We thank Dani Schmid, Marta Adamuszek and Marcin Dabrowski for their short comment (referred to here as Schmid et al., 2016) which will help us to clarify certain statements concerning their theoretical model for Large Amplitude Folding (LAF; Adamuszek et al., 2013) during the revision of our manuscript.

The aim of our statement (line 454 on) concerning the "not dramatic" improvement of the LAF model with respect to the Exponential Single Waveform Solution (ESWS) of Biot et al. (1961) displayed in our Eqn. (27) (and in this reply again in Eqn. (1)) was not to discredit the LAF model, but to underline the usefulness and strength of the simple ESWS. Therefore, this statement came before the description of the LAF model, but we agree that the statement can be misunderstood and we will modify the text. The LAF model is important for understanding finite fold amplification, because it combines three major features of fold amplification: (i) the wavelength dependent amplification rates based on the stability analysis, (ii) the shortening of waveform components and the variation of the corresponding amplification rates ("preferred wavelength"), and (iii) the difference between the shortening rate of the layer's arc length and the bulk shortening rate. The LAF model is described by a coupled system of ordinary differential equations (time derivatives) and is hence considerably simpler than a full two-dimensional (2D) fluid mechanics model described by a system of partial differential equations which has to be solved numerically by, for example, the finite element method.

As clear in our original Fig.16 and highlighted in the Fig. 1 in Schmid et al. (2016), for the geometry and rheology taken as an example, the simple ESWS reproduces the finite fold geometry of the amplified bell-shaped initial perturbation rather well up to a shortening value of ca. 20% (here ~25° limb dip), but the fit then rapidly worsens. The LAF model continues to

provide a better approximation up to ca. 25% shortening (~45° limb dip), but even at this stage the predicted fold shape does differ noticeably from the numerical model. If accuracy is to be maintained, the transition must be made at some stage to two-dimensional (2D) numerical modelling: in the case of the simple ESWS at ca. 20% shortening, for the significantly more complicated LAF solution at around 23-25%. For the example we presented in Fig. 16, this was the basis of our statement that in this case the improvement was "not dramatic". By recently providing the Folder package (Adamuszek et al., 2016), the authors have made the switch to 2D numerical modelling of such an isolated perturbation (or indeed any layer shape) very easy and straightforward, so that the transition from (semi-)analytical to numerical has never been easier.

A major aim of our review is to focus on simple analytical solutions and to show that these simple solutions are extremely useful to get fundamental insight into folding mechanics and to make first order estimates. The ESWS of Biot et al. (1961) is a simple and comprehensible solution which can accurately predict fold amplification up to limb dips of around 25°. At such limb dips the general final fold shape (e.g. localized or regular, symmetric or asymmetric etc.) can be anticipated. If the Fourier transform of the initial geometrical perturbation of a layer is known, then the fold amplification can simply be calculated with the ESWS by modifying this Fourier transform. For an initial bell-shaped function the fold shape (represented here by the vertical coordinate of the central line in the folding layer, y) can be calculated by the ESWS equation:

$$y(x,t) = ab \int_0^\infty \exp\left(\underbrace{\alpha t}_{\substack{\text{exponential} \\ \text{time evolution}}} - ak \right) \cos(kx) dk \tag{1}$$

The fold shape given by $y(x,t)$ can be calculated for any time with Eqn. (1) by simply including an exponential growth term, αt (α is amplification rate and t is time), into the Fourier

transform of the initial perturbation which is given by Eqn. (1) for $\alpha t = 0$. The ESWS is described by a single and simple equation and we hence consider the ESWS as a comprehensible (or transparent) solution which provides fundamental insight into fold shape evolution. The LAF solution is more accurate than the ESWS, but in turn the LAF model is much less transparent because it is described by a coupled system of ordinary differential equations for which the time evolution must be calculated numerically.

We have a similar reply concerning the comparison of the LAF solution with the Finite Amplitude Solution (FAS) of Schmalholz and Podladchikov (2000) which is displayed in our Eqn. (29) and is given here again:

$$\frac{L_0}{L} = \underbrace{\left(\frac{A}{L}\frac{L_0}{A_0}\right)^{\frac{1}{2+\alpha_d}}}_{\text{Exponential solution}} \underbrace{\left(\frac{S}{L}\frac{L_0}{S_0}\right)^{\frac{\alpha_d}{2+\alpha_d}}}_{\substack{\text{Finite amplitude} \\ \text{correction}}} \quad \text{with} \quad S = L + \pi^2 \frac{A^2}{L} \tag{2}$$

The first term on the right hand side including amplitude, A (subscript 0 indicates initial values), and wavelength, L, is the classical exponential solution (e.g. Johnson and Fletcher, 1994, also showed such "power-law version" of the exponential solution; α_d is the dominant amplification rate) and the second term represents the finite amplitude correction due to the change of the arc length, S, with respect to the wavelength, L. The FAS is an analytical solution and provides an algebraic relationship between shortening (quantified by L_0 / L) and the fold amplification (quantified by A / L). The FAS provides hence fundamental insight into finite fold amplification and can be used, for example, to analytically derive algebraic expressions to estimate the limb dip at which the exponential solution breaks down (see our review). Again, the LAF model provides more accurate results than the FAS but the finite amplitude evolution is much more transparent from Eqn. (2) than from the ordinary differential equations of the LAF model.

Simple analytical solutions provide fundamental insight into fold amplification and are useful to make first order estimates. If accurate fold shape calculations are needed, then 2D (or 3D) numerical simulations are required. Between the simple and approximate analytical solutions and the accurate numerical simulations exist several mathematical models of "intermediate accuracy" such as the LAF model or the third-order analysis presented in Johnson and Fletcher (1994). All models have their justifications, can reveal different aspects of fold amplification and are essential for a thorough understanding of fold amplification. Nevertheless, the computational power has increased dramatically in the last decades. The best example is the free software Folder by Adamuszek et al. (2016) which enables 2D finite element simulations of folding and necking, and also the accurate, finite element based calculation of amplification rates for nearly any model configuration on a standard laptop within few seconds to several minutes. Therefore, if accurate fold shapes or amplification rates must be calculated, then 2D or 3D numerical simulations are most useful, but when fundamental insights and simple first order predictions are needed, then the simple analytical solutions are most useful.

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