A new theoretical interpretation of Archie's saturation exponent

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Abstract. This paper describes the extension of the concepts of connectedness and conservation of connectedness that underlie the generalised Archie`s law for *n* phases to the interpretation of the saturation exponent. It is shown that the saturation exponent as defined originally by Archie arises naturally from the generalised Archie's law. In the generalised Archie`s law the saturation exponent of any given phase can be thought of as formally the same as the phase (i.e., cementation) exponent, but with respect to a reference subset of phases in a larger *n*-phase medium. Furthermore, the connectedness of each of the phases occupying a reference subset of an *n*-phase medium can be related to the connectedness of the subset itself by $G_i = G_{ref}\, S_i^{n_i}$. This leads naturally to the idea of the term $S_i^{n_i}$ for each phase *i* being a fractional connectedness, where the fractional connectednesses of any given reference subset sum to unity in the same way that the connectednesses sum to unity for the whole medium. One of the implications of this theory is that the saturation exponent of any phase can be now be interpreted as the rate of change of the fractional connectedness with saturation and connectivity within the reference subset.

1 Introduction

Currently, there is no well-accepted physical interpretation of the saturation exponent other than qualitatively as some measure of the efficiency with which electrical flow takes place within the water occupying a partially saturated rock. Some might say that the meaning is not important as long as one can reliably obtain the water saturation of reservoir rocks with sufficient accuracy to calculate reserves. According to the 2016 BP Statistical Review of World Energy (BP, 2016), the world had proved oil reserves at the end of 2015 of 1.6976 trillion (million million; Tbbl.) barrels, slightly down on the value at the end of 2014 (1.7 Tbbl.) and significantly above the respective values at the end of 1995 (1.1262 Tbbl.) and 2005 (1.3744 Tbbl.). The same source lists proven natural gas reserves of 186.9 trillion cubic metres (Tcm) at the end of 2015, slightly lower than at the end of 2014 (187.0 Tcm) and significantly and progressively higher than the values at the end of 1995 (119.9 Tcm) and 2005 (157.3 Tcm). This represents combined oil and gas reserves of approximately 78.4 trillion US dollars combined at end December 2015 prices (using WTI crude and Henry Hub).

Even a tiny uncertainty of, say, 0.01 in a saturation exponent of 2 (i.e., 0.5% or 2±0.01) would result in an error in the reserves of about ±254.36 billion US dollars; the equivalent of 82 Queen Elizabeth class aircraft carriers or one mission to Mars. This calculation has been carried out by calculating the percentage change in hydrocarbon saturation resulting from an error of 2±0.01 in the value of the saturation exponent. Since the calculated change in hydrocarbon saturation also depends on other parameters in Archie's equations, typical representative values for these parameters have been used; R_T = 500 Ω.m, R_w = 1 Ω.m, ϕ = 0.1 and m = 2. When these values are used with n=2±0.01 a change of ±0.3245% was calculated for the hydrocarbon saturation, allowing the change in global reserves to be calculated. However, the degree to which we can carry out the real calculations does not match this precision. Uncertainties in input parameters, over how representative seismic and petrophysical parameters are and difficulties with heterogeneity and anisotropy, to name but a few, result in the real calculations having uncertainties in the order of ±20-40%.

Within the hydrocarbon industry it is extremely common to assume that the saturation exponent is about 2 for most rocks. However, it is worthwhile thinking about the 254 billion dollar global shortfall in revenue if it really is equal to 2.01 instead.

These frightening large financial values make it extremely important that the physical interpretation of the saturation exponent in the classical Archie's law is well understood. This paper attempts to provide a new theoretical and physical interpretation. The classical Archie`s laws (Archie, 1942) link the electrical resistivity of a rock to its porosity, to the resistivity of the water saturating its pores, and to the fractional saturation of the pore space with the water. They have been used for many years to calculate the hydrocarbon saturation of the reservoir rock and hence hydrocarbon reserves. The classical Archie's laws contain two exponents, *m* and *n*, which Archie called the cementation exponent and the saturation exponent, respectively. The conductivity of the hydrocarbon saturated rock is highly sensitive to changes in either exponent.

Like the cementation exponent, and despite its importance to reserves calculations, the physical meaning of the saturation exponent is difficult to understand from a physical point of view, which leads to petrophysicists not giving it the respect it deserves. It is common, for example, to hear that, in the absence of laboratory measurements, the saturation exponent has been taken to be equal to 2, which it has just been noted is bound to lead to gross errors. While it is true that there seems to be a strong preference for values of saturation exponent near 2±0.5 for most water-wet rocks, oil-wet rocks show much higher values (4-5) (Montaron, 2009; Sweeney and Jennings, 1960), and there is evidence that the saturation exponent changes with saturation, with the type of rock microstructure and with saturation history, leading to hysteresis in the plot of resistivity index as a function of water saturation.

When a saturation exponent is derived from laboratory measurements, it is commonly done by fitting a straight line to resistivity data where the *y*-axis is the logarithm of the Resistivity Index and the *x*-axis is the logarithm of the water saturation. The Resistivity Index is the ratio of the measured rock resistivity at a given water saturation S_w divided by the resistivity of the same rock when the pore space is completely saturated with water (i.e., S_w=1). The problem is that the saturation exponent varies with water saturation, becoming significantly smaller at low saturations, leading to an uncertainty in which value to use. This observation also gives us the first hint that it is the connectedness of the water phase that is controlling the saturation exponent just as it did for the phase exponent in the generalised Archie's law.

It is clear that the physical understanding of the saturation exponent needs to be improved. The purpose of this paper is to investigate the elusive physical meaning of the saturation exponent, where it is shown that the saturation exponents are intimately linked to the phase exponents in the generalised Archie's model.

2 Traditional interpretations

Considering the classical form of Archie`s laws; the first Archie law relates the formation factor *F*, which is the ratio of the resistivity of a fully saturated rock ρ_o (R_o) to the resistivity of the fluid occupying its pores ρ_f (R_w), to the rock porosity ϕ and a parameter he called the cementation exponent *m*, where the symbols in parentheses are those traditionally used in the hydrocarbon industry. Archie's first law can be expressed as $F = \rho_o / \rho_f = \phi^{-m}$ using resistivities (Archie, 1942), or as $G = \sigma_o / \sigma_f = \phi^{+m}$ using conductivities. In the latter case *G* is called the conductivity formation factor or the connectedness (Glover, 2009). It can easily be seen that the effective resistivity and effective conductivity of the fully saturated rock can be expressed as $\rho_o = \rho_f \phi^{-m}$ and $\sigma_o = \sigma_f \phi^{+m}$ using resistivities or conductivities, respectively. It should be noted that this work does not consider the form of Archie's law which includes the so-called 'tortuosity factor' *a*, which was developed by Winsauer et al. (1952). The role of this parameter is discussed fully in Glover (2016).

Archie's second law considers that the rock is not fully saturated with a conductive fluid, but is partially saturated with a fractional water saturation S_w. It relates the resistivity index *I*, which is the ratio of the resistivity of a partially saturated rock ρ_{eff} to the resistivity of the fully saturated rock ρ_o, to the water saturation S_w and a parameter he called the saturation exponent

n. Archie's second law can be expressed as $I = \rho_{eff} / \rho_o = S_w^{-n}$ using resistivities, or $1/I = \sigma_{eff} / \sigma_o = S_w^{+n}$ using conductivities.

The two laws may be combined to give $\rho_{eff} = \rho_f \phi^{-m} S_w^{-n}$ using resistivities, and $\sigma_{eff} = \sigma_f \phi^{+m} S_w^{+n}$ if conductivities are used. In reserves calculations, the resistivity of the partially saturated rock, the resistivity of the pore water, the porosity of the rock and the two exponents are "known" from logging or laboratory measurements. This enables the water saturation S_w and hence the hydrocarbon saturation $S_h = (1 - S_w)$ and, consequently, the reserves to be calculated.

Archie's laws require that both the rock matrix and all but one of the fluid phases that occupy the pores to have infinite resistivity. Hence, it is a model for the distribution of one conducting phase (the pore water) within a rock sample consisting of a non-conducting matrix and other fluids which also have zero or negligible conductivity. Problems arise when there are other conducting phases in the rock, such as clay minerals. These problems have generated a huge amount of research in the past (e.g., Waxman and Smits, 1968; Clavier *et al.*, 1984), which are reviewed in Glover (2015). The classical Archie`s laws were based upon experimental determinations. However, there has been progressive theoretical work (Sen *et al.*, 1981; Mendelson and Cohen, 1982) showing that for at least some values of cementation exponent Archie's law has a theoretical pedigree while hinting that the law may be truly theoretical for all physical values of cementation exponent. A study has recently shown that the Winsauer et al. (1952) modification to Archie's law is only needed to compensate for systematic errors in the measurement of its input parameters and has no theoretical basis (Glover, 2016). Meanwhile independent modifications to the original Archie's law have allowed it to be used when both the pore fill and the matrix have significant electrical conductivities (Glover et al., 2000a; Glover, 2009), such as the case when a rock melt occupies spaces between a solid matrix in the lower crust (Glover et al., 2000b). This has culminated in a generalised Archie`s law which is valid for any number of conductive phases in the three-dimensional medium which was published in 2010 (Glover, 2010).

3 The generalised Archie's law

The generalised Archie's law (Glover, 2010) extends the classical Archie's law to a porous medium containing *n* phases. It is based on the same concept of connectedness that was introduced in the author's previous interpretation of the cementation exponent (Glover, 2009). It should be noted that from this point in this paper that the symbol ϕ refers not just to the porosity of the rock, but to the volume fraction of a particular phase, whether it be the matrix, the water, hydrocarbon or whatever other phase may be present. It will either be used for a specific phase such as water (e.g., ϕ_f) or for a set of phases (e.g., ϕ_i). The unsubscripted symbol continues to refer to conventional porosity, where $\phi = \sum_i \phi_i - \phi_m$, and ϕ_m is the phase fraction of the rock matrix (conventionally equal to 1-ϕ).Occasionally, the unsubscripted symbol will also be used when the general properties of phase fractions are being discussed, such as in the following two equations.

In the 2009 paper the connectedness was defined as

$$G \equiv \frac{\sigma_o}{\sigma_w} = \frac{1}{F} = \phi^m \quad , \tag{1}$$

where *F* is the formation factor. The connectedness of a given phase is a physical measure of the availability of pathways for conduction through that phase. The connectedness is the ratio of the measured conductivity to the maximum conductivity possible with that phase (*i.e.*, when that phase occupies the whole sample). This implies that the connectedness of a sample composed of a single phase is unity. Connectedness is not the same as connectivity. The connectivity is defined as the measure of how the pore space is arranged in its most general sense as that distribution in space which makes the contribution of the specific conductivity of the material express itself as a different conductance (see Glover, 2010). The connectivity is given by

$\chi = \phi^{m-1}$ and depends upon the porosity and the classical Archie's cementation exponent m. It should be noted that the connectedness is also given by

$$G = \phi\chi \,, \tag{2}$$

and then it becomes clear that the connectedness depends both upon the amount of pore space (given by the porosity) and the arrangement of that pore space (given by the connectivity).

The generalized Archie's law was derived by Glover (2010) and is given by

$$\sigma = \sum_i \sigma_i \phi_i^{m_i} \qquad \text{with} \qquad \sum_{i=1} \phi_i = 1 \,, \tag{3}$$

where there are n phases, each with a conductivity σ_i, a phase volume fraction ϕ_i and an exponent m_i. The porosity and cementation exponent in the classical Archie's law are the same as the pore space phase volume fraction and pore space phase exponent in the generalized Archie's law, respectively. However, the pore space and the matrix may be subdivided into any number of other phases as required. Indeed, the generalized Archie's law will not contain a term that represents the pore space unless the pore space is only occupied by a single phase.

In the generalized law the phase exponents can take any value from zero to infinity. Values less than unity represent a phase with an extremely high degree of connectedness, such as that for the solid matrix of a rock. Connectedness decreases as the phase exponent increases. Phase exponents that tend towards 1 are associated with a highly connected phase which is analogous to the low cementation exponents occurring in the traditional Archie's law for networks of high aspect ratio cracks. Phase exponents about 2 represent the degree of connectedness that one might find when the phase is partially connected in a similar way to which the pore network in a sandstone is connected, and which is, again, analogous to that scenario in the traditional Archie's law. By extension, higher values of phase exponents represent lower phase connectedness, such as that in the traditional Archie's law for the pores in a vuggy limestone.

It is clear that the classical and generalized laws share the property that the exponents modify the volume fraction of the relevant phase with respect to the total volume of the rock. However the exponents in the generalized law differ from the classical exponent because some of them have values which are not measureable because their phases are composed of materials with negligible conductivity. Despite this, each phase has a well-defined exponent providing (i) it has a non-zero volume fraction, and (ii) the other phases are well-defined.

It should be noted that higher phase exponents tend to be related to lower phase fractions, although this relationship is not implicit in the generalized Archie's law as it is currently formulated.

The generalized Archie's law as formulated by Glover (2010) hinges upon the proposal that the sum of the connectednesses of the phases in a three-dimensional n-phase medium is given by

$$\sum_i \phi_i^{m_i} = \sum_i G_i = 1 \,. \tag{4}$$

It is important to consider Equation 1 and Equation 4 together to develop a fuller understanding of the model. There is an infinite number of solutions to Equation 4 even in the most restrictive 2 phase system. However, there is only a small subset of solutions if both Equation 1 and Equation 4 are to be fulfilled together, as the model requires. The problem of having enough degrees of freedom is not problematic for 3 phases or more, and is trivial for one phase. Consequently, if there is to be a problem with the Glover (2010) model, it should be clearest for a two phase system.

Considering a two phase system. Equation 1 gives $\phi_1 = 1 - \phi_2$ while Equation 4 can be written as $\phi_1^{m_1} + \phi_2^{m_2} = 1$. Substituting we obtain either $(1 - \phi_2)^{m_1} + \phi_2^{m_2} = 1$ or $(1 - \phi_1)^{m_2} + \phi_1^{m_1} = 1$. These equations are formally the same. They each have trivial solutions when each of the volume fractions tends to unity, the other volume fraction consequently tending to zero. Another solution occurs when $m_1 = m_2 = 1$, which is the simple parallel conduction model. Only one other

solution exists for the general case where the volume fractions are variable, and that requires $m_1 > 1$ when $m_2 < 1$ or vice versa. Consequently the non-trivial solution for a 2-phase medium falls into one of the following classes:

(i) $m_1 = m_2 = 1$, the phases, whatever their volume fractions, are arranged in parallel and both have a unity exponent.

(ii) $m_1 > 1$ and $m_2 < 1$. This implies that Phase 1 has a path across the 3D medium that is less connected than a parallel arrangement of that phase. Since we have a two phase medium Phase 2 must have a path across the medium which is more connected than a parallel arrangement, hence forcing $m_2 < 1$.

(iii) $m_1 < 1$ and $m_2 > 1$. Since the system is symmetric. This scenario is formally the same as (ii) above, but with the phase numbers switched around.

Consequently, for a two-phase medium, defining the porosity and connectedness (or exponent) of one of the phases immediately fully defines the other phase. For higher numbers of phases, there are more solutions, but if the porosity and connectedness (or exponent) of n-1 of the phases is known, the n^{th} phase is also fully defined in the same way. The logical extension of this idea is that both the sum of the volume fractions of the n-phases is unity and the sum of the connectednesses of the n-phases is also unity, or that both volume fraction and connectedness are conserved in a three-dimensional n-phase mixture.

Another, more intuitive way of looking at this is as follows. It has already been shown that the connectedness of a system that contains only one phase is unity as a result of Equation 1, i.e., if there is one phase $\phi = 1$ and hence $G = 1$. Let us imagine that a second phase is introduced. Intuitively, it seems reasonable that as the phase fraction of the new phase increases, its connectedness will increase, and that when this happens both the volume fraction and connectedness of the first phase will decrease. The same would be true if any number of new phases were introduced – all the phases would compete for a fixed amount of connectedness, its increase for one phase being balanced by a decrease in at least one of the other phases. In other words there is a fixed maximum amount of connectedness possible in a three-dimensional sample, expressed by Glover (2010) as Equation 4.

Figure 1 is an illustrative example of the idea of a fixed amount of connectedness, using a 2D slice for simplicity and clarity. Hence, Figure 1 shows a two dimensional slice through a 3D 4-phase water-wet medium composed of detrital quartz grains, a string of clay, and a porosity that is partially filled with water, at near irreducible saturation and oil. The figure should be read in two columns. The left hand column shows an arbitrary arrangement of the four phases that together completely make up the medium (Fig. 1a). In this case I have chosen to represent the detrital quartz as sub-angular detrital grains with a grain size distribution, the clay as a stringer, the near-irreducible water as covering the quartz grain surfaces and the oil as occupying the centre parts of the pores as these geometries can be found in typical water-wet shaly sandstone reservoirs. It should be noted, however, that the equations make no such distinction and what follows is true for any geometrical set of 4-phases composing the 3D medium completely. Reading downwards, Figure (c), (e), (g) and (i) show each of the quartz, clay, water and oil phases alone and respectively. One can imagine that each phase has a certain phase fraction and a certain connectedness. Some of the phases look disconnected in the figure, but it should be remembered that there will be a greater connectedness in reality because there will be connection in the third dimension that are not shown in the figure. If we imagine hydraulic flow or electrical flow from the bottom to the top of the medium, the quartz seems to have a relatively high phase fraction and a moderate connectedness, the clay to have a moderate phase fraction and a high connectedness, the water to have a low phase fraction but a relatively high connectedness due to the multiple pathways formed by the thin 'ribbons' of water, and the oil has a moderate phase fraction, but a relatively low connectedness as the patches of oil are relatively isolated. The right hand part of the figure represents the same medium but with the small addition of a quartz grain, labelled 'Q', and its accompanying thin film of surface water. The addition of this makes a miniscule increase in the phase fractions of the detrital quartz and water phase fractions, and, literally, an equally small decrease in the phase fractions of the clay and oil. Reading the distributions for the quartz, clay, water and oil phases alone (Figure (d), (f), (h) and (j)) shows that the addition has made a significant increase in the connectedness of the quartz as well as some increase in that of the water, which was well connected

anyway. The low connectedness of the oil will have changed little, but the addition has blocked the main pathway through the clay, leaving only a minor secondary pathway, and consequently resulting in a significant decrease in the clay connectedness. Consequently Figure 1 shows the principle behind the idea of the conservation of connectedness given in Equation (4), but not a proof, the latter of which is considered in Glover (2010).

In summary, both the sum of the volume fractions and the sum of the connectednesses of the phases composing a 3D medium is equal to unity. The corollary is that connectedness is conserved; if the connectedness of one phase diminishes, there must be an increase in the connectedness of one or more of the other phases to balance it.

It is interesting to consider the role of percolation effects within the generalized model (see Glover (2010) for a full treatment). In percolation theory, the bulk value of a given transport property is only perturbed by the presence of a given phase with a well-defined phase conductivity after a certain phase volume fraction has been attained. This critical volume fraction is called the percolation threshold. This works well for a two-phase system when one phase is nonconductive, with a percolation threshold occurring near the 0.3316 to 0.342 (Montaron, 2009). For such a system, one non-conducting and one conducting phase, the effective conductivity of the medium depends only on the conductivity of the conducting phase, its volume fraction and how connected it is. It is intuitive, therefore, that there may exist a phase volume fraction below which the conducting phase is not connected and for which the resulting effective conductivity will be zero. The concept of a percolation factor becomes unclear if the matrix phase has a non-zero conductivity or one or more additional, either solid or fluid conducting phases are added. Under these circumstances a percolation threshold may not exist. Glover (2010) went further than this claiming that Equation (4) in this work (which is Equation (26) in Glover (2010)) contains enough information to make the explicit inclusion of percolation effects unnecessary.

4 Origin of the saturation exponent

Within the framework of the classical Archie's laws it is possible to envisage the cementation exponent as controlling how the porosity is connected within the rock sample volume, and to envisage the saturation exponent as controlling how the water is connected within that porosity. The cementation exponent is defined relative to the total volume of the rock, while the saturation exponent is defined relative to the pore space, which is a subset of the whole rock. This is an important concept for what follows.

The water is one of two phases within the porosity, while that porosity is one of two phases within the rock. Hence, there exists a three phase system to which the generalised Archie's law can be applied. In fact, the generalised Archie's law can be used to show that the saturation exponents arise naturally and have a physical meaning: they are defined in the same way as the phase exponents but are expressed relative to the pore space instead of the whole rock.

By writing the generalized law (Equation 4) for three defined phases; let's say matrix, water and hydrocarbon gas, and assuming that neither the matrix nor the gas is conductive, *i.e.*, $\sigma_m = 0$ and $\sigma_h = 0$, but allowing the pore space to be partially saturated with water such that $\phi_h \neq 0$, it is possible to obtain $\sigma_{eff} = \sigma_f \phi_f^{m_f}$. This is a re-expression of Equation (4), which is the sum of three terms, one for each phase, two of which are zero because the conductivity of the material which makes up each of those is zero (*i.e.*, the matrix and hydrocarbon). The exponent m_f is the phase exponent of the fluid phase, which is the only phase contributing to the effective conductivity of the three phase medium. Since $\phi_h \neq 0$, the pore space is partially saturated with hydrocarbon and partially saturated with water. It is also possible to write $\phi_f = \phi S_w$, and hence obtain

$$\sigma_{eff} = \sigma_f \phi^{m_f} S_w^{m_f} \quad . \tag{5}$$

Comparison with the classical Archie`s laws, which can be written as $\sigma_{eff} = \sigma_f \phi^m S_w^n$ (Tiab and Donaldson, 2004) shows structural similarity. However, the exponent m_f in Equation 5 is expressed relative to the whole rock because it is the phase

exponent for the fluid that appears in Equation 4. By contrast, although the cementation exponent *m* in the classical first Archie`s law is expressed relative to the whole rock, the saturation exponent *n* is related to the pore space which is a subset of the whole rock. The distinction between whether the exponent is expressed relative to the whole rock or relative to a subset of the rock, such as the pore space, can be made easily by imagining whether the saturation exponent is independent of any changes one might make to the rock matrix. In this case it is possible to see that the saturation exponent is independent of the rock matrix, and is only sensitive to changes occurring within the pore space. Consequently, it is expressed relative to the pore space rather than the whole rock.

Accordingly, both equations provide a valid measure of the effective rock conductivity, so they may be equated as $\sigma_f \phi^{m_f} S_w^{m_f} = \sigma_{eff} = \sigma_f \phi^m S_w^n$, hence obtaining $\phi^m = \phi^{m_f} S_w^{(m_f - n)}$. It can be recognized that the classical Archie saturation exponent refers to saturation with water and is hence renamed as n_f, giving

$$\phi^m = \phi^{m_f} S_w^{(m_f - n_f)}. \tag{6}$$

It is important to realize that the exponent n_f is a 'saturation' exponent that refers to the arrangement of the water phase within the pore space. In other words it is expressed with respect to the pore space, not the whole rock, and is found experimentally by varying the saturation of the water in the pore space, the latter of which is assumed to always remain unchanged.

Now it is possible to write Equation (6) in terms of connectednesses. The left hand side of Equation (6) is simply the connectedness of the pore space, as defined by Equation (1). It is the phase volume fraction of the pore space, i.e., the classical porosity, raised to the power of the phase exponent that contains the information about how that pore space is distributed, which is the classical cementation exponent *m*. Consequently we can write $G_{pore} = \phi^m$, and Equation (6) becomes

$$G_{pore} = \phi^{m_f} S_w^{(m_f - n_f)}. \tag{7}$$

The right hand side of the equation may be rewritten as $(\phi S_w)^{m_f} / S_w^{n_f}$ which allows Equation (7) to be written as

$$G_{pore} S_w^{n_f} = (\phi S_w)^{m_f}. \tag{8}$$

The term in brackets is simply the phase fraction of the water with respect to the whole rock, i.e., $\phi_f = \phi S_w$ and the exponent m_f is simply the phase exponent of the fluid phase with respect to the whole rock. Consequently, Equation (1) can be applied for the fluid phase leading to

$$G_f = \phi_f^{m_f} = (\phi S_w)^{m_f}, \tag{9}$$

which, when substituted into Equation (8) and rearranged gives

$$G_f = G_{pore} S_w^{n_f}. \tag{10}$$

This equation is for one fluid phase, i.e., water, occupying the pore space. Since the system is symmetric Equation (10) can be generalized for any of the fluid phases occupying the pore space

$$G_j = G_{pore} S_j^{n_j}, \tag{11}$$

where G_j is the connectedness of fluid *j*, S_j is its saturation, and the exponent n_j is a 'saturation' exponent that refers to the arrangement of the water phase within the pore space. In other words n_j is expressed with respect to the pore space, not the whole rock.

However, there is nothing geometrically special about the entity we call the pore space or any distinction between solid and fluid phases that compose the whole rock. Consequently, Equation (11) is only a partial generalization, and it is possible to extend the result in Equation (10) to any phase of *i* phases composing a three-dimensional medium each of which partially or fully occupies a saturation S_i of a subset of the medium whose connectedness is given as G_{ref}, according to

$$G_i = G_{ref} S_i^{n_i} . \quad (12)$$

The pore connectedness is relabelled as the reference connectedness because the equation is valid not only for multiple phases that fill the porosity, but multiple phases composing any other phase.

Equation 12 gives the connectedness of the i^{th} phase in an n-phase 3D medium as depending on both its fractional saturation S_i within a larger volume which has a connectedness G_{ref} and that reference connectedness. The distribution of that saturation is taken into account by the exponent n_i, which will have a general functional form.

If one considers the whole 3D n-phase medium (i.e., one where $\sum_i \phi_i = 1$), Equation 1 states that the connectedness of each phase is the volume fraction of that phase raised to the value of its phase exponent, and Equation 4 states that the sum of those connectednesses is unity.

If a subset of a whole n-phase medium (i.e., one where $\sum_i \phi_i < 1$) is considered, and labelled the reference subset, the reference subset will have a connectedness $G_{ref} = \phi_{ref}^{m_{ref}}$ relative to the whole rock, and the connectedness of any phase which partially occupies the reference subset (e.g., water within the pore space, clay within the rock matrix etc.) is equal to the connectedness of the reference phase multiplied by the volume fraction of the phase within the reference subset (i.e., the saturation relative to the reference subset) raised to the value of its saturation exponent.

The definition above is somewhat complex due to the requirement to be both completely general and precise, and that there are two reference frames here. The first is the whole 3D n-phase medium. The second is the 3D reference subset which may contain between 2 and n-1 phases. Conversion between the two reference frames can be carried out using the relationship

$$\phi_i^{m_i - n_i} = \phi_{ref}^{m_{ref} - n_i} , \quad (13)$$

It can also be shown that (Glover, 2010)

$$\sum_i S_i^{n_i} = 1, \quad (14)$$

where the sum is carried out over all the phases within the reference subset.

It should be noted that Equation 14 is formally the same as Equation 4 except that Equation 14 is valid for the reference subset of phases, while Equation 4 is valid for the whole n-phase medium. Hence it is possible to use $S_i = \phi_i / \phi_{ref}$ to write both Equation 4 and Equation 14 as

$$\sum_i \left(\frac{\phi_i}{\phi_{ref}} \right)^{m_i} = 1. \quad (15)$$

- For a whole n-phase medium $\phi_{ref} = 1$ and Equation 15 becomes equal to Equation 4.
- For a subset of the n-phase medium $\phi_{ref} < 1$ and Equation 15 becomes equal to Equation 14.

The distinction between the phase exponent and saturation exponent becomes trivial; they each control how connected the phase is relative to the reference volume fraction. In other words, the transformation

$$1 \leftrightarrow \phi_{ref} \text{ leading to } \phi_i \leftrightarrow S_i \text{ and } m_i \leftrightarrow n_i . \quad (16)$$

Figure 2 illustrates the concept of a subset of an n-phase medium using a 2D slice from a 3D medium. Figure 2(a) shows a simple 2 phase situation where Phase 1 is brown and Phase 2 is yellow. Both phase are connected across the medium from top to bottom, and were they not in the 2D slice, they would likely be connected through the third dimension. Phase 1 (brown) can be considered as the solid matrix of a rock, and Phase 2 (yellow) is considered to be the pore spaces in the rock for the purposes

of this illustration, but the distinction is arbitrary. The rock matrix has a phase fraction ϕ_1 and a connectedness $G_1 = \phi_1^{m_1}$ and the pore space has a phase fraction ϕ_2 and a connectedness $G_2 = \phi_2^{m_2}$ (Eq. (1)). Both of these are expressed with respect to the whole medium that is bounded in the figure by the dashed box. Consequently, $\phi_1 + \phi_2 = 1$ and $G_1 + G_2 = 1$ (Eqs. (3) and (4)).

The pore space may be occupied by any number of miscible or immiscible fluids. Let us assume there are 2 immiscible fluids completely occupying the pores, which are water and oil, and which we will assign the names Phase 3 and Phase 4. Figure 2(b) shows this situation. Once again the phase fraction and connectedness of each of the three phases that compose the medium can be defined as phase fractions ϕ_1, ϕ_3 and ϕ_4 and $G_1 = \phi_1^{m_1}$, $G_3 = \phi_3^{m_3}$ and $G_4 = \phi_4^{m_4}$ for the solid matrix, water and oil, respectively. Since these parameters are being considered with respect to the whole medium it is possible to write $\sum_{i=1,3,4} \phi_i = 1$ and $\sum_{i=1,3,4} G_i = 1$.

However, it is possible to use a different reference medium for calculations. For example the classical Archie's second law is expressed in terms of saturations, which uses the pore space as a reference space in order to express the amount of water and hydrocarbons, not with respect to the total volume of the rock, but as a fraction of the pore space. Let us, therefore also take the pore space as a convenient reference sub-space of the whole medium. This situation is shown in Figure 2(c) where the dotted line delineated the extent of the reference space. In this space (i) what was the whole medium, represented by unity in the transform given in Eq. (16) becomes the volume fraction of the reference space $1 \leftrightarrow \phi_{ref}$ (i.e., the pore space in this example, (ii) the volumes of the different phases are more efficiently described using saturations Si with respect to the reference space (i.e., the pore space) than using phase volume fractions which are defined relative to the whole medium $\phi_i \leftrightarrow S_i$, and (iii) the whole medium connectednesses $G_i = \phi_i^{m_i}$ are replaced by the entity $S_i^{n_i}$, which uses the saturation exponent in place of the phase exponent $m_i \leftrightarrow n_i$. It will be seen that the entity $S_i^{n_i}$ has its own properties in the next section and will be labelled the fractional connectedness. Topologically, the occupation of the fluids within the pore space (Figure 2(c)) is identical to the occupation of the whole medium by the matrix and pore space (Figure 2(a)), which leads to the symmetry in the mathematical equations.

The transformation given in Eq. (16) is perhaps not immediately clear when expressed in these most general terms. Let us take an illustrative example. Imagine a three-dimensional 5-phase medium where the phases are (i) detrital quartz (*dq*), (ii) calcite cement (*cc*), (iii) distributed clay (*dc*), (iv) saline water (*sw*), and (v) hydrocarbon gas (*hg*), where the subscripts that will be used for each phase are given in parentheses. First let us consider the whole medium (i.e., $\phi_{ref} = 1$). Each of the phase volume fractions are given by ϕ_{dq}, ϕ_{cc}, ϕ_{dc}, ϕ_{sw}, and ϕ_{hg}, respectively. Each of their connectednesses is equal to their phase volume fraction raised to the power of their phase exponents (according to Equation (1)), where the phase exponents contain the information about how each of the five phases is distributed in the medium. The connectednesses are $G_{dq} = \phi_{dq}^{m_{dq}}, G_{cc} = \phi_{cc}^{m_{cc}}, G_{dc} = \phi_{dc}^{m_{dc}}, G_{sw} = \phi_{sw}^{m_{sw}}, G_{hg} = \phi_{hg}^{m_{hg}}$, respectively. Equation (15) can be used, setting $\phi_{ref} = 1$, to give

$$\phi_{dq}^{m_{dq}} + \phi_{cc}^{m_{cc}} + \phi_{dc}^{m_{dc}} + \phi_{sw}^{m_{sw}} + \phi_{hg}^{m_{hg}} = 1. \qquad (17)$$

This is the same result as applying Equation (4) directly. It is expressed in terms of the parameters (i) $\phi_{ref} = 1$ (i.e., the whole medium), (ii) individual phase fractions (ϕ_i), and (iii) individual phase exponents (m_i); the latter two of which are expressed relative to the whole medium. These are the conditions and parameters expressed by the left hand components of the transformation given by Equation (16).

Now consider the subset of the whole medium which comprises just its solid parts. The reference fraction ϕ_{ref} is the sum of the solid phase fractions (i.e., $\phi_{dq}^{m_{dq}} + \phi_{cc}^{m_{cc}} + \phi_{dc}^{m_{dc}}$), which is less than unity. Rewriting Equation (15) for the reference subset gives

$$\left(\frac{\phi_{dq}^{m_{dq}}}{\phi_{dq}^{m_{dq}}+\phi_{cc}^{m_{cc}}+\phi_{dc}^{m_{dc}}}\right)^{n_{dq}}+\left(\frac{\phi_{cc}^{m_{cc}}}{\phi_{dq}^{m_{dq}}+\phi_{cc}^{m_{cc}}+\phi_{dc}^{m_{dc}}}\right)^{n_{cc}}+\left(\frac{\phi_{dc}^{m_{dc}}}{\phi_{dq}^{m_{dq}}+\phi_{cc}^{m_{cc}}+\phi_{dc}^{m_{dc}}}\right)^{n_{dc}}=1, \quad (18)$$

which can be written in terms of 'saturations' (i.e., fractional volumes of the reference subset) as

$$S_{dq}^{n_{dq}}+S_{cc}^{n_{cc}}+S_{dc}^{n_{dc}}=1, \quad (19)$$

because $S_{dq}=\phi_{dq}^{m_{dq}}/\left(\phi_{dq}^{m_{dq}}+\phi_{cc}^{m_{cc}}+\phi_{dc}^{m_{dc}}\right)$ etc.

There are two important aspects about Equation (19) to note. First, there are no terms for the saline water and hydrocarbon gas in the equation because these phases are not present in the reference subset. Second, that the phase exponents that were used when considering the whole medium have been replaced by saturation exponents because we are now considering the distribution of each of the phases within the reference subset rather than within the whole medium. Third, both Equation (17) and Equation (19) are simultaneously true and may be equated.

Equation (19) is clearly the same as Equation (14). Under the transformation that considers a subset of the whole medium (in this case the solid fractions only) where $1\leftrightarrow\phi_{ref}$ the individual phase fractions rlating to the whole medium are replaced by saturations relative to the subset (i.e., $\phi_i\leftrightarrow S_i$) and the original phase exponents, which were related to the whole medium are now 'saturation' exponents that are related only to the reference subset (i.e., $m_i\leftrightarrow n_i$).

Both the phase (cementation) exponent and the saturation exponent control how the phase is connected. The phase exponent does this with reference to the whole rock, while the saturation exponent does it with reference to a subset of the whole rock. The underlying physical meaning of the saturation exponent is the same as that of the phase (cementation) exponent, it is only the reference frame that changes. The implication is that the general Archie's law replaces both of the classical Archie's laws. For an application to a sandstone gas reservoir, one would use a 3 phase generalized Archie law.

Equation 12 is easily transformed to provide a calculable value for the saturation exponent by taking the logarithm of both sides of Equation (12) and rearranging the result before substituting Equation (1) for the relevant connectednesses and using the relationship $S_i=\phi_i/\phi_{ref}$ to obtain

$$n_i=\frac{\log(G_i)-\log(G_{ref})}{\log(S_i)}=\frac{m_i\log(\phi_i)-m_{ref}\log(\phi_{ref})}{\log(\phi_i)-\log(\phi_{ref})}. \quad \mathbf{(20)}$$

This equation may be illustrated using a three phase medium. Imagine a reservoir rock with a 20% porosity. The pore space contains only oil and water with a water saturation of 0.25. We want to calculate the saturation exponent of the water if the phase exponents of the matrix and the oil are 0.2 and 1.68, respectively. It is simple to calculate the volume fractions of matrix oil and water to be 0.8, 0.15 and 0.05, respectively. The connectednesses of matrix and oil can be calculated using Equation (1) to be 0.956 and 0.0413, respectively. Using Equation (4) we obtain the connectednesses of the pores and water to be 0.0436 and 0.00236, respectively. In this case the reference subset is the pore space so $G_{ref}=G_{pore}=0.0436$. Equation (20) can now be used with $G_{water}=0.00236$, $G_{ref}=0.0436$ and $S_w=0.25$ to give $n_w=2.105$. The saturation exponent of the oil can also be calculated as $n_o=0.1931$. There is no value for the matrix as the matrix is not included in the pore space reference subset.

There is a reiterative symmetry in this transformation where both the whole medium phase fractions and the reference subset saturations are both volume fractions with respect to the whole medium and the reference subset, respectively. Similarly, the phase exponents and the saturation exponents are also defined with respect to the whole medium and the reference subset, respectively. This would, therefore, allow the calculation of a reference subset of a subset of a whole medium if required, and so on. There is of course the possibility that the whole *n*-phase medium is itself a subset of a larger medium with more phases. In this case Equation 15 still holds, but with $\phi_{ref}>1$. The implication is that the definition of the original whole medium is arbitrary and can be defined to make the solution of the problem more tractable.

5 Physical interpretation of the saturation exponent

This section provides a physical interpretation for the saturation exponent in a perfect analogy to that derived for the cementation exponent by Glover (2009).

The connectedness G is the inverse of the Archie's formation factor and is central to the generalized Archie's law. The inverse of the Archie's resistivity (saturation) index $1/I = S_w^n$ is also rather important. It relates the connectednesses of each phase with respect to the whole rock to the connectedness of the reference subset in Equation 12, and when summed over all the phases that occupy the reference subset it produces unity as in Equation 14. In this paper the inverse of the Archie's resistivity (saturation) index has been given the symbol H_i and defined as

$$H_i \equiv S_i^{n_i}. \tag{21}$$

Just as the saturation of any given phase S_i is the ratio of the volume fraction of the phase to that of the all the phases making up any reference set of phases, H_i is the ratio of the connectedness of the phase to that of the all the phases making up any reference set of phases. The parameter H_i is in fact a fractional connectedness.

We follow the approach of Glover (2009) in the analysis of the physical interpretation of the cementation exponent. In this work Glover (2009) showed that the cementation exponent was the differential of the connectedness with respect to both porosity and pore connectivity. Following the same methodology, differentiating the fractional connectedness with respect to the phase saturation S_i gives

$$\frac{\partial H_i}{\partial S} = n_i S_i^{n_i-1}. \tag{22}$$

By analogy we recognize that $S_i^{n_i-1}$ represents the connectivity of Phase i with respect to the reference subset and define this connectivity as

$$\psi_i = S_i^{n_i-1}, \tag{23}$$

to give

$$\frac{\partial H_i}{\partial S} = n_i \psi_i. \tag{24}$$

A further differentiation, this time with respect to the connectivity ψ_i allows us to obtain

$$n_i = \frac{\partial}{\partial \psi}\left(\frac{\partial H_i}{\partial S}\right). \tag{25}$$

Consequently, the saturation exponent is the rate of change of fractional connectedness with respect to both phase saturation and phase connectivity in a similar way that Glover (2009) found that the physical interpretation of the cementation exponent was the rate of change of connectedness with respect to phase fraction (porosity) and its connectivity. This shows once again the symmetry between phase fractions and saturations and between phase exponents and saturation exponents.

The fractional connectedness is also the product of the saturation and the connectivity with respect to the reference subset

$$H_i = S_i \psi_i. \tag{26}$$

Hence, the saturation exponents obey the same laws as the phase (cementation) exponents, but whereas the phase exponents are defined relative to the whole rock, the saturation exponents are defined relative to some subset of the rock. Table 1 shows the relationships of the generalised Archie's law expressed relative to the whole rock and with respect to a reference subset of the whole rock.

For petrophysicists the reference subset has been the porosity and there has only been one conducting phase that partially saturates that porosity – the pore water. Now we are not restricted to that model. The reference subset could be, for example, the solid matrix in which a number of separate mineral phases can be defined, one of which might be, say, a target ore or a clay phase. Let us take a 4-phase medium as an example. Imagine a 4 phase medium composed of 65% quartz matrix with a

phase volume exponent of 0.3, 15% clay. Consequently, the medium's porosity is ϕ=0.2. The porosity is occupied with gas and saline water with saturations S_g=0.625 and S_w = 0.375, respectively and the classical cementation exponent m = 1.8 and the classical saturation exponent is n = 2.05. Imagine needing to calculate the resistivity of the rock if the resistivity of the clay and the water are known, ρ_{clay} = 50 Ω.m and ρ_{water} = 5 Ω.m, say. Equation (1) can be used to calculate G_{quartz} = 0.8788 and G_{pore} = 0.0552. Using Equation (4) provides G_{clay}= 0.0660 with no need to consider the various saturations of the fluids occupying the pores. The phase exponent of the clay can be found to be m_{clay}=1.43. The contribution of the clay to resistivity can be calculated using Equation (3), rewritten as $\rho_{contclay} = \rho_{clay}\phi_{clay}^{-m_{clay}} = \rho_{clay}/G_{clay}$, as ρ_{clay} = 757 Ω.m, noting that this value takes full account of its volume fraction and its geometrical distribution. Now we must consider the relative distributions of water and gas in the medium. Calculations can be carried out in terms of connectednesses G or fractional connectednesses H. In this case we use the connectednesses G. Equation (11) or (12) can be used to calculate G_{water}=0.00739, and Equation (4) applied to give G_{gas}= 0.0478. Once again Equation (1) may be applied, but this time in the rearranged form $m_i = \log G_i / \log \phi_i$ in order to calculate the respective phase exponents m_{water} = 1.895 and m_{gas} = 1.462. Now, the contribution of the saline water to the overall resistivity can be calculated using Equation (3), rewritten as $\rho_{contcwater} = \rho_{water}\phi_{water}^{-m_{water}} = \rho_{water}/G_{water}$, as ρ_{water} = 677 Ω.m, noting that this value takes full account of its volume fraction and its geometrical distribution. The resistivity of the rock can now be calculated by simply summing the contributions to conductivity as implied by Equation (3) to give ρ_{eff}= 357 Ω.m. In this particular example, the conductivity of the medium is controlled by the clay and water fractions in approximately equal measure. It should also be noted that there are a number of different pathways for obtaining the same result using the equations contained in this paper.

6 Conclusions

The main conceptual steps in this paper are summarised as:

- The classical Archie saturation exponent arises naturally from the generalised Archie's law.
- The saturation exponent of any given phase can be thought of as formally the same as the phase (i.e., cementation) exponent, but with respect to a reference subset of phases in a larger n-phase medium.
- The connectedness of each of the phases occupying a reference subset of an n-phase medium can be related to the connectedness of the subset itself by $G_i = G_{ref}\, S_i^{n_i}$.
- The sum of the connectednesses of a 3D n-phase medium is given by $\sum_i \phi_i^{m_i} = 1$, mirroring the relationship for phase volumes $\sum_i \phi_i = 1$.
- Connectedness is conserved in a 3D n-phase medium. If one phase increases in connectedness the connectedness of one or more of the other phases must decrease to compensate for it, just as phase volumes are conserved with the decrease in one leading to the increase of another phase.
- The sum of the fractional connectednesses (saturations) of an n-phase medium is given by $\sum_i S_i^{n_i} = 1$.
- Fractional connectedness is conserved in a 3D n-phase medium.
- The saturation exponent may be calculated using the relationship $n_i = \dfrac{m_i \log(\phi_i) - m_{ref} \log(\phi_{ref})}{\log(\phi_i) - \log(\phi_{ref})}$.
- The connectivity of any phase with respect to the reference subset is given by $\psi_i = S_i^{n_i - 1}$.

- The connectedness of a phase with respect to a reference subset (also called the fractional connectedness) is given by $H_i = S_i \psi_i$ and depends upon the fractional volume of the phase divided by that of the reference subset (i.e., its saturation) and the arrangement of the phase within the reference subset (i.e., its connectivity with respect to the reference subset).
- The rate of change of fractional connectedness with saturation $\frac{dH_i}{dS_i} = n_i \psi_i$ depends upon the connectivity with respect to the reference subset ψ_i and the saturation exponent n_i.
- Hence, the saturation exponent is interpreted as being the rate of change of the fractional connectedness with saturation and connectivity within the reference subset, $n_i = \frac{d^2 H_i}{d\psi_i dS_i}$.

While this paper represents a theoretical treatment of the saturation exponent and attempts to develop a theoretical interpretation that should offer insight into the physical meaning of the saturation exponent, it does not contain a physical proof of these equations. That can only come from targeted experimental work on multi-phase media which are difficult to carry out and represent one of our research goals.

Table 1. Comparison of all the parameters in the classical and generalised Archie's laws.

Parameter	Generalised Archie's law		Classical Archie's law	
	With respect to the whole medium	**With respect to a reference subset of the whole medium**	**First law**	**Second Law**
Phase volume fraction	ϕ_i $\phi_i = \phi_{ref}\, S_i$	S_i $S_i = \phi_i / \phi_{ref}$	ϕ $V_f = V_{pore}\, S_w$	S $S_w = V_f / V_{pore}$
Exponent	$m_i = \dfrac{d}{d\chi}\left(\dfrac{dG_i}{d\phi}\right)$ $m_i = \dfrac{\log(\sigma_i) - \log(\sigma_f)}{\log(\phi_i)}$	$n_i = \dfrac{d}{d\psi}\left(\dfrac{dH_i}{dS}\right)$ $n_i = \dfrac{m_i \log(\phi_i) - m_{ref} \log(\phi_{ref})}{\log(\phi_i) - \log(\phi_{ref})}$	m $m = \dfrac{\log(\sigma_{eff}) - \log(\sigma_f)}{\log(\phi)}$	n $n = \dfrac{\log(\sigma_{eff}) - \log(\sigma_{100})}{\log(S_w)}$
Connected-ness	$G_i \equiv \phi_i^{m_i}$ $G_i = \phi_i \chi_i$ $G_i = 1/F_i$ $G_i = G_{ref}\, H_i$	$H_i \equiv S_i^{n_i}$ $H_i = S_i \psi_i$ $H_i = 1/I_i$ $H_i = G_i / G_{ref}$	undefined	undefined
Connect-ivity	$\chi = \phi_i^{m_i - 1}$	$\psi = S_i^{n_i - 1}$	$\chi = \phi^{m-1}$	undefined
Rate of change of connected-ness	$\dfrac{dG_i}{d\phi_i} = m_i \chi_i$	$\dfrac{dH_i}{dS_i} = n_i \psi_i$	undefined	undefined
Sum of phases	$\sum_{i=1} \phi_i = 1$ $\sum_i S_i > 1$	$\sum_{i=1} \phi_i < 1$ $\sum_i S_i = 1$	$\phi_{pore} + \phi_{matrix} = 1$	$S_w + S_o + S_g = 1$
Sum of connected-nesses	$\sum_i \phi_i^{m_i} = \sum_i G_i = 1$ $\sum_i \left(\dfrac{\phi_i}{\phi_{ref}}\right)^{m_i} = 1$ The transformation $1 \leftrightarrow \phi_{ref}$ leads to $\phi_i \leftrightarrow S_i$ and $m_i \leftrightarrow n_i$	$\sum_i S_i^{n_i} = \sum_i H_i = 1$	undefined	undefined
Effective conduct-ivity	$\sigma_{eff} = \sum_i \sigma_i \phi_i^{m_i}$	$\sigma_{eff} = \sum_i \sigma_i \phi_{ref}^{m_i} S_i^{m_i}$	$\sigma_{eff} = \sigma_f\, \phi^m$	$\sigma_{eff} = \sigma_f \phi^m S_w^n$

Data availability

This work is entirely theoretical and contains no data.

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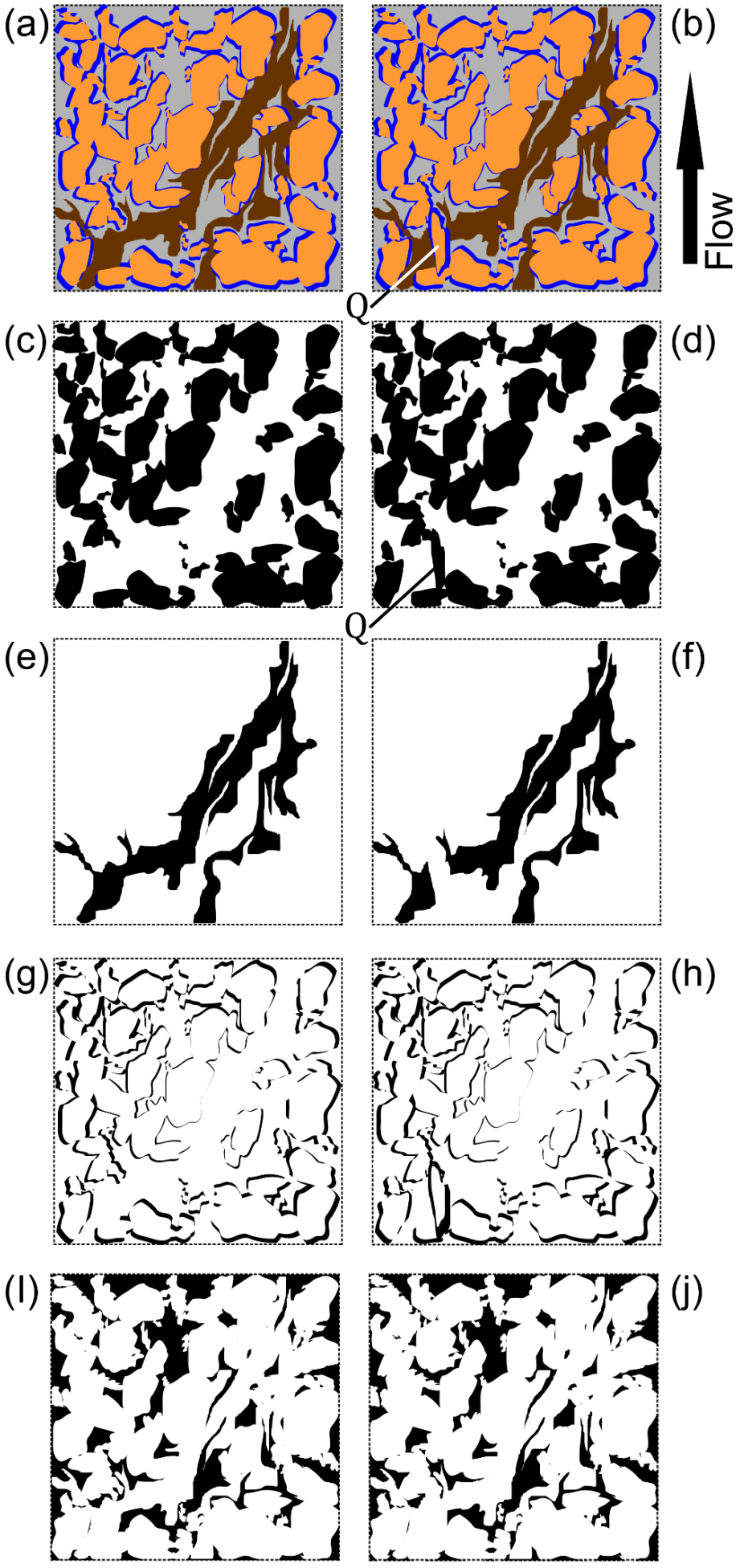

Figure 1. Distribution of a 4-phase clay-rich water-wet sandstone saturated with water and oil (quartz, orange; clay, brown; water, blue and oil, gray) represented by a 2D slice through a 3D medium. The left hand column differs from the right hand column by the addition of a single grain of quartz with its associated surface water, labelled Q. Consequently the figure should be read vertically comparing the two columns. (a) and (b) complete medium, (c) and (d), quartz distribution; (e) and (f), clay distribution; (g) and (h), water distribution; and (i) and (j), oil distribution.

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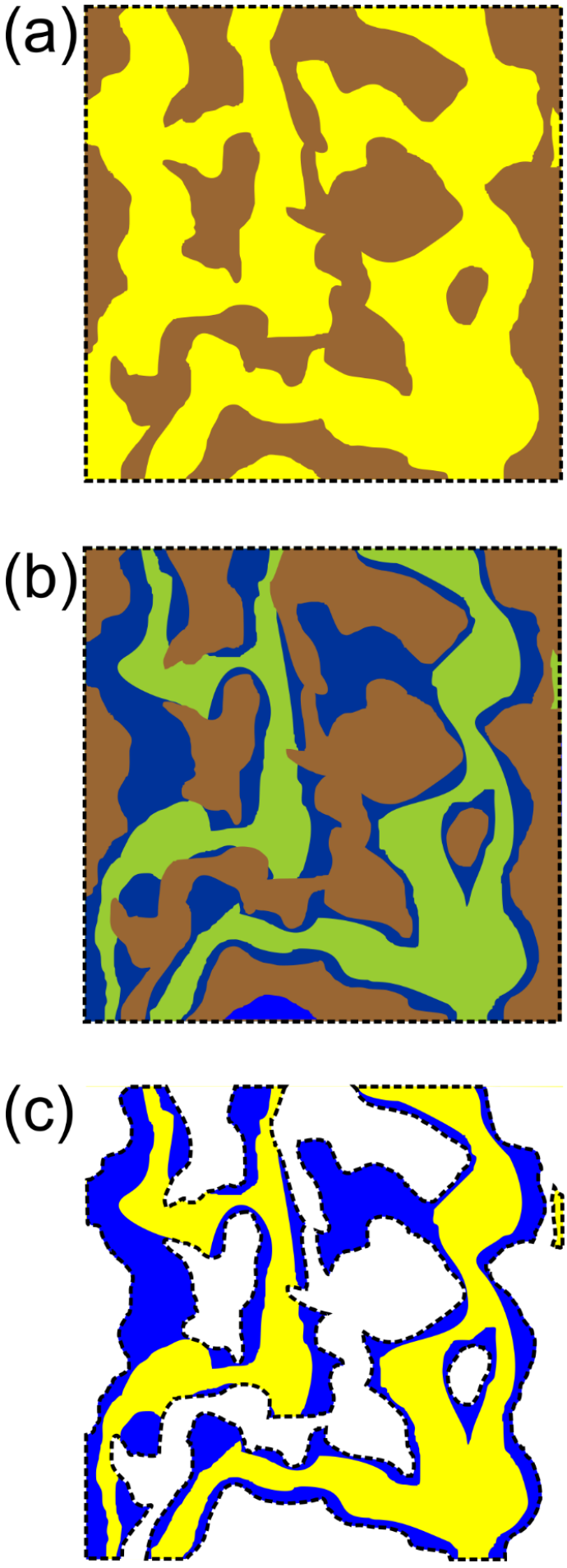

Figure 2. Sets and subsets of a 3-phase medium using a 2D slice to represent the whole 3D medium. (a) 2 phases – Phase 1, brown representing solid matrix; Phase 2, yellow, represents pore space, with unspecified fill. Phase fractions and connectednesses can be defined for each phase with respect to the whole medium (dotted box). (b) 3 phases created by filling (replacing) the porosity with 2 phases – Phase 1, brown representing solid matrix as before; Phase 2, blue representing water; Phase 3, green representing oil. Phase fractions and connectednesses can be defined for each of the three phases with respect to the whole medium (dotted box). (c) If only the pore space is considered by considering Phase 1 to be unchanging, what remains is a 2-phase subset of the 3-phase situation. Phase fractions and connectednesses can be defined for the two fluid phases with respect to the subset which is the porosity (inside the dotted interface).