r_c(t) - r_\delta\right]^3}{\kappa_0 \eta_0} = \frac{\rho_0^2 \alpha_0 \Delta T_0 g_0 c_{p0} \left[r_c(t) - r_\delta\right]^3}{k_0 \eta_0}, \\ H(t) &\equiv \frac{\Omega \left[r_c(t) - r_\delta\right]^2}{M_m \Delta T_0 \kappa_0 c_{p0}} = \frac{\Omega \rho_0 \left[r_c(t) - r_\delta\right]^2}{M_m \Delta T_0 k_0}, \\ \zeta(t) &\equiv \frac{r_c(t) - r_\delta}{r_c(t)}, \end{aligned} \tag{5}$$

where ρ_0 is the reference density; α_0, the reference thermal expansion coefficient; ΔT_0, the reference temperature difference across the inner core; g_0, the reference gravitational acceleration; c_{p0}, the reference specific heat at constant pressure; k_0, the reference thermal conductivity; and M_m, the mass of the mantle (Table 1). Note that I ignore the secular cooling of the whole inner core, because the cooling rate and the resulting time change of the temperature difference across the inner core can not be estimated a priori. However, the absolute time change in the temperature difference across the inner core should be small throughout the inner core formation, and the effect of the time change on the magnitude of thermal Rayleigh number (Eq. 5) is negligibly small compared to other physical values. Therefore, I consider that this assumption would be justified at least in the framework of this numerical model (see also discussion in Section 5). Furthermore, although the gravity acceleration is almost linearly increasing from 0 at the Earth’s centre to ~4.4 m s^{-2} at the present ICB (Dziewonski & Anderson 1981), I treated it as a constant with radius for the simplicity.

Thermal conductivity has received a lot of attention in recent mineral physics studies, and the main finding is that it may be much larger than the value of 36 W m^{-1} K^{-1} from Stacey and Davis (2008), i.e., 100–200 W m^{-1} K^{-1} (de Koker *et al.* 2012; Pozzo *et al.*

2012, 2014). Therefore, two end-member models with k_0 = 36 and 200 W m^{-1} K^{-1} are investigated in this study.

Because the radioactive potassium may be major heat source in the inner and outer core, and there is a large possibility that the role of potassium is important in determining the history of growth of inner core and geodynamo power (e.g., Nimmo *et al.* 2004). However, I ignore explicitly radioactive heating in the present model because the amount of radioactive potassium in the inner and outer core would be still under the debate in mineral physics, geodynamics, geomagnetism and seismological communities, and instead consider primordial heating in the heat source term of the conservation equation of energy. This is because the temperature the inner core is normalized by the characteristic temperature variation, although there is another way for normalizing the temperature the inner core by the amount of heat production.

The amount of approximated primordial heat that has existed since Earth's formation, Ω, is taken as a free parameter estimated from the heat released by mantle cooling in the present Earth (Turcotte & Schubert 2014):

$$\Omega = -\frac{4}{3}\pi r_e^3 \rho_e c_{pe0} \left.\frac{dT}{dt}\right|_{t=t_0}, \tag{6}$$

where r_e is the radius of Earth; ρ_e, the average density of Earth; and c_{pe0}, the average specific heat at constant pressure for Earth. Here $\left. dT/dt \right|_{t=t_0}$ is the present cooling rate of the mantle (Turcotte & Schubert 2014):

$$\left.\frac{dT}{dt}\right|_{t=t_0} = -3\lambda\left(\frac{RT_m^2}{E_a}\right), \tag{7}$$

where λ is the average radiogenic decay constant in the mantle; R, the gas constant, T_m, the mean mantle temperature; and E_a, the activation energy of dry olivine. Using the values in Table 1, Ω is 11.3 TW, which is a reasonable value compared with the total heat release by mantle cooling estimated from a global heat-flow balance (Lay *et al.* 2008).

Following the work of Buffett *et al.* (1992), who studied an analytical model for solidification of the inner core, the global heat balance of the outer core is

$$\frac{4\pi}{3}\left(r_b^3 - r_c^3\right)\rho c_p \frac{dT_s(r)}{dt} = 4\pi r_c^2 f_i(t) - 4\pi r_b^2 f_m(t), \tag{8}$$

where $T_s(r)$ is the "solidification temperature", and $f_i(t)$ ($\equiv F_i(t)/(4\pi r_c^2)$) and $f_m(t)$ ($\equiv F_m(t)/(4\pi r_b^2)$) are the heat fluxes across the ICB and the core-mantle boundary (CMB), respectively (Fig. 1b). The potential temperature in the well-mixed liquid outer core is assumed to be spatially uniform and slowly decreases with time, and the ICB is assumed to be in thermodynamic equilibrium with the surrounding liquid. Under these assumptions, the temperature through the outer core is uniquely defined by the solidification temperature, $T_s(r)$, as a function of pressure or depth (see the explanation in Buffett *et al.* (1992) for details). Here, the heat sources associated with solidification of the inner core (i.e., the release of latent heat and gravitational energy) are ignored. Although Buffett *et al.* (1992) implicitly evaluated that these effects play a secondary role in the growth of the inner core, most of other studies kept these effect. For example, the modeling studies of the core evolution by Gubbins *et al.* (2003) and Nimmo *et al.* (2004) revealed that the latent heat plus gravitational energy is larger than the specific heat for the present Earth, and once the inner core starts to freeze, the core temperature decreases significantly with time, which has probably influence on the growth speed of the inner core and the generation and maintenance of geodynamo.

According to Eq. (4) in Buffett *et al.* (1992), the radial dependence of T_s is expressed as

$$T_s(r) = T_s(0) - \frac{2\pi}{3} G\rho_0^2 r^2 \frac{\partial T_s}{\partial p}, \quad (9)$$

where G is the gravitational constant, and $\partial T_s/\partial p$ is the solidification profile (Table 1).

Substituting Eq. (9) into Eq. (8), an expression for the radius of the inner core is obtained:

$$r_c(t) = r_b \left[\frac{1}{4\pi r_b^2 N} \int_0^t F_m(t)dt \right]^{\frac{1}{2}}, \quad (10)$$

where the model constant N is expressed as

$$N = \frac{2\pi}{9} r_b^3 c_{p0} \rho_0^2 G \frac{\partial T_s}{\partial p}. \quad (11)$$

Assuming that the heat flux across the CMB is constant throughout Earth's history (Buffett *et al.* 1992), a model constant, F_m', is obtained:

$$F_m' = \frac{4\pi N}{t} \left(r_c - r_c^i \right)^2, \quad (12)$$

where r_c^i is an arbitrary value that represents the initial radius of the inner core at the beginning of the simulation. When the age of Earth's core is assumed to be 4.5 Ga (Lister & Buffett 1998), the solidification of the inner core is assumed to have begun at this age, and r_c^i is taken to be 21.5 km so that r_c–r_c^i is exactly 1200 km, F_m' is 2.56×10^{12} W, which would be a lower limit value for the real Earth considering an even relationship between the total plume buoyancy flux observed at the Earth' surface and the inferred total CMB heat flow (e.g., Davies 1988; Sleep 1990; Davies & Richards 1992). Eq. (12) indicates that the relationship between the total CMB heat flow and the radius of the growing inner core is $F_m' \propto r_c^2$, which means that the radius of the inner core is proportional to the square root of F_m'. It should be noted that there is a trade-off between the choices of F_m' and r_c^i, which are critical for identifying the age of the present-day inner core in the real Earth. In the present model, I set r_c^i to a significantly small value to see the behavior of inner core convection over the longest geological time, i.e., 4.5 Gyr (see Section 4 for discussion).

Finally, the time-dependent radius of the growing inner core used in the present simulation is expressed as

$$r_c(t) = \left(\frac{F_m'}{4\pi N}\bar{t}\right)^{\frac{1}{2}} + r_c^i, \tag{13}$$

where the dimensional time is scaled as $\bar{t} = \left(\bar{r}_c(t)^2 / \kappa_0\right)\cdot t'$ using the radius of the inner core at the previous time step of the simulation, $\bar{r}_c(t)$. Eq. (13) means that r_c increases with the square root of age, and eventually reaches r_c = 1221.5 km after 4.5 Gyr, which matches the present radius of Earth's inner core.

At the beginning of the simulation, the initial condition for dimensionless temperature with significant small-scale lateral perturbations is given as

$$T'(r,\theta,\phi) = 0.5 + \omega\cdot Y_{34}^{17}(\theta,\phi)\cdot\sin\left[\pi\frac{r_1' - r'}{r_1'}\right], \tag{14}$$

where $Y_\ell^m(\theta, \phi)$ is the fully normalized spherical harmonic function of degree ℓ and order m and ω (=0.1) is the amplitude of perturbation.

The viscosity of the inner core materials, η_T', in this model depends on temperature according to a dimensionless formulation (Yoshida 2014):

$$\eta_T' = \eta_0'\cdot\exp\left[2T_{ave}'(t)\left(\frac{E'}{T' + T_{ave}'(t)} - \frac{E'}{2T_{ave}'(t)}\right)\right], \tag{15}$$

where $T_{ave}'(t)$ is the dimensionless average temperature of the entire sphere at each time step. A model parameter, E', controls the viscosity contrast between the ICB with T' = 0 and the interior of the inner core. In the present model, E' = ln($\Delta\eta_T$) varies from ln(10^0) = 1 (i.e., no laterally variable viscosity) to ln(10^5) = 11.51. The essence of the viscosity equation in Eq. (15) is just a simple relationship between the temperature and viscosity under the assumption that non-Newtonian rheology does not work in the iron-nickel alloy unlike in the rocky mantle (e.g., Karato 2003, 2008).

3 Results

Fig. 2 shows the time evolution of the convection pattern in the inner core for models with $\eta_0 = 10^{17}$ Pa s, $k_0 = 36$ W m^{-1} K^{-1}, and with $\Delta\eta_T = 10^0$ (Fig. 2a–d), $\Delta\eta_T = 10^3$ (Fig. 2e–h), $\Delta\eta_T = 10^{3.5}$ (Fig. 2i–l), and $\Delta\eta_T = 10^4$ (Fig. 2m–p). When $\Delta\eta_T = 10^0$ (i.e., the viscosity of the inner core is homogeneous) the convection pattern kept a short-wavelength structure over almost all of the simulation time and the "present" inner core at 0.0 Ga has numerous downwellings uniformly distributed in the sphere (Fig. 2d). On the other hand, when $\Delta\eta_T$ is large enough to make the surface thermal boundary layer stagnant ($\Delta\eta_T \geq 10^4$), the convection pattern tends towards a short-wavelength structure with age, and secondary cold plumes from the bottom of the highly viscous lid are evenly distributed in the sphere (Fig. 2p). A remarkable change in the convection pattern is found for moderate values of $\Delta\eta_T$: when $\Delta\eta_T = 10^3$ and $10^{3.5}$, the convection patterns shift towards a long-wavelength structure with increasing age, and eventually the longest-wavelength thermal structures develop for the inner core at 0.0 Ga (Fig. 2h, 2l)

To quantitatively assess the variations in thermal and mechanical heterogeneities in the inner core with time, Fig. 3 shows the power spectra of the temperature and root-mean-square velocity fields throughout the modelled inner core at each time step. On the other hand, when $\Delta\eta_T$ is $\leq 10^{2.5}$, it is found that the scales of the thermal and mechanical heterogeneities generally tend to shorten with increasing age, although long-wavelength structures develop just after the beginning of the simulation ("A" in Fig. 3a, c) in spite of an initial temperature condition with a short-wavelength structure (Eq. (14)). When $\Delta\eta_T$ is moderate (i.e., $\Delta\eta_T = 10^3$ and $10^{3.5}$) it appears that the scales of the thermal and mechanical heterogeneities generally tend to shorten with increasing age before c. 2.0 Ga, but the degree-one structure begins to develop after 1.0 Ga (see "B" in Fig. 3e–h).

Even when $\eta_0 = 10^{16}$ Pa s and $k_0 = 36$–200 W m^{-1} K^{-1}, these conclusions remain essentially unchanged: the degree-one thermal/mechanical structure only appeared for a limited range of parameter values for lateral viscosity variations, i.e., $\Delta\eta_T = 10^4$ and $10^{4.5}$ for the model with $k_0 = 36$ W m^{-1} K^{-1} ("A" in Fig. 4) and $\Delta\eta_T = 10^3$ and $10^{3.5}$ for the model with $k_0 = 200$ W m^{-1} K^{-1} ("A" in Fig. 5). These results imply that the degree-one structure is found in the models with a wide range for the Rayleigh number, as also shown in mantle convection simulations (McNamara & Zhong 2005; Yoshida & Kageyama 2006).

This degree-one convection pattern is similar to the familiar "sluggish-lid regime" or the "transitional regime" in thermal convection of the mantle that has already been found in numerical simulations of mantle convection (e.g., Solomatov 1995) (Fig. 6b). In this regime, the flow velocities of downwelling plumes from the sluggish-lid are large compared with the interior of the inner core, and the global flow pattern in the sphere corresponds to the temperature distribution. This is because temporal changes in thermal heterogeneity roughly correlate with those in mechanical heterogeneity, as shown in Fig. 3, 4 and 5. Also, once the degree-one structure is formed, the sluggish-lid regime generates the largest magnitude of flow velocity and the most laterally heterogeneous velocity field in the inner convecting region (Fig. 6b and 7a) when compared with other regimes such as the mobile-lid regime (Fig. 6a) and the stagnant-lid regime (Fig. 7b).

When $\Delta\eta_T$ is 10^4 or larger, the scales of the thermal and mechanical heterogeneities under the stagnant-lid, whose thickness is approximately 100 km, are quite small (the dominant degrees are below 16) even after 4.5 Gyr (Fig. 3i–l). Although the relatively long-wavelength mode is intermittently dominant during the simulation even when $\Delta\eta_T$ is quite large ("C" in Fig. 3k–l), it is immediately damped over a short time-scale of ≤ c. 0.5 Gyr. As a result, the stagnant-lid regime is maintained for almost the entire simulation time.

4 Discussion

The degree-one structure of the inner core suggested by various seismological evidence may have been realized by an "exogenous factor" from outside the ICB, rather than by an "endogenic factor" due to the internal rheological heterogeneity. The exogenous origin for the hemispherically asymmetric structure of the inner core and the related hemispherical difference in the degree of crystallization, i.e., freezing and melting, of the inner core materials is consistent with a previous suggestion that planetary-scale thermal coupling among the lower mantle, the outer core, and the inner core plays a primary role in the growth and evolution of the inner core (Aubert *et al.* 2008; Gubbins *et al.* 2011; Tkalčić 2015). More recently, a new isotopic geochemical analysis study (Iwamori & Nakamura 2015; Iwamori *et al.* 2015) revealed that such planetary-scale thermal coupling operates in the whole-Earth system through so-called "top-down hemispheric dynamics", in which the hemispheric supercontinent-ocean distribution at Earth's surface controls the thermal convection system in the whole Earth via the mantle, outer core, and inner core over the course of Earth's history (see Fig. 13 of Iwamori and Nakamura (2015)).

It should be noted that the style of convection envisioned in this study is only one form of degree-one convection. Another form involves melting and solidification processes at the ICB, as suggested by Alboussière *et al.* (2010) and Monnereau *et al.* (2010), who concluded that the inner core translates laterally as a rigid body and the return flow effectively occurs in the fluid outer core. Because the present study adopts impermeable boundary conditions at the ICB, this second form of degree-one convection is not permitted. This scenario provides a strong alternate candidate for the form of degree-one convection through top-down hemispheric dynamics, if degree-one convection due to internal rheological heterogeneity is not possible. As another new idea, some authors suggested that the hemispheric seismic heterogeneity in the inner core would be caused by the magnetic field (Takehiro 2011; Lasbleis *et al.* 2015; Takehiro 2015). The effects of magnetic field on the 3D numerical model of thermal convection in the growing inner core should be investigated in future. As a start, we are developing a new 3-D numerical simulation code for multiple-layered thermal convection with a large viscosity contrast among each layer, considering the methodology used in the present study and several issues discussed in this Section. The relationship between the degree-one thermal convection and characteristic of geomagnetic field may be an interesting topic not only in earth science but also in computational fluid dynamics.

The age of the inner core is one of the most controversial issues in Earth Science. As mentioned in Section 2, the value of the total heat flow at the CMB is a key parameter that controls the speed of growth of the inner core. A previous study (Cottaar & Buffett 2012) has shown that the minimum heat flow for inner core convection is 4.1 TW, giving a maximum inner core age of 1.93 Ga. Heat flow greater than 6.3 TW leads to the present convection regime, but the inner core age is then less than 1.26 Ga. Following their analysis, if the total CMB heat flow is c. 5–15 TW (Lay et al., 2008 and references therein), the age of inner core should be younger (i.e., less than 1 Ga). If the value of the total CMB heat flow is significantly larger than that used in this study, the possibility of the formation of degree-one structure by an endogenic factor due to rheological heterogeneity becomes less likely, because it takes c. 3.0 Gyr to form the degree-one structure when $\eta_0 = 10^{17}$ Pa s, $k_0 = 36$ W m^{-1} K^{-1}, and $\Delta\eta_T$ is 3.5 (Fig. 3g). At the very least, the conclusion that the convective motion in the inner core is maintained even for the present Earth can be drawn from the models studied here.

Almost all of recent paleomagnetic and geodynamics studies have suggested that the age of inner core would be less than 1.3 Ga to 2.0 Ga (Labrosse *et al.* 2001; Nimmo *et al.* 2004; Davies 2015; Nimmo 2015). According to a more recent paleogeomagnetic study, the inner core formed at ~1.5 Ga (Biggin *et al.* 2015). This “young” inner core age is consistent with the indirect evidences from seismology and mineral physics that the CMB heat flow is larger than previously thought (Hernlund *et al.* 2005; Lay *et al.* 2006; van der

Hilst *et al.* 2007). In the present numerical model, the CMB heat flow is assumed to be constant, $F_m' = 2.56\times10^{12}$ W, based on the model constants used in Buffett *et al.* (1992) and the initial radius of the inner core arbitrarily set. As stated in Section 2, this value would be a lower limit value for the present Earth considering an even relationship between the total plume buoyancy flux observed at the Earth' surface and the inferred total CMB heat flow (e.g., Davies 1988; Sleep 1990; Davies & Richards 1992) and a minimal power requirement for maintenance of the geodynamo (Buffett 2002). However, the CMB heat flow in the past Earth would be lower than that in the present Earth, because the average mantle temperature may increase as the Earth older. More than that, there is a possibility that the CMB heat flow may have a significant variation throughout the Earth's history. Although the implementation of time-dependent CMB heat flow is beyond the framework of the present simple numerical model, it might be a serious problem for the growing speed of the inner core. However, if the inner core grows faster than the present model, the degree-one structure would only appeared for a further limited range of $\Delta\eta_T$ than the results presented in this paper. Therefore I believe that the conclusion on the less possibility of an endogenic origin for the degree-one thermal/mechanical structure of the inner core is justified.

Seismic observations provide evidence for a seismic velocity discontinuity about 200 km below the ICB separating an isotropic layer in the uppermost inner core from an underlying anisotropic inner core (e.g., Song & Helmberger 1998). The results presented here may imply that this "inner core transition zone" represents the boundary between a sluggish, highly viscous, cold layer and an underlying hot convection region (Fig. 6b and 7a). Sumita and Yoshida (2003) predicted that there may be a characteristic structure in the topmost section of the inner core that is similar to the plate tectonic mechanism at Earth's surface, which is explained by the existence of a crust-like, thin, low-degree partial-melting layer and an underlying asthenosphere-like, high-degree partial-melting layer. The ICB is, by definition, at melting temperature, which means that it is possible that the viscosity could be lowest at the top and gradually increase with depth into the interior of the inner core. However, because the temperature across the inner core is never very far from the melting temperature (Stacey & Davis 2008), depending in detail on the contribution from light element impurities, viscosity variations in the underlying hot convection region should be small.

Here I need to mention the lack and limitation of the present simple numerical model. First, the framework of present model is based on mantle convection simulations to deal with convection within the growing inner core. In contrast to mantle convection simulations, I used a time-dependent inner core radius to get dimensionless equations Eq.

(2–4) to account for the growing inner core. However, Eq. (2-4) is build based on an Eulerian specification that is fixed on space. With increasing inner core radius, the grid is actually slowly expanding through time. Thus, strictly speaking, mass, momentum, and energy may be no longer conserved with this expending grid, although the growth rate could be small enough to make it negligible. However, considering the significant accumulating growth of the inner core during the whole simulation, improvement of the present numerical model should be required.

Next, on the issue of secular cooling raised in Section 2, the dimensionless internal heating production shown in Eq. (5) is incomplete because the inner core got cooling down rapidly once the inner core gets started growing in the present model. The exact way is for the secular cooling to be addressed as the boundary condition problem, not the internal heating production, because the initial temperature of inner core should be determined by the solidus temperature of iron-alloy, adiabatic heat flux across the ICB, the latent heat release and gravitational energy caused by light element release. Furthermore, the molten core might have initial accretion energy before the inner core started growing, but not in the inner core. Although the integration of these two serious problem into the present numerical model is numerically difficult and is beyond the present study on the possibility of the formation of the degree-one structure with lateral viscosity variations, they should be considered in the future model.

5 Conclusions

The systematic numerical simulations conducted in this study investigated the possibility of the formation of a degree-one structure with lateral viscosity variations in thermal convection within the age of Earth for a simulated inner core confined in a sphere. The degree-one thermal/mechanical structure, however, only appeared for a limited range of parameter values for lateral viscosity variations. Considering the uncertainties in the exact magnitude of lateral temperature variations, the rheology, and the composition of Earth's inner core materials, the formation of a degree-one structure with lateral viscosity heterogeneity under the limited geophysical conditions confirmed here would be considered possible but not probable for the real Earth. If the value of the total CMB heat flow is significantly larger than that used in this study, the possibility of the formation of degree-one structure by an endogenic factor due to rheological heterogeneity becomes less likely, because it takes c. 3.0 Gyr to form the degree-one structure. Namely, the degree-one structure of the inner core may have been realized by an "exogenous factor" from outside the ICB, rather than by an "endogenic factor" due to the internal rheological heterogeneity.

In future, the evolution of the inner core should be resolved by numerical simulations of the whole-Earth thermal convection system because it is highly possible that the growth rate of the inner core is determined by heat flow at the CMB, which largely depends on the behavior and style of mantle convection (Buffett *et al.* 1992; Sumita & Yoshida 2003). The combined effects of the thermal and compositional buoyancies (Lythgoe *et al.* 2015) and the effects of non-uniform heat flux boundary condition at the ICB, rather than a fixed temperature condition on the style and regime of inner core convection, should also be studied numerically in future.

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591 **Table 1.** Physical parameters for the simulation used in this study.

Symbol	Definition	Value	Unit	Refs.
α_0	Thermal expansion coefficient*	9.74×10^{-6}	K^{-1}	a, b
c_{p0}	Specific heat at constant pressure*	7.03×10^{2}	$J\ kg^{-1}\ K^{-1}$	a
g_0	Gravitational acceleration*	4.4002	$m\ s^{-2}$	a
ρ_0	Density*	1.27636×10^{4}	$kg\ m^{-3}$	a
k_0	Thermal conductivity*	36 or 200	$W\ m^{-1}\ K^{-1}$	a/c, d, e
κ_0	Thermal diffusivity*	4.01×10^{-6}	$m^2\ s^{-1}$	a
η_0	Viscosity*	10^{17} or 10^{16}	Pa s	f
-	Temperatures at the inner core boundary	5000	K	a
ΔT_0	Characteristic temperature variation*	30	K	a
M_m	Mass of the mantle	4.043×10^{24}	kg	g
τ_c	Formation age of the core	4.5	Ga	h
c_{pe0}	Average specific heat at constant pressure for Earth	9.2×10^{2}	$J\ kg^{-1}\ K^{-1}$	g
ρ_{e0}	Average density of Earth	5.520×10^{3}	$kg\ m^{-3}$	g
T_m	Mean mantle temperature	2250	K	g
E_a	Activation energy for dry olivine	540×10^{3}	$J\ mol^{-1}$	g, i
λ	Average decay constant for the mixture of radioactive isotopes in the mantle	2.77×10^{-10}	yr^{-1}	g
$dT/dt\|_{t=t_0}$	Present cooling rate of the mantle	64.7	$K\ Gyr^{-1}$	Eq. (7)
G	Gravitational constant	6.67384×10^{-11}	$m^3\ kg^{-1}\ s^{-2}$	g
R	Gas constant	8.3144621	$J\ mol^{-1}\ K^{-1}$	g
$\partial T_s/\partial p$	Solidification profile	7×10^{-9}	$K\ Pa^{-1}$	j, k
r_e	Radius of Earth	6.371×10^{6}	m	g
r_b	Radius of the outer core	3.48×10^{6}	m	l
$r_c(t)$	Radius of the inner core	Time-dep.	m	-
r_c	Radius of the present inner	1.2215×10^{6}	m	l

	core			
r_c^i	Initial radius of the inner core	2.15×10^{4}	m	x
Ω	Model constant (see text)	1.13×10^{13}	W	Eq. (6)
N	Model constant (see text)	2.01×10^{16}	J m^{-2}	Eq. (11)
F_m'	Model constant (see text)	2.56×10^{12}	W	Eq. (12)
		Dimensionless parameters		
$Ra(t)$	Thermal Rayleigh number	Time-dep.	-	Eq. (5)
$H(t)$	Internal heat-source number	Time-dep.	-	Eq. (5)
$\zeta(t)$	Spherical-shell ratio number	Time-dep.	-	Eq. (5)

Definition: "*" indicates reference values for the inner core. References: a, Stacey and Davis (2008); b, Vočadlo *et al.* (2003); c, de Koker *et al.* (2012); d, Pozzo *et al.* (2012); e, Pozzo *et al.* (2014); f, Karato (2003); g, Turcotte and Schubert (2014); h, Lister and Buffett (1998); i, Karato and Wu (1993); j, Verhoogen (1980); k, Buffett *et al.* (1992); l, Dziewonski and Anderson (1981). "x" indicates arbitrary values (see text).

Figure captions

Figure 1. (a) Illustration for the numerical model of inner core convection. (b) Schematic profile of the potential temperature in the cooling core of the Earth used in an analytical model for solidification of the inner core (after Buffett et al., 1992). The solid line represents the temperature in the upper part of the thick solid inner core, the liquid outer core, and the lower mantle at some instant. The thick dashed line represents the subsequent evolution of temperature as heat is continuously extracted from the core. Heat fluxes f_m and f_i pertain to the core-mantle and inner-core boundaries (CMB and ICB), respectively. The solidification temperature $T_s(r_c)$ is defined at the ICB, and $T_s(r, t)$ is the temperature within the inner core.

Figure 2. Time evolution of the convection pattern in the inner core for models with $\eta_0 = 10^{17}$ Pa s, $k_0 = 36$ W m^{-1} K^{-1}, and (a–d) $\Delta\eta_T = 10^0$, (e–h) $\Delta\eta_T = 10^3$, (i–l) $\Delta\eta_T = 10^{3.5}$, and (m–p) $\Delta\eta_T = 10^4$. Blue and copper isosurfaces indicate regions with lower and higher than average temperatures at each depth. (a–d and m–p) blue: –0.3 K; copper: +0.3 K. (e–f and i–k) blue: –0.6 K; copper: +0.6 K. (g, h, and l) blue: –1.5 K; copper: +1.5 K.

Figure 3. Temporal changes in the heterogeneity of temperature and root-mean-square velocity fields in the inner core for the models with $\eta_0 = 10^{17}$ Pa s, $k_0 = 36$ W m^{-1} K^{-1}, and (a–b) $\Delta\eta_T = 10^0$, (c–d) $\Delta\eta_T = 10^2$, (e–f) $\Delta\eta_T = 10^3$, (g–h) $\Delta\eta_T = 10^{3.5}$, (i–j), $\Delta\eta_T = 10^4$, and (k–l) $\Delta\eta_T = 10^5$. The logarithmic power spectra are normalized by the maximum values at each elapsed time. The four black wedges on the panels (a), (e), (g), and (i) represent the ages that correspond to Fig. 2.

Figure 4. Temporal changes in the heterogeneity of temperature and root-mean-square velocity fields in the inner core for the models with $\eta_0 = 10^{16}$ Pa s, $k_0 = 36$ W m^{-1} K^{-1}, and (a–b) $\Delta\eta_T = 10^0$, (c–d) $\Delta\eta_T = 10^2$, (e–f) $\Delta\eta_T = 10^3$, (g–h) $\Delta\eta_T = 10^{3.5}$, (i–j), $\Delta\eta_T = 10^4$, and (k–l) $\Delta\eta_T = 10^5$. The logarithmic power spectra are normalized by the maximum values at each elapsed time.

Figure 5. Temporal changes in the heterogeneity of temperature and root-mean-square velocity fields in the inner core for the models with $\eta_0 = 10^{16}$ Pa s, $k_0 = 200$ W m^{-1} K^{-1}, and (a–b) $\Delta\eta_T = 10^0$, (c–d) $\Delta\eta_T = 10^2$, (e–f) $\Delta\eta_T = 10^3$, (g–h) $\Delta\eta_T = 10^{3.5}$, (i–j), $\Delta\eta_T = 10^4$, and (k–l) $\Delta\eta_T = 10^5$. The logarithmic power spectra are normalized by the maximum values at each elapsed time.

Figure 6. Cross sections of temperature and velocity fields for the models with $\eta_0 = 10^{17}$ Pa s, k_0 = 36 W m^{-1} K^{-1}, and (a) $\Delta\eta_T = 10^0$ and (b) $\Delta\eta_T = 10^3$ at 0.0 Ga. The top panels show the cross sections at the central depth of the inner core (i.e., radius of 610.8 km) and bottom panels show the cross sections cut along the great circles shown by dashed lines in the top panels. The contour interval in the temperature plots is 3 K. These figures correspond to (a) Fig. 2d and Fig. 3a, and (b) Fig. 2h and Fig. 3e.

Figure 7. Cross sections of temperature and velocity fields for the models with $\eta_0 = 10^{17}$ Pa s, k_0 = 36 W m^{-1} K^{-1}, and (a) $\Delta\eta_T = 10^{3.5}$ and (b) $\Delta\eta_T = 10^4$ at 0.0 Ga. The top panels show the cross sections at the central depth of the inner core (i.e., radius of 610.8 km) and bottom panels show the cross sections cut along the great circles shown by dashed lines in the top panels. The contour interval in the temperature plots is 3 K. These figures correspond to (a) Fig. 2l and Fig. 3g, and (b) Fig. 2p and Fig. 3k.