AUTHOR'S RESPONSE TO REVIEWER'S COMMENTS FOR SOLID EARTH SE-2017-5

by Paul Glover

This document is structured in the following way. There are 3 reviewers who submitted their reviews chronologically. Each one is treated in turn, first quoting the comments of the reviewer, and then responding to them.

Reviewer 1's comments (Harald Milsch)

Review on paper manuscript se-2017-5

Summary
In this paper and based on earlier findings (Glover, 2009; 2010), the author derives a new theoretical interpretation of the saturation index contained in Archie's second law. The essence of this interpretation is the extension of the "generalized Archie's law" outlined by the author in Glover (2010), where the saturation index is viewed as being "formally the same as the phase exponent, but with respect to a reference subset of phases in a larger n-phase medium".

The author carried out an important task with implications for fundamental rock physics and industrial applications alike. The paper is well structured and, in my perception, mathematically sound and may definitely be suitable for publication in Solid Earth (SE).

However, there are a number of substantial issues outlined in the following that I encourage the author to address before the paper can be recommended for publication.

General comments
1. It should be noted that (1) what is attempted here is a physical interpretation of an empirical parameter, which I find per se problematic. Also, it should be noted that (2) the outlined interpretation comes as an ad hoc approach and that (3) no proof is presented that this approach and the resulting interpretation is physically correct. Please comment and clarify within the manuscript.

2. The motivation for performing this particular theoretical investigation is well presented in Section 1. However, this discussion also implicitly suggests that reserves calculations can now be performed with unprecedented precision. There is no proof that this is the case. It should also be noted that there will still be experiments and/or analyses to be performed to parameterize the newly introduced equations. How these experiments/analyses should look like and what type of data is required should be included in the text.

3. The theoretical approach is, mathematically, not very demanding but it appears abstract and hard to grasp. I therefore would wish to see (1) some of the equations to be developed in more detail (e.g. in some appendix), (2) one or more graphical representations of the model to better depict the theory, and, not least, (3) a few example calculations where for some type of rock with some kind(s) of fluid(s) some saturation index is derived and then is compared to existing (experimental) data. Please see also comments below.

Specific comments:

- Section 3; Lines 148- 152: I wonder if this is correct. What about percolation or a percolation threshold? Please comment. This comment also applies to Line 260.
- Section 3: In this section a first illustrating sketch should be introduced.
- Section 4; Line 171: Please clarify from where this equation arises.
- Section 4; Lines 178-180: Reasoning unclear. Please improve.
- Section 4; Lines 182-185 and 202: The equations contained here should be fully derived, e.g. within the section or some appendix.
- Section 4; Lines 199 and following: Here, a second illustrating sketch should be introduced.
- Section 4; Lines 213-214: Can this transformation be exemplified or illustrated?
- Section 4; Eq. (10): This equation should be fully derived and also (numerically) exemplified for a 3-phase medium like the one mentioned before in Line 221.
- Section 5: The motivation for this section is somewhat unclear and should be outlined.
- Section 5; Line 234: Please briefly recall the approach of Glover (2009).
- Section 5; Eq. (12), (13), and (14): In my opinion the derivation should be improved/expanded and also inverted such that Eq. (12) is the final outcome (as in Section 6).
- Section 5; Eq. (12): This equation is only correct if one can assume that $n_i \neq f(\Psi_i)$. Please show that this is the case.
- Section 5; Eq. (12): Please show that Eq. (12) yields Eq. (10) or vice versa.
- Section 5; Lines 248-250: To illustrate this statement and by applying either equation I would wish to see an example calculation / numerical evaluation for a 4-phase porous medium (e.g. quartz, clay, water, gas).

Technical corrections

- The expression “rate of change” suggests some time dependence/derivative and should be replaced throughout the manuscript including in Table 1 by some other, more appropriate, expression.
- Lines 51-52: please check if statement is correct.
- Glover (2016) not in reference list.
- The use of “ϕ” (phi) for both porosity and phase volume fractions may lead to confusion. Please reconsider.
- Line 125: Equation 4 (?), please check. If correct move Eq. (4) in Line 115 up in text.
- Line 131: please check indices in equation.
- Line 191: Equation 1 (?), please check.
- Line 206: Equation 7 (?), please check.
- Lines 237-238: Index “i” missing in “Ψ” (psi).

References:

Glover, P. W. J.: What is the cementation exponent? A new interpretation, The Leading Edge, 82–85, doi: 10.1190/1.3064150, 2009.

Glover, P. W. J.: A generalised Archie’s law for n phases, Geophysics, 75(6), E247-E265, doi: 10.1190/1.3509781, 2010.

Author's response Reviewer 1's comments

General comments

Reviewer's comment	Author's response
1. It should be noted that (1) what is attempted here is a physical interpretation of an empirical parameter, which I find per se problematic …	*Point (1): I disagree with Harald on a philosophical level. I believe that we should ALWAYS seek to find physical interpretations for empirical parameters since (i) it helps understand what they represent, and (ii) it may lead to a deeper understanding of the fundamental theoretical basis underlying the experimental science. It should be noted that most of classical theoretical physics began as an interpretation of empirical observations.*
… Also, it should be noted that (2) the outlined interpretation comes as an ad hoc approach and that …	*Point (2): I agree with Harald that the original MS did not contain a sufficiently robust set of derivations and could have been perceived as ad hoc. This was due to me not recognising that my familiarity with manipulations drove me to cut corners in derivations, not realising that most of my colleagues who are not familiar with these equations due to their novelty would need all of the steps to be explicit – they now are!* *I have heeded the advice of the reviewer and expanded the derivations considerably. There are now 11 extra display equations (an increase of 79%) and numerous extra in-line equations. The associated descriptive text has added about 3664 words, an increase of 73% in the length of the paper, as well as two requested figures with considerable additional explanatory text. The derivations are now extremely robust and clear. Consequently, the paper now represents a theoretical proof of the equations it contains.*
… (3) no proof is presented that this approach and the resulting interpretation is physically correct. Please comment and clarify within the manuscript.	*Point (3): Whereas the last point considered the theoretical proof of the equations in the paper, which is now explicit in the revised MS, this point considers the physical proof. That can only come from targeted experimental work, which we are currently asking for funding to carry out, but could also be carried out by colleagues who read this new interpretation. This has now been clarified within the revised MS.*
2. The motivation for performing this particular theoretical investigation is well presented in Section 1. However, this discussion also implicitly suggests that reserves calculations can now be performed with unprecedented precision. There is no proof that this is the case.	*A sentence of 52 words has been added to the relevant section to make this point clear.*
It should also be noted that there will still be experiments and/or analyses to be performed	*The need for experimental measurements has now been covered in the conclusions of the MS,*

to parameterize the newly introduced equations. How these experiments/analyses should look like and what type of data is required should be included in the text.	*and has already been commented upon in the reviewer's point above (Point (3)). The detail of the parameters and methodology for such experiments is reserved for the appropriate follow-up paper.*
3. The theoretical approach is, mathematically, not very demanding but it appears abstract and hard to grasp. I therefore would wish to see (1) some of the equations to be developed in more detail (e.g. in some appendix), …	*Harald is perfectly right here. The maths is not complex but the concepts are rather harder to grasp. This point has already been covered in the response to the first general comment above. However, to reiterate: Point (1): This has been done in the text rather than in an appendix resulting in 11 extra display equations (an increase of 79%) and numerous extra in-line equations as well as associated extra descriptive text to clarify some of the difficult conceptual jumps (about 3664 words, an increase of 73% in the length of the paper).*
(2) one or more graphical representations of the model to better depict the theory,	*Point (2): Two graphics have been added with over 1000 words of explanatory text.*
… and, not least, (3) a few example calculations where for some type of rock with some kind(s) of fluid(s) some saturation index is derived and then is compared to existing (experimental) data. Please see also comments below.	*Point (3): The paper now contains 4 separate example calculations; a 2-phase, a 3-phase, a 4-phase and a 5-phase example, at various points in the paper.*

Specific comments:

Reviewer's comment	**Author's response**
Section 3; Lines 148- 152: I wonder if this is correct. What about percolation or a percolation threshold? Please comment.	*This comment also applies to Line 260. I see Harald's point here but the problem is already known and considered at length in Glover (2010). In rereading that treatment, I find that I have nothing to add to it and cannot find a better way to say what I said concerning percolation thresholds in that paper. Consequently, I have included a short paragraph (222 words) near the first statement in order to discuss the percolation problem, including an exhortation for the reader to read the relevant parts Glover (2010) if, for them, the issue needs further clarification.*
Section 3: In this section a first illustrating sketch should be introduced.	*A new figure has been included, together with 521 words of explanatory text.*
Section 4; Line 171: Please clarify from where this equation arises.	*63 words of clarification added.*
Section 4; Lines 178-180: Reasoning unclear. Please improve.	*47 words of clarification have been added to or modified the existing text.*
Section 4; Lines 182-185 and 202: The equations contained here should be fully derived, e.g. within the section or some appendix.	*A full step-by step derivation has been added in the text, amounting to a significant addition of text and 6 display equations. The mathematics is simple but some of the conceptual steps were not. Hence, I thank the reviewer for flagging up why this derivation would not be understood easily as it was originally stated.*

Section 4; Lines 199 and following: Here, a second illustrating sketch should be introduced.	*A new figure has been included, together with an extra 525 to describe the figure as clearly as possible.*
Section 4; Lines 213-214: Can this transformation be exemplified or illustrated?	*A five phase medium has been added as an illustration, taking 497 additional words and 3 display equations.*
Section 4; Eq. (10): This equation should be fully derived and also (numerically) exemplified for a 3-phase medium like the one mentioned before in Line 221.	*This is a very simple mathematical manipulation for someone au fait with logarithms. Consequently only 27 words were added in order to explain how the equation is developed step-by-step. No extra equations were necessary. A worked example has been given (184 additional words).*
Section 5: The motivation for this section is somewhat unclear and should be outlined.	*30 words have been added including a change of section heading.*
Section 5; Line 234: Please briefly recall the approach of Glover (2009).	*This has been done in 48 additional words.*
Section 5; Eq. (12), (13), and (14): In my opinion the derivation should be improved/expanded and also inverted such that Eq. (12) is the final outcome (as in Section 6).	*This has been carried out with inversion and the addition of an extra step.*
Section 5; Eq. (12): This equation is only correct if one can assume that ni≠ f (Ψi). Please show that this is the case.	*This comment is not correct and, consequently, I have made no further changes. The new Eq. (23) shows that n_i is a function of ψ_i, but since the differential is with respect to ψ this is a hidden functionality that does not invalidate the use of differentiation nor the resulting differential equation. This mathematical nicety is now clearer thanks to the reordering the equations implemented at the request of the reviewer in the last comment.*
Section 5; Eq. (12): Please show that Eq. (12) yields Eq. (10) or vice versa.	*A proof has not been inserted because it is already implicit in the paper. In the new numbering Eqs. (10) and (12) are now Eqs. (20) and (25). Taking each separately, Eq. (20) is derived from Eqs. (6) to (11), which produces Eq. (12) from which Eq. (20) is derived). This process is now much more explicit than the original MS resulting from a response to the reviewer's previous comment, and which has led to a significant improvement in the MS. Eq. (25) is derived in the paper explicitly in Eqs. (21) to (24). Consequently, taking these two explicit derivations together gives the proof for which the reviewer is looking. It is 13 display equations long, and given that it is already in the paper I take the view that to repeat it would be unnecessary.*
Section 5; Lines 248-250: To illustrate this statement and by applying either equation I would wish to see an example calculation /	*A 4-phase example has been added (380 words), which in my view shows the power of the equations well. I am grateful to the reviewer for suggesting their inclusion. The paper now*

numerical evaluation for a 4-phase porous medium (e.g. quartz, clay, water, gas).	*contains 4 separate example calculations; a 2-phase, a 3-phase, a 4-phase and a 5-phase example, at various points in the paper.*

Technical corrections

Reviewer's comment	**Author's response**
The expression "rate of change" suggests some time dependence/derivative and should be replaced throughout the manuscript including in Table 1 by some other, more appropriate, expression.	*"Rate of change" is mentioned in the original manuscript 5 times. I disagree with the reviewer on this point. In the first place I do not consider that a "rate of change" need necessarily imply some time dependence, i.e., a rate of change with respect to time, even if the reference variable is not explicit. However, in this paper the rate of change is given explicitly with respect to either saturation or connectedness in all 5 mentions, and therefore the mathematics is very clearly described. I have not made changes because I believe the reviewer has a view of what "rate of change" means which is more restrictive than the general use.*
Lines 51-52: please check if statement is correct.	*Whoops! I gave the derivation for the cementation exponent by mistake. I have changed it so it is an accurate. Many thanks to the reviewer.*
Glover (2016) not in reference list .	*Now inserted into the reference list.*
the use of "ϕ" (phi) for both porosity and phase volume fractions may lead to confusion. Please reconsider.	*I had a long think about this. The trouble is that porosity is a phase volume fraction and so using 2 different symbols would make an artificial distinction which is not real. I think that there is not such a difficulty with leading the reader into confusion because the classical porosity is only used up until Equation 2 and then the phase volume fraction terminology takes over, generalising phase volume fractions and incorporating porosity into that structure. In order to avoid confusion I have added 95 words of clarification early in Section 3 explaining the retention of ϕ for porosity and the additional use of ϕ_i for phase volume fractions. So in the modified form, porosity is considered and used using the symbol ϕ up until line 111, and during this time there is no mention of phase volume fractions. Phase volume fractions ϕ_i are defined on Line 111 and are used exclusively for the rest of the paper. Consequently I think that the process of generalising porosity into phase volume fractions is carried out in a smooth and clear fashion.*
Line 125: Equation 4 (?), please check. If correct move Eq. (4) in Line 115 up in text.	*Corrected. Equation 4, which was actually on line 155 has been moved to line 125 in the*

	original numbering scheme. This is so that references to both equations 1 and 4 that occurred in line 125 do not have to look ahead. This problem was created by adding the 2 phase system example after the rest of the paper have been written, and adding it a little too early.
Line 131: please check indices in equation.	*One subscript corrected.*
Line 191: Equation 1 (?), please check.	*This is correct, but I have modified the sentence to make it clearer.*
Line 206: Equation 7 (?), please check.	*This should read Equation 8, and has been corrected.*
Lines 237-238: Index "i" missing in "Ψ" (psi).	*This has been corrected.*

References
Both of the suggested references were already listed in the original submission.

Referee 2's comments

The classical Archie's law is an important expression to describe the relationship between electrical property and the porosity of rocks. In this paper, the author builds a new theory to extend Archie's law and make it more completely, so that it can be applied to the n-phase medium. Although there is a Table listed to make a comparison of these two theories, I do not think it is enough to show the advances and validities of the new theory. It is better to give a simulated analysis in the paper at least. However, the author has presented that there is no data here, so hope to see the related paper soon, which interests me the best.

Author's response Reviewer 2's comments

The reviewer's only substantive comment/wish is to see a simulated analysis. The changes and the three extra examples made in the response to other reviewers, particularly Reviewer 1, should satisfy this.

Referee 3's comments (Graham Heinson)

An interesting and worthwhile paper on the importance and calculation of Archie's Law saturation exponent. I have little background in this area of petrophysics other than accepting the well-known and simple empirical relationship between resistivity, pore fluid, porosity and saturation. Glover explains both the mathematics in a careful manner and also the context for developing such a theoretical approach. The argument about estimated reserves is both dramatic and perhaps a bit ambit, but it does provide a good reason why a redefinition might matter. Of course, anything connected with such large reserves and value will have a significant effect as a small percentage.

- Line 29 starting "Since..." seems to be missing part of the sentence.
- The sentence from Line 82 - 86 is quite long and could be re-phrased.

- The flow of logic is reasonably well presented. I'm a bit confused on Line 125 that Equation 1 and 4 are mentioned, but Equation 4 does not get defined until line 155. The example for a two-phase system from Line 130 is good in highlighting a simple case.

- The sentence on Line 176 "By contrast..." could be rephrased. My take is that the exponent is related to the fractional volume of pores filled with the fluid rather than being a related to the whole rock. There is a bit of confusing sentence structure.

The conclusions are a nice summary of the paper, but need the paper to make sense of the equations. Thus, they could not really be read stand-alone. Not sure if this is a problem.

Author's response Reviewer 3's comments

The reviewer requires no general or substantial changes, modifications or additions. However, all his specific comments have been acted upon and listed below:

Reviewer's comment	**Author's response**
Line 29 starting "Since..." seems to be missing part of the sentence.	*The sentence was originally part of the following sentence and were accidentally separated during editing. The 2 sentences have now been combined again into a correct form.*
The sentence from Line 82 - 86 is quite long and could be re-phrased:	*The sentence has been split into at least 4 sentences, and now treats each concept separately.*
The flow of logic is reasonably well presented. I'm a bit confused on Line 125 that Equation 1 and 4 are mentioned, but Equation 4 does not get defined until line 155. The example for a two-phase system from Line 130 is good in highlighting a simple case.	*This problem arose because "the 2 phase system example" that the reviewer likes was a late addition that was placed too early. Consequently, Equation 4 has been moved forwards to ensure that logical steps are retained.*
The sentence on Line 176 "By contrast..." could be rephrased. My take is that the exponent is related to the fractional volume of pores filled with the fluid rather than being a related to the whole rock. There is a bit of confusing sentence structure:	*The sentence has been rephrased using some of the terminology suggested by the reviewer.*
The conclusions are a nice summary of the paper, but need the paper to make sense of the equations. Thus, they could not really be read stand-alone. Not sure if this is a problem.	*Since the reviewer considers the conclusions to be a nice summary of the paper, I have made no changes here because the conclusions make sense. If I were to remove the equations and replace them with text, the conclusions would be as long as the paper.*