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Interactive comment on "Characterisation of subglacial water using a constrained transdimensional Bayesian Time Domain Electromagnetic Inversion" *by* Siobhan F. Killingbeck et al.

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We thank Anandaroop Ray for his time reading our manuscript and his constructive comments and suggestions. We greatly appreciate all feedback provided. All comments are addressed as completely as possible, and we consider our manuscript much-improved as a result. In our text below, the reviewers' comments are first, with our response directly beneath. The page and line numbers refer to the updated, edited, manuscript, which is attached as a pdf.

Specific comment 1: Page 2 Line 12: It is better to expand TEM as Transient EM instead of Time Domain EM. If the authors wish to say Time Domain EM it is more appropriate to say TDEM.

Response: We have changed "time-domain" to "transient".

Specific comment 2: Page 2 Line 18: "electromagnetic fields to investigate subsurface resistivity structure INDIRECTLY by measuring TRANSIENT eddy currents"

Response: We have updated this sentence with the added words above.

Specific comment 3: Page 2 Line 20: Please change eddy currents to "transient decay"

Response: This has been changed.

Specific comment 4: Page 3 Line 15: TEM / TDEM confusion

Response: Changed "time-domain" to "transient"

Specific comment 5: Page 3 Line 24: More time domain / transient confusion. For example, GPR can also be considered a time domain method, when analysis is done in the time domain. However, quasi-static diffusive EM geophysical methods, when analysed in the time domain, typically involve transients, hence the more appropriate "Transient EM" or TEM.

Response: Changed "time-domain" to "transient". We have also changed the title from "Time-Domain" to "Transient" to reflect this change.

Specific comment 6: Page 3 Line 29: Change "time dependence" to "switch off"

Response: This has been changed.

Specific comment 7: Page 3 Line 30: The eddy currents PRODUCE a secondary EM field

Response: This has been changed.

Specific comment 8: Page 3 Line 31: The receiver TYPICALLY measures the induced . . . in the off periods. We can measure during on-periods too, it is harder to model.

Response: "Typically" has been added.

Specific comment 9: Page 4 Line 3: The authors could mention somewhere around here that conductive material implies slower transient decay (e.g., Figure A4 of the manuscript) and sustenance of the induced subsurface eddy currents

Response: ". . ... implying a slower transient decay" has been added to the end of this sentence.

Specific comment 10: Page 4 Line 6: Equation 1 is an approximation I believe, should be mentioned.

Response: "estimated" has been changed to "approximated"

Specific comment 11: Page 4 Line 22: Equations 2 t is not necessary in the data vector as time is not an observable. Just to be clear here, the authors should mention here that the mean recording in a stack window is used as data, and the variance of the mean (i.e., variance of the measurements divided by the number of measurements in the window) is the variance of the data. Population variance is not the variance of the mean. Also, the stack, through central limiting admits the use of a Gaussian likelihood.

Response: t has been removed from the data vector. The sentence before Equation 2 has been adapted to highlight the points above and make them clearer, Page 4 line 18: "The data input, d, to MuLTI-TEM are the voltages (v) at each of the N timegates (t), measured as the mean recording in a stack window. The mean, through central limiting, is assumed to be normally distributed with a variance σ2, the variance of the measurements divided by the number of measurements in the stack window, so that the data and uncertainties can be written as:"

Specific comment 12: Equations 3) We do not use the evidence constant for trans-D. Better to leave out p(d) and say p(m|d) \propto p(d|m) p(m)

Response: This has been changed.

Specific comment 13: Page 5 Line 14: Using depth dependent priors in a trans-D formulation is not strictly allowed (see Bodin and Sambridge 2009 for why this is so in the development of the prior). One can however use trans-D with depth independent priors and transform to depth dependent values before modelling.

Response: It is true that most authors assume that the prior subsurface structure is depth independent so that the prior analytically separates completely into its constituent parts (e.g. Bodin & Sambridge, 2009). However, as shown in Killingbeck et al. (2018), equation (7), in fact this is not necessary and the prior on the model (where the model includes both the depth and resistivity for each nucleus) can be written in terms of conditional probabilities. In this formulation, the prior is completely specified (but does not have a straight forward analytic form) and the usual trans-D methodology can be applied as usual. We have added the paragraph to page 5 line 14: "For the choice of prior distribution in transdimensional calculations, it is worth noting that usually the geophysical properties of the cells (here the resistivity) and the cell depths are assumed independent, allowing a simple separated analytic form for the prior distribution (e.g. Bodin and Sambridge, 2009). This is followed in our simplest geometry with no GPR constraints, for which the prior distribution on the resistivity is depth-independent and uniform with wide bounds on log(R) (e.g., R between 100-105 Ωm), to convey the fact that no prior information (beyond that which can be reasonably assumed for typical materials) is known about the subsurface. However, by interpreting any GPR-derived layers as different materials (table 2) with much more narrowed ranges of resistivity, it is clear that a broad depth-independent prior distribution is no longer appropriate. Here we allow the prior distribution of resistivity to depend on depth, by defining for each layer a different uniform distribution that reflects the tightened bounds from lithological information. This restricted prior distribution then significantly decreases the number of permissible models describing resistivity with depth, reducing model ambiguity from any given set of data. In terms of the model parameters, the prior of the resistivity for any nucleus is given by the specific layer that the nucleus is within (Killingbeck et al. (2018). Although a closed form expression for the depth-dependent prior distribution cannot be easily formulated, in the algorithm only the ratios of prior distributions are needed."

Specific comment 14: Page 12 Lines 20 onwards: As the authors point out on the following page, it is not surprising that better resolution is available when conductivity thickness tradeoffs are restricted. The authors could look at another approach of conditioning the posterior after inversion (Ray and Key 2012). However, I would recommend mentioning that the fixed interface depths may be allowed to vary, as the uncertainties on GPR interfaces are not as low as purported in the manuscript (see Ray et al 2016 for uncertainties on seismic reflectors and analogous wave physics, for example).

Also, the authors may try proposing from the prior for birth for better convergence (Dosso et al 2014).

Response: Thank you for these very interesting suggestions. In our case study, we are mainly interested in understanding the whole subglacial resistivity structure, which is completely unknown to us. Therefore, we do not have any specific hypothesis we wish to test at this stage. However, we understand the approach presented in Ray and Key (2012) could be a very useful analysis methodology for further studies. We have noted these suggestions in the manuscript's discussion section, Section 6.2. We have added the following sentence to page 22, line 6: "We note other methods could be used to enhance the efficiency of the transdimensional inversion, potentially providing better convergence rates, such as proposing the birth parameters from the prior (instead of a Gaussian distribution) e.g., Dosso et al., 2014. Further still, having access to the full posterior distribution enables subsets of the posterior model probabilities to be selected, testing various hypothesis about the model structure (Ray and Key, 2012)."

With regards to the fixed interface depths in MuLTI-TEM, this is a very good point we omitted to mention in our discussion, apologies. In our specific case study, the un-

certainty in the constrained depths (derived from GPR) are very low compared to the uncertainty associated with the TEM inversion, hence why we fixed the depth constraints (page 5 line 2). However, we understand this is not always the case as pointed out in Ray et al., 2016. Therefore, we are currently developing the algorithm to be able to input the mean and standard deviation depths of each layer. We are working on a methodology where the layer depths are randomly perturbed (creating an extra perturbation step) within a normal distribution of the inputted mean and standard deviation, during numerical sampling of the posterior. We have added the sentence below to page 22, line 20: "In our Midtdalsbreen case study, the uncertainty in the depth constraints applied is negligible (deciimeter-accuracy from GPR data) compared to the observed data uncertainty (meter accuracy from TEM), motivating us to fix the internal interface depths. However, there remains a finite resolution in GPR data hence we are considering a modification to the MuLTI-TEM code to make it compatible with uncertain interface depths. This would also benefit depth constraints supplied from more uncertain data sources, thus making MuLTI-TEM more broadly applicable."

Specific comment 15: Page 15 Section 5.2 and Figure 8: Marginal uncertainties along a 2D line can also be displayed instead of showing modal models, as shown by Ray et al 2014, Figure 11.

Response: This is a really interesting point and the probability cubes displayed in Ray et al., 2014 are a very good visualization method for displaying and understanding the full solution, including its uncertainties, from the Bayesian inversion. As shown in Ray et al., 2014, this tool is very useful for characterising an anomalous target from a constant background resistivity e.g., a gas reservoir. Here, were are interested in the whole resistivity structure of the subglacial material, along all lines acquired, therefore we display the mode solutions. However, we have added the sentence below to the end of Section 5.2, page 16 line 22: "We note that marginal uncertainties along a 2D line can also be displayed as a 3D probability cube, with axes representing resistivity, line distance and depth, and colour bar representing the probability (e.g., Ray et al., 2014). This aids visualisation of the Bayesian solution and its uncertainty, particularly useful when characterising an anomalous target from a constant background resistivity e.g., a subglacial aquifer or lake underlain by bedrock."

Concluding comment: In conclusion, since the authors have carried out probabilistic inversions of EM and shear wave dispersion data, I would recommend they try and present their conclusions on the facies classifications of the geology also in a probabilistic manner (or at least mention that this can be done). This requires some thought but will provide a much more informative set of displays than Figure 10.

Response: Thank you for your interesting suggestion and improvement for our concluding figure. It is possible to combine the facies classification in a probabilistic manner, however, we would need the joint probability distribution of Vs and R. If we assumed Vs and R were independent variables, we could combined the normalised pdf values, associated to the mode solutions for Vs and R, by calculating the product. However, realistically Vs and R are not independent variables, as they both depend on the same underlying subsurface and we do not have access to these conditional distributions, we only have the marginal distributions for each separate variable. Although, with a direct joint Vs-R Bayesian inversion we would be able to estimate this joint probability distribution and output the facies classifications in a probabilistic manner, this is currently being investigated as further work. We have adapted the following sentence in the manuscript to emphasis this point, Page 22, line 31: "Future extensions of this interpretative strategy could include petrophysical relationships to obtain and/or guide interpretations of the volumetric proportions of water, ice and air in the subsurface (e.g., Hauck et al., 2008). A further promising extension would be a modification to calculate the joint distribution of resistivity and Vs (rather than only the marginal distributions discussed in this paper) which could lead to a more accurate understanding of the subsurface structure (utilizing the structural similarities between resistivity and seismic velocity (e.g., Wisén and Christiansen, 2005). Such a combined approach would also lead resultto in more detailed analysis of the Midtdalsbreen

margin, including a probabilistic facies classification, leading to a framework by which aquifer properties, such as porosity, water content and pore fluid conductivity/salinity, beneath large ice masses could be quantified."

Please also note the supplement to this comment:
https://www.solid-earth-discuss.net/se-2019-126/se-2019-126-AC2-supplement.pdf

Interactive comment on Solid Earth Discuss., https://doi.org/10.5194/se-2019-126, 2019.