Authors' response to Griera review

Here we respond to the comments made by referee Griera. Interactive comment on "The Mohr–Coulomb criterion for intact rock strength and friction – a re-evaluation and consideration of failure under polyaxial stresses"
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After presenting a brief summary of the key points made in the paper Prof Griera raises a number of questions, which we have numbered below for ease of reference. The points are very constructive and helpful and all will be taken into account in the preparation of a revised manuscript.

1) In Fig. 4, in which we compare the intact and residual strengths of Pennant and Darley Dale sandstones in both axial extension and shortening configurations. In the same way as for measurements of friction coefficient on sawcut samples, friction coefficient on surfaces produced by failure of the intact rock is similarly lower in axial extension than shortening. However, the reviewer draws attention to the fact that there is no such marked trend when comparing the so-called 'internal' friction angle between extensional and shortening loading regimes. We do not have a particular explanation for this. It is true that for a given least principal stress, differential stress at intact rock failure is higher in extension than in shortening, and this is likely attributable to the higher mean stress in extension , in accord with what has been previously reported. But for frictional sliding the effect goes the other way.
2) The question is asked "How do you reconcile the deviation between the failure envelope (defined from the combination of shear stress and normal stress on the failure planes produced) compared to the tangent to the envelope of Mohr circles at failure?". This relates to fig.5. In the case of both rock types, in axial shortening the fault angle is larger than the imaginary plane defined by the normal to the tangent to the Mohr circles, and vice-versa in extension. The difference suggests that the ultimate fault plane may be defined by a linkage between an en-echelon array of small scale shear surfaces that are perhaps formed the peak of the stress-strain curve. We cannot prove this because the damage done in fault formation destroys any microstructural evidence. A series of tests terminated at various stages through the failure process on a very stiff machine would be required to investigate this question.

3) This point raises the question of the axial ratio of the sample in our results. For the reasons the referee cites, we used a length : diameter ratio = 2.5 : 1, as is common practice, to reduce the impact of specimen end-effects on ultimate failure.

4) This question argues that our discussion of the effects of specimen mechanical anisotropy is not supported by experimental results. This is correct, as in this study we have not investigated the effects of anisotropy by means of deformation of specimens cored in different directions relative to bedding. We report that that whilst these rocks show no visible anisotropy it does not mean that mechanical anisotropy may be totally absent. We include the (short) section on anisotropy to point out that by changing the orientation of maximums stress relative to bedding (extension vs shortening tests), there may be present a hidden effect of anisotropy in the results in addition to the effect of the difference in stress state. It also seemed opportune to point out that a more complete investigation of anisotropy will require tests on cores taken in different orientations in addition to changes in the relative orientation of principal stresses, and that there might be a non-unique best fitting failure criterion even for a given rock type.

5) The reviewer points out that equation 7 makes no physical sense. This is a good point. The intention was to find an arbitrary function based on the Mogi empirical approach that would reconcile the extensional and contractional results. In retrospect it is not useful to seek a simple reconciliation, which disguises the fact that whilst we know end-member friction coefficients, we do not know anything about how they vary between these extremes in advance of making experiments with different values of the intermediate principal stress.
In the light of the referee's comment we have therefore adopted the following approach:
To show how σ_2 relates to σ_1 and σ_3 we can usefully define a ratio C according to
$\sigma_2 = \sigma_3 + C(\sigma_1 - \sigma_3)$, from which $C = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$
For now we must make the simplest possible assumption, that the friction coefficient $\mu(C)$ varies linearly with σ_2 between μ_1 (contraction tests) and μ_2 (extensional tests), as shown in new Fig. 15a, from which we can write
$\mu(C) = \mu_1(1 - C) + \mu_2 C$
Sliding will be activated on any weak plane such that for given values of σ_1 and σ_3 and C the orientation of the weak plane plots along the friction line $\mu(C)$.
Jaeger (1964) describes the extension of the Mohr circle construction into 3 dimensions. Fig. 15b shows the relations that exist between the stress state and the frictional sliding line $\tau = \mu(C)\,\sigma_n$, with an example of one slip plane upon which the stress state (τ, σ_n) will meet the slip condition. Taking the reference frame to coincide with the principal stress directions

σ_1, σ_2 and σ_3, any slip plane is described by a set of corresponding direction cosines of the normal to the plane, $l = \cos \gamma$, $m = \cos \nu$ and $n = \cos \delta$. For convenience, thinking of σ_1 as vertical, γ is measured in the vertical plane from σ_1 and the other two angles to the normal to the plane from σ_2 and σ_3 respectively. Resolved maximum shear stress and normal stress (τ , σ_n) on the slip plane are the coordinates of the point of intersection of the two Mohr circles defined by angles 2γ and 2δ, respectively measured from σ_1 and σ_3 on Fig. 15b.

The slip vector is expected to be parallel to the maximum resolved shear stress, thus in general oblique slip is expected in a 3D stress field. Bott (1955) and Jaeger (1964) derived equations for the resolved dip- and strike-parallel shear stress components, τ_{dip} and τ_{strike} . Expressed in terms of C and $(\sigma_1 - \sigma_3)$ these are:

$$\tau_{dip} = n\,(m^2 C(\sigma_1 - \sigma_3) - (1 - n^2)\,(\sigma_1 - \sigma_3)) / \sqrt{(l^2 + m^2)}$$

$$\tau_{strike} = l\, m\, C\,(\sigma_1 - \sigma_3) / \sqrt{(l^2 + m^2)}$$

The maximum shear stress τ is given by $\tau = \sqrt{(\tau_{strike}^{\ 2} + \tau_{strike}^{\ 2})}$.

The pitch angle ω between the horizontal on the plane (strike) and the slip vector is given by $\tan \omega = \tau_{dip} / \tau_{strike} = nmC / (l\,(m^2 C - (n^2 + m^2)))$. In this way the variability of the friction coefficient can be incorporated to describe slip propensity in a polyaxial stress field.

6) Remaining minor points that need comment:

It is not clear what the reviewer means by ‘residuals not shown on Fig. 12’, but we have tried to clarify the caption as appropriate.

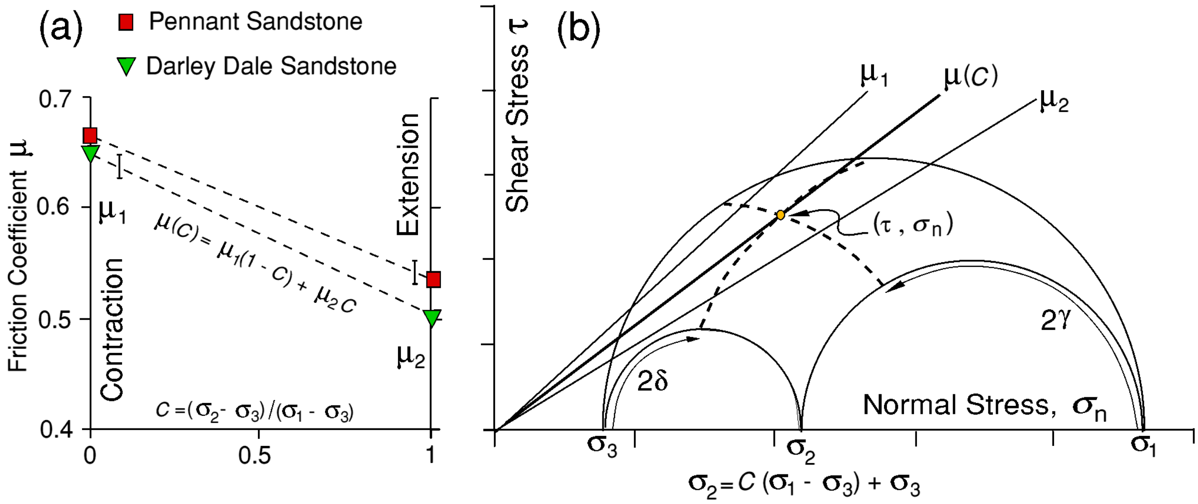
The reviewer finds our reference to the shape of the Mogi (1971) failure criterion as being in the form of ‘inclined ellipses’ and wonders if this means the failure criterion may not be convex. Yes, it is possible for stresses at failure to be two-valued in this criterion, as discussed by Colmenares and Zoback (2002).

The lack of statements of correlation coefficients on linear fits is criticised. This is because r-squared is so close to unity for good linear fits the value does not tell us anything. We state that errors of measurement (as standard error on the dependent variable) is generally smaller than the point size shown. This is more meaningful for strongly linear correlations.

The determination of beta parameter for the Mogi 1967 fits should be clear from the caption to fig. 11, as the value that brings the best fits for the extensional and contractional data into coincidence.

On fig. 10 the dashed lines are tie lines to indicate association between pairs of points.

New fig. 15



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