Solid Earth Discuss.,
https://doi.org/10.5194/se-2018-109-AC3, 2018

Interactive comment on "Power Spectra of Random Heterogeneities of the Solid Earth" *by* Haruo Sato

Sato

haruo.sato.e2@tohoku.ac.jp

Received and published: 5 December 2018

I am grateful to Dr. Margerin for his precise comments. I will revise the paper reflecting his comments and suggestions.

General question: Figure 8 is a central result of the compilation .... If the power-law is universal then does it contain information on the dynamic processes that are at the origin of the heterogeneity? Indeed, one would expect that different tectonic processes occurring in different envelopes of the Earth leave different imprints in the power spectrum of heterogeneities at small-scale. And to some extent, this is what seismological observations -recalled by the author- also suggest. For example, the observation of guided waves indicate the presence of small-scale heterogeneities with

anisotropic scale-lengths in subducting slabs. In Japan, Pulse broadening is wildly different between fore-arc and back-arc regions. Yet Figure 8 seems to imply that the same power spectrum can match completely different geological environments at different scales. Therefore, I wonder whether the important information is really contained in the exponent of the power spectrum or if it should be used in conjunction with other measurements like frequency dependent attenuation, vp/vs ratio, etc. . . or if the model should be complexified (introduction of anisotropic scale lengths)?

Reply: Most of kappa values are distributed between 0 and 0.5 and mechanisms which create random heterogeneities are thought to vary in different environments. Figure 8 simply shows that the envelope of various power spectra well obeys a power law with kappa =0 for a wide range of wave-numbers. Not that the gray line is not the average spectrum but the spectral envelope. So far I do not have adequate answer to the above question. But, I still believe it is geophysically important to measure the power spectrum of random heterogeneities in each environment.: the location the corner wavenumber, how the flat level extends into the lower wavenumber, and how the decaying branch extends into higher wavenumbers. It is also necessary to examine the validity of mathematics used in each analysis. We will have to study more about the anisotropy of randomness, intrinsic absorption, and scattering contribution of cracks/pores, which were not considered in this review.

Technical questions: (1) The author rightfully points out that the use of the Born approximation (BA) is problematic at high-frequency. Indeed BA predicts an increase of attenuation without limit as ka tends to infinity. The author also suggests that the limit of applicability of BA is the same as the limit of applicability of the Bourret approximation from mean-field theory. Yet, the catastrophic increase of attenuation predicted by BA does NOT occur in the Bourret approximation. Bourret approximation does in fact predict the same geometrical limit as the phase screen approximation, although in a much less transparent way since it involves the solution of an implicit equation for the wavenumber of the mean field. It is only when the solution of this implicit equation is simplified by employing the BA that the Bourret approximation fails. But conceptually, I think that the limit of validity of the 2 approximations should be distinguished.

Reply: Thank you for the above comments on the Bourret approximation. I will restrict myself to point out the problem of the conventional Born app. in this paper. I will delete the sentence about the Bourret approximation on page 8 ". . ... . . which is the same as the criterion of the Bourret approximation (Rytov et al., 1989)." in the revision.

(2) The author explains that the Phase Screen Approximation cannot model the coda. It is not clear to me how the sentence should be interpreted. Certainly the method cannot model wide-angle scattering. At the same time, if the random walker takes a large number of steps, it may eventually come back to its starting point, thereby generating a coda. In optics, this situation is very common. For instance, light diffusion in tissues is in a regime of very strong forward scattering, where the transport mean free path (the length scale of the diffusion process) is much larger than the mean free path (the length of a single step of the random walker). Could the author elaborate a bit on this point?

Reply: I understand that you are talking about the coherent back scattering phenomena. I agree with the reviewer's comment. As shown in Figure 8 of Sato and Emoto (GJI, 2018), a decaying coda is shown according to the RTT with the phase screen app. In the revised version, I will rephrase this as follows: " The phase screen approximation is not adequate for wide angle scattering but for narrow angle scattering."

(3) In the text, the author refers to various estimates of the power spectrum of heterogeneities based on different sampling of the random process. Some estimates are based on 1-D sampling, others on 2-D sampling. It would be interesting to briefly recall how one extrapolates from either a 1-D or 2-D power spectrum to a 3-D one. What are the necessary assumptions (isotropy?) and what is the relation between the measured 1-D or 2-D power law and its 3-D extrapolation?

Reply: I assume isotropic randomness in the conversion from 1D to 3D. For von Kar-

man type, kappa value is common to different dimensions. Attached pdf shows the mathematics (after Sato et al. 2012).

(4) Would it be possible to explain in simple terms why the classical BA and the phase screen approximation disagree for ka < 0.2 in the example shown in Figure 5?

Reply: In the case k a <1, the conventional Born app. is applicable but the phase screen app. is not appropriate since it is based on the parabolic app.

More generally is there a simple criterion which could be employed to know whether one should employ the BA or its distorted-wave version?

Reply: The criterion epsˆ2 aˆ2 kcˆ2 «O(1) at the bottom of page 8 is a kind of extrapolation from the deterministic scattering case. So far I cannot say the criterion in a simple manner.

Interactive comment on Solid Earth Discuss., https://doi.org/10.5194/se-2018-109, 2018.

Evaluation of PSDF in 1-D from that in 3-D When 3-D random media are isotropic, we can evaluate the PSDF along a line by taking samples along the z-axis at $x = y = 0$:

$$\begin{aligned} P_{1D}(m_z) &\equiv \int_{-\infty}^{\infty} R_{3D}(0,0,z)\, e^{-im_z z} dz \\ &= \int_{-\infty}^{\infty} \left[\frac{1}{(2\pi)^3} \iiint_{-\infty}^{\infty} P_{3D}(\mathbf{m}')\, e^{im'_z z} d\mathbf{m}' \right] e^{-im_z z} dz \\ &= \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} P\left(m'_x, m'_y, m_z\right) dm'_x dm'_y. \end{aligned} \tag{1}$$

In the case of isotropic von Kármán type,

$$\begin{aligned} P_{1D}(m_z) &= \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} \frac{8\pi^{3/2}\varepsilon^2 a^3 \Gamma(\kappa + 3/2)}{\Gamma(\kappa)\left[1 + a^2\left(m'^2_x + m'^2_y + m_z^2\right)\right]^{\kappa+3/2}} dm'_x dm'_y \\ &= \frac{2\pi^{1/2}\Gamma(\kappa + 1/2)\,\varepsilon^2 a}{\Gamma(\kappa)\left(1 + a^2 m_z^2\right)^{\kappa+1/2}}. \end{aligned} \tag{2}$$

This relation between different dimensions is a key for the interpretation of well logs, which are 1-D sample data.

Fig. 1.