To the Reviewer #1

Many thanks for the referee's valuable comments and his/her time spent in reviewing our paper (SE-2016-12). I would like to mention that in the revised paper, all the points have been taken into consideration. Following, I refer the comments made by the referee with their corresponding answers as italic font. The changes are highlighted in the revised paper (in yellow color).

Comment#1: Does the Giesekus constitutive relationship that they use actually apply to rocks? This relationship was derived specifically for polymers. Rocks are not polymers – they are crystalline. Are there any laboratory experiments showing that rock minerals, or indeed any type of crystalline material, has nonlinear viscoelasticity that matches the Giesekus constitute relationship? Or any field observations? Unless the authors can cite some evidence that this is actually relevant, then this manuscipt has no place in an Earth science journal.

Response: *It is pleasure for me that it is prepared an opportunity for us to clarify some uncertainties. The main comment of respectable reviewer is: Just linear viscoelastic models (such as Maxwell model) are suitable for modeling the mantle convection and Giesekus constitutive equation is not a good choice due to difference of the class of materials. I should present some clarifications that should be useful for readers. Since 1950, a movement in rheology is begun to present constitutive equations for viscoelastic materials especially the polymeric materials. <u>Not only the Giesekus constitutive equation but also all of famous linear and nonlinear constitutive equations (such as **Maxwell model**, power-law equation, cross equation and ...) have been presented for polymers</u>. These models have been used in other branches of science for solving the flow and deformation of viscoelastic materials such as geology, biotechnology, soil engineering, chemical engineering, food engineering and so on. For example, the power-law model is a simple nonlinear constitutive equation that can be model the nonlinear shear dependent viscosity using the second invariant of shear rate tensor to define the generalized shear rate. This model is widely used for solving the flow of non-Newtonian liquids due to its simplicity. <u>**The power-law model was also used for modeling the mantle convection**.</u> A summary for the type of models that used in previous studies is presented in following Tables (tables located after the response to this comment). In these Tables, **6 works** are listed that used the power-law model as the constitutive equations.*

The respectable reviewer should notice to this problem: why did the previous researchers use the ***power-law*** *equation for* ***modeling the mantle convection*** *(as a too simple nonlinear model which is basically presented for* ***polymeric liquids****)? The answer is: Not only the class of material but also the* ***type of deformation*** *is important in selecting a constitutive equation for any rheological problems. The previous researchers used the power-law model to solve the mantle convection due* ***to large scale deformation*** *(The flow is a large scale deformation). A same approach has been performed to solve the mantle convection problem using the linear viscoelastic model to study the effect of material* ***elasticity*** *on the problem but the result of these models are not proper for large scale deformation. The main motivation of present study is answering to this question: "is it possible to study the both effect of material elasticity and nonlinear viscosity on mantle convection?" The answer is using the nonlinear viscoelastic constitutive equations such as Giesekus model. The model can present simultaneously a fractional* ***nonlinear viscosity*** *(similar to power-law model) and* ***elasticity*** *(similar to linear differential viscoelastic models). Because of using the convective coordinate system in its definition (using complicated upper convected derivations instead of simple time derivations), it is so suitable for modeling the large scale deformations that is the main advantage of this model on linear models. In other word, the model is able to keep the memory of deformation like as linear integral viscoelastic models. It is important to mention that the Giesekus model can be simplified to* ***linear Jeffries model*** *for small deformations. Therefore, the authors believe that this complicated nonlinear constitutive equation can better model the mantle convection (as a nonlinear-viscoelastic-large scale flow) by selecting the suitable constants of this model (viscosity, relaxation time and mobility factor). This finding was also mentioned by Prof. Harder (1991) in conclusion of his study about the modeling of mantle convection as a suggestion for future studies:*

> "*This study has demonstrated the major differences between convection with Maxwellian versus Newtonian rheology and shown that thermal convection is a very suitable test case for numerical methods simulating viscoelastic flow. It has been possible to extend the simulation up to Deborah numbers De = 1.0, which is sufficient to induce significant changes in the flow fields. A main new feature at high De is the presence of a normal stress singularity along the boundary, which is absent in Newtonian or in low-De flow. Since this singularity is starting from the stagnation points at the cell corners, this behaviour is presumably related to the well known singularity of the upper convected Maxwell model in a pure shear*

flow [4,5]. It is common experience [4,7,13] that rheological models with more realistic response are numerically easier to handle. Examples are the Jeffreys model and its non-linear generalisations, i.e. the ***Giesekus*** *and Leonov models (Harder (1991))*".

The work of Harder (1991) is reported in the revised manuscript and some explanations about selecting the nonlinear Giesekus model is inserted to the revised paper (refer to pages 4, 5 and 7).

Constitutive model	**References**
Maxwell	*OzBench, M., Regenauer-lieb, K., Stegman, D.R., Morra, G., Farrington, R., Hale, A., May, D.A., Freeman, J., Bourgouin, L., Muhlhaus, H., Moresi, L., 2008. A model comparison study of large-scale mantle-lithosphere dynamics driven by subduction. Phys. Earth Planet. Int. 171, 224–234.
	*Thielmann, M., Kaus, B.J.P., Popov, A.A., 2015. Lithospheric stresses in Rayleigh– Bénard convection: effects of a free surface and a viscoelastic Maxwell rheology. Geophys. J. Int., 203, 2200–2219.
	*Harder, H., 1991. Numerical simulation of thermal convection with Maxwellian viscoelasticity. Journal of Non-Newtonian Fluid Mechanics, 39, 67–88.
	*Moresi, L., Dufour, F., Muhlhaus, H.B., 2002. Mantle Convection Modeling with Viscoelastic/Brittle Lithosphere: Numerical Methodology and Plate Tectonic Modeling. Pure Appl. Geophys. 159, 2335–2356.

* The asterisk sign means that the reference has been mentioned in our paper.

Constitutive model	References
Power-law	King, S.D., Gable, C.W., Weinstein, S.A., 1992. Models of convection-driven tectonic plates: a comparison of methods and results. Geophys. J. Int., 109, 481-487.
	* Gerya, T.V., Yuen, D.A., 2007. Robust characteristics method for modelling multiphase visco-elasto-plastic thermo-mechanical problems. Phys. Earth Planet. Int. 163, 83–105.
	* Christensen, U., 1983. Convection in a variable-viscosity fluid: Newtonian versus power-law rheology. Earth and Planetary Science Letters, 64, 153–162.
	* Cserepes, L., 1982. Numerical studies of non-Newtonian mantle convection. Physics of the Earth and Planetary Interiors, 30, 49–61
	* Van den Berg, Arie P., Yuen, D.A., Van Keken P.E., 1995. Rheological transition in mantle convection with a composite temperature-dependent, non-Newtonian and Newtonian rheology. Earth and Planetary Science Letters, 129, 249–260.
	Gerya, T.V., Yuen, D.A., 2003. Characteristics-based marker-in-cell method with conservative finite-differences schemes for modeling geological flows with strongly variable transport properties. Physics of the Earth and Planetary Interiors, 140, 293–318.
	Muhlhaus, H.S., Regenauer-Lieb, K., 2005. Towardsa self-consistent plate mantle model that includes elasticity: simple benchmarks and application to basic modes of convection. Geophys. J. Int., 163, 788–800.

* The asterisk sign means that the reference has been mentioned in our paper.

Constitutive model	References
Newtonian, temperature- or pressure-dependent	* Yanagawa, T.K.B., Nakada, M., Yuen, D.A., 2004. A simplified mantle convection model for thermal conductivity stratification. Phys. Earth Planet. Int. 146, 163–177.
	Stemmer, K., Harder, H., Hansen, U., 2006. A new method to simulate convection with strongly temperature-and pressure-dependent viscosity in a spherical shell: Applications to the Earth's mantle
	* Moresi, L.N., Solomatov, V.S., 1995. Numerical investigation of 2D convection with extremely large viscosity variations. Phys. Fluids 7, 2154–2162.
	* Pla, F., Herreroa, H., Lafitte, O., 2010. Theoretical and numerical study of a thermal convection problem with temperature-dependent viscosity in an infinite layer. Physica D 239, 1108–1119.
	* Hansen, U., Yuen, D.A., Kroening, S.E., Larsen, T.B., 1993. Dynamical consequences of depth-dependent thermal expansivity and viscosity on mantle circulations and thermal structure. Physics of the Earth and Planetary Interiors, 77, 205–223
	Kronbichler, M., Heister, T., Bangerth, W., 2012. High accuracy mantle convection simulation through modern numerical methods. Geophys. J. Int., 191, 12–29.
	* Kameyama, M., Kageyama, A., Sato, T., 2008. Muligrid-based simulation code for mantle convection in spherical shell using Yin-Yang grid. Phys. Earth Planet. Int. 171, 19–32.
	Yoshida, M., 2010. Preliminary three-dimensional model of mantle convection with deformable, mobile continental lithosphere. Earth and Planetary Science Letters, 295, 205–218
	* Kellogg, L.H., King, S.D., 1997. The effect of temperature dependent viscosity on the structure of new plumes in the mantle: results of a finite element model in a spherical, axymmetric shell. Earth Planet. Sci. Lett. 148, 13–26.

* The asterisk sign means that the reference has been mentioned in our paper.

Comment#2*:* Secondly, in a journal like SE one expects a strong link between the presented results and the actual Earth, but instead the authors present a nondimensional parameter study with no attempt to make quantitative inferences or predictions regarding the actual Earth. This is in contrast to earlier studies on mantle convection with viscoelasticity, which the authors don't seem to know about since they don't cite them (Beuchert and Podladchikov, 2010; Harder, 1991; Moresi et al., 2003; Moresi et al., 2002) (also there are many papers on lithosphere & crust deformation that include viscoelasticity). The usual view is that in the mantle elasticity is unimportant at geological time scales because the viscoelastic relaxation time is short compared to the deformation time scale (viscosity/shear modulus ~1e21/1e11 ~ 300 years), while in the lithosphere, viscosity can be orders of magnitude higher so elasticity can be important. It is not clear that the authors reach this regime. As it is, this paper belongs more in a journal like Journal of Non-Newtonian Fluid Mechanics.

Response: Using the dimensionless group (analogy) is useful for studying any fluid flow because the scale of different types of forces (by defining the Reynolds, Weissenberg and Rayleigh numbers) can be specified using this type of report and the results can be simply changed to the scale with real dimensions using the reference parameters. It is also so useful for experimental studies based on the analogy and making too smaller models (setups).

Comment#3*:* Thirdly, the investigative method. If the authors want to demonstrate that nonlinear viscoelasticity is important then they need to show corresponding solutions for (i) nonlinear viscoelasticity (ii) linear viscoelasticity and (iii) viscous flow, and identify how and where they are different.

Response: *The mobility factor (α) of Giesekus model helps us to adjust the nonlinearity degree of the model and it is discussed in the paper (refer to section 4.2.4 of revised paper). The model is linear for* $\alpha = 0$ *and the shear dependency is increased by increasing the mobility factor up to 0.5.*

Comment#4: Fourthly, they claim that they include viscous dissipation "for the first time" but in fact there are several 10s of papers on mantle convection that include this term – basically anything study that uses a compressible approximation. Here are just a few examples to get them started: (Balachandar et al., 1993; Glatzmaier, 1988; Jarvis and McKenzie, 1980; Leng and Zhong, 2008; Tackley, 1996).

Response: *Thank you for your comment. Your indication is correct. Actually, author's purpose is: the conjugated effect of nonlinear viscoelasticity and viscous dissipation is considered in the present study for the first time. This is corrected in the revised paper (refer to page 1 of revised paper).*

Comment#5: they claim that including variation of "g" through the mantle is an important new aspect of their study: actually the variation with depth is very small – only a few C2 SED Interactive comment Printer-friendly version Discussion paper % - as you can see in the widely-used PREM (Preliminary Reference Earth Model) (e.g. http://geophysics.ou.edu/solid_earth/prem.html). This is why it is almost always ignored.

Response: *In order to present a better simulation, we considered the variation of "g" in our CFD simulation. Actually, this effect is not considered in previous studies and it changes around 1.07% the maximum of velocity of vortices. This finding may be useful for future modeling to ignore or consider the changing in the gravitational acceleration with depth.*

Best,

M. Norouzi