

## ***Interactive comment on “Pinch and swell structures: evidence for brittle-viscous behaviour in the middle crust” by R. Gardner et al.***

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Comments on Pinch and swell structures: evidence for brittle-viscous behaviour in the middle Crust. By R. Gardner, S. Piazzolo, and N. Daczko.

An anonymous reviewer has made very detailed comments on this paper and I do not intend to go through the same exercise although I agree with many of the comments made by that reviewer. The paper is nicely written and makes an important contribution. However it needs tightening up with respect to the concepts used. My comments involve four main points: (i) The paper needs to make clear to the reader what Mohr-Coulomb constitutive behaviour actually means. In its present form the paper mixes up the concepts of Coulomb-Navier-Mohr fracture criterion (a very old concept, 18th-19th centuries; see Jaeger 1969) and Mohr-Coulomb constitutive behaviour (a relatively

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recent concept; see Vermeer and de Borste, 1984). (ii) The paper needs to clarify why Mohr-Coulomb behaviour is relevant. (iii) Mesh sensitivity needs to be explored or commented upon. (iv) What is the influence of gravity in the crustal scale model?

Mohr-Coulomb constitutive behaviour. The authors apparently do not understand what Mohr-Coulomb behaviour is. The behaviour reported in Figure 2(a) of the paper is the classical Coulomb-Navier-Mohr criterion for fracture best discussed by Jaeger (1969) where the normal to the plane of fracture makes an angle with the direction of  $\sigma_1$ . Notice that the paper claims (erroneously) this relation to be for the angle between the plane and  $\sigma_1$  so that equation 2 needs to be corrected. In doing so Figure 2(a) needs to be re-drawn so that the “Mohr envelopes” are straight lines reflecting the constant friction angle with no tension cut-off as assumed in most of this paper. The Coulomb-Navier-Mohr concept is a criterion for fracture and is the one upon which classical Andersonian fault mechanics is based. By contrast the Mohr-Coulomb relation (see Vermeer and de Borste, 1984 and Hobbs and Ord, 2015, pp 168-173, for details) is not a criterion for fracture; it is a constitutive relation that describes how pressure sensitive flow occurs with and without localisation. An important part of the constitutive framework is the presence of a flow rule (equations 16 and 18 of Moresi and Muhlhaus, 2006). This distinguishes the behaviour from classical Coulomb-Navier-Mohr behaviour. Mohr-Coulomb behaviour involves a criterion for localisation but that zone of localisation may not necessarily be a single fracture; it is a localised zone of brittle deformation with no comment on the detailed microstructure of the zone. It might for example not be a discrete fracture but a zone of crushed grains. The zones of localisation predicted by Mohr-Coulomb constitutive behaviour are not mode II fractures as implied by this paper. They are shear zones where compatibility of deformation is matched across the boundary between the localised and adjacent non-localised material. It is this compatibility requirement that controls the angle between the shear zone normal and  $\sigma_1$  (Rudniki and Rice, 1975). In other words they are not faults with discontinuities on their boundaries. The paper has to be reworded to remove this connotation. In this particular implementation of Mohr-Coulomb no elasticity is included so

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that the behaviour is (unrealistic) rigid-plastic behaviour. This places severe constraints on compatibility between the localised and non-localised material so that the boundary is a plane of zero strain. This probably means that in these models the angles predicted by equation 14 of Moresi and Muhlhaus do not occur. Some comment based on observations would be useful.

As indicated above, the initial angle between the normal to the plane of localisation and  $\bar{\alpha}_1$  is given by equation (14) of Moresi and Muhlhaus : where is the friction angle ( $\mu = \tan \bar{\alpha}_1$ ). This angle is, in general, different to that predicted by the Coulomb-Navier-Mohr criterion. Moresi and Muhlhaus discuss the way in which this angle changes with strain. It would add to the paper if some discussion was included regarding the initial orientation of the shear bands and how this changes with strain. I can see no systematic variation but it is difficult to analyse this with the figures presented. At the very least the paper should include a comparison between predicted and observed orientations.

It should also be noted that the behaviour of Mohr-Coulomb materials is intrinsically unstable because of the corners on the yield surface. Thus localisation is an intrinsic part of the behaviour of Mohr-Coulomb materials. The material used in this paper is also unstable because it has non-associative constitutive behaviour (the dilation angle is presumed to be zero, although this is never stated, so that the dilation angle and the friction angle are not equal). This means that the yield and potential surfaces (as discussed by Moresi and Muhlhaus) are not coincident. These are important points that describe why localisation occurs in these simulations and they should be emphasised. In principle Mohr-Coulomb materials do not need to feature softening behaviour in order to localise, they are intrinsically unstable. In fact, Rudniki and Rice (1975) show that these materials can localise in the hardening regime. Thus a lot of the discussion in this paper justifying weakening behaviour misses the mark. Weakening is sufficient but it is not necessary to produce the modelled behaviour in non-associated pressure sensitive materials.

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One should also note that although the authors go to great pains to insist that no initial irregularities or perturbations are in their models, one needs a perturbation of some kind to set the instability off. In their case the perturbation comes from rounding errors in the computations.

Another point concerns references to the values of  $c_1$  and  $c_s$  on page 1527. The authors claim these are dimensionless. This cannot be true and still remain consistent with the formulation of Moresi and Muhlhaus. They must have the units of stress. Their ratio of course is dimensionless. However one should note the implications of a ratio  $c_1/c_s = 100$ . If  $c_1 = 50$  MPa (a reasonable value) then  $c_s = 0.5$  MPa, a ridiculously small value for a cohesion. This is approaching the behaviour of a cohesionless Mohr-Coulomb material which is thermodynamically inadmissible (Hobbs and Ord, 2015, p 170).

The relevance of Mohr-Coulomb behaviour. An important emphasis in this paper is the claim that Mohr-Coulomb behaviour is important throughout the crust. This is not the first time such a relation has been explored for crustal behaviour (see Ord, 1991) and that paper should be referred to not simply as an example of Mohr-Coulomb material on a crustal scale but also as a reason for using 0.6 as a value for the internal friction coefficient. The dominating effect of Mohr-Coulomb behaviour needs to be toned down. An important point is that the experimental evidence for Mohr-Coulomb constitutive behaviour is very weak and other forms of brittle crustal behaviour are to be preferred (See discussion in Hobbs and Ord, 2015, pp168-173). The only reason I can see for promoting Mohr-Coulomb behaviour in this paper is that it is available for use in Underworld. Even if fracturing is documented that does not necessarily indicate that the localisation leading to pinch and swell features is controlled by Mohr-Coulomb constitutive behaviour. The same behaviour could arise in a material that is deforming essentially by viscous flow and following a viscous constitutive law but where energy dissipated by local fracturing leads to viscosity weakening and hence localisation. This is the type of behaviour reported by Hobbs et al. (2008: viscosity weakening due to

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thermal feedback), Hobbs et al. (2010: viscosity weakening due to dissipation arising from chemical reactions) and by Peters et al. (2015: viscosity weakening due to dissipation from grain size reduction). Any process (including local fracturing) that dissipates energy will lead to localised structures of some kind simply from strain-rate (that is, viscosity) softening and need not specifically involve a brittle-type constitutive relation directly. In this regard, the discussion in the last paragraph of page 1535 is incomplete. Softening resulting in a decrease in stress is important (but not necessary for localisation) in rate insensitive materials such as Mohr-Coulomb but in rate sensitive materials (such as viscous materials) the important process is strain-rate (viscosity) weakening. This is because viscous materials (with  $n$  not equal to 1) are strain rate hardening (a positive perturbation in strain-rate leads to an increase in stress) and in order to weaken them one needs a coupled process that decreases the viscosity with an increase in strain-rate. The authors need to flesh this out and admit that the model they present is one way of producing what is observed and not push the line that their results unambiguously show that Mohr-Coulomb behaviour is present throughout the crust. Even if one accepts that brittle behaviour controls what we see in these structures, the authors also need to indicate why Mohr-Coulomb behaviour is likely rather than some other brittle constitutive relation such as Drucker-Prager. Drucker-Prager behaviour is more stable than Mohr-Coulomb because there are no corners on the yield surface. However in the absence of dilatancy such materials still localise and would produce very similar results to those reported in this paper.

Mesh dependency. Localisation in Mohr-Coulomb materials is well known to be mesh dependent because there is no intrinsic length scale in the constitutive relation and the only length scale in the model is the mesh size. This means that the spacing and thickness of shear zones depends on the mesh size. I have checked with Moresi and he confirms that mesh dependency exists for Mohr-Coulomb behaviour in Underworld. It would be nice to see two models run under identical conditions except for the mesh size to see the effect. Certainly if mesh dependency exists then nothing can be said about the details of pinch and swell shapes without a detailed analysis.

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Gravity and pressure. The authors imply that it is not necessary to consider pressure in their models. They mistakenly quote equation (3) of Moresi and Muhlhaus to support this. This particular equation describes the coupling between the motion of deforming material and the effects of thermal expansion upon the density of material during mantle convection. It has absolutely nothing to do with the effect of pressure on the mechanical behaviour of Mohr-Coulomb materials and is true for any material. In fact the effect of pressure on the flow stress of Mohr-Coulomb materials is very large. Pressure can also have an influence on the cohesion and friction angle (and the dilation angle) of Mohr-Coulomb materials (see Ord, 1991). The point made here is particularly relevant with respect to the crustal scale models. As far as I can determine, gravity is not turned on in the crustal scale models reported here. If one does this then for an average crustal density of  $2700 \text{ kg m}^{-3}$  at a depth of say 20 km the normal stress on a plane of localisation would be of the order of 500 MPa. Using equation 9 of Moresi and Muhlhaus, a pressure independent value of the cohesion of 50 MPa and a pressure independent value of  $\tan \phi = 0.6$ , as assumed by the authors, one obtains a shear stress necessary to initiate failure of 350 MPa; at 40 km the failure stress is 700 MPa. This is quite high and the issue is whether in Underworld, with realistic values of viscosity, failure of Mohr-Coulomb materials can occur at these depths. I doubt it. Hence, if the authors have already included gravity then they should say so and I am wrong. If they have not included gravity they should do so and see if I am correct. As the paper stands at present this part of the modelling needs clarification or needs to be redone.

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