



# 1 The relative contributions of scattering and viscoelasticity to the 2 attenuation of S waves in Earth's mantle

3 Susini deSilva<sup>1</sup>, Vernon F. Cormier<sup>1</sup>

4 <sup>1</sup>Department of Physics, University of Connecticut, 196 Auditorium Road, Storrs, CT 06269, USA

5 *Correspondence to:* Vernon F. Cormier (vernon.cormier@uconn.edu)

6 **Abstract.** The relative contributions of scattering and viscoelastic attenuation to the apparent attenuation of seismic  
7 body waves are estimated from synthetic and observed S waves multiply reflected from Earth's surface and the core-  
8 mantle boundary. The synthetic seismograms include the effects of viscoelasticity and scattering from small-scale  
9 heterogeneity predicted from both global tomography and from thermodynamic models of mantle heterogeneity that  
10 have been verified from amplitude coherence measurements of body waves observed at dense arrays. Assuming  
11 thermodynamic models provide an estimate of the maximum plausible power of heterogeneity measured by elastic  
12 velocity and density fluctuations, we predict a maximum scattering contribution of 43 % to the total measured  
13 attenuation of mantle S waves having a dominant frequency of 0.05 Hz. The contributions of scattering in the upper  
14 and lower mantle to the total apparent attenuation are estimated to be roughly equal. The relative strength of the  
15 coda surrounding observed ScSn waves from deep focus earthquakes is not consistent with a mantle having zero  
16 intrinsic attenuation.

17

## 18 1 Introduction

19

20 Seismic tomography reveals a laterally heterogeneous velocity structure in the mantle. Constraining the locations  
21 and dimensions of such elastic heterogeneities is critical to understanding the intricate details of the dynamic mixing  
22 process of the mantle, which is closely tied to the plate tectonic evolution of the Earth. Large-scale (~ 1000 km)  
23 heterogeneities are likely caused by the buoyancy differences that drive thermal-chemical convection. The effects of  
24 thermal diffusion, however, limit small-scale (~ 1 to 100 km) heterogeneities to chemical variations. Small-scale  
25 heterogeneities can scatter 0.1 to 1 Hz. body waves, transferring energy from body wave pulses observed at a  
26 receiver to later time windows and receivers (Shearer, 2015). Mantle attenuation measured from P and S waves will  
27 hence always be a summation of a scattering and an intrinsic viscoelastic attenuation. The viscoelastic dispersion of  
28 dominantly intrinsic attenuation successfully explains the lower velocities of Earth models derived from low  
29 frequency free oscillations observed in the millihertz band from those derived from 1 Hz body waves (Dziewonski  
30 and Anderson, 1981). Yet some extrapolations of the scale lengths and intensities of heterogeneity inferred from  
31 high frequency body waves have suggested attenuation in the mantle may instead be dominated by scattering  
32 (Ricard et al., 2014, Sato, 2019).

33



34 The apparent attenuation of multiple ScS waves is an excellent observable to untangle the relative contributions of  
35 scattering and intrinsic attenuation. Many previous studies have used ScS and its reverberations within the mantle to  
36 obtain path averaged values for the mantle attenuation. These attenuation measurements are usually represented in  
37 terms of a quality factor ( $Q$  or  $Q_{ScS}$  for ScS-based measurements). The estimates of these apparent attenuation  
38 measurements include both the intrinsic or viscoelastic attenuation of the wave amplitude and the attenuation caused  
39 by scattering effects. In this work we will consider the apparent attenuation ( $\frac{1}{Q_{ScS}}$ ) to be the addition of intrinsic  
40 attenuation ( $\frac{1}{Q_{intr}}$ ) and scattering attenuation ( $\frac{1}{Q_{scat}}$ ) for path averaged observations of SH waves reflected from the  
41 free surface and core-mantle boundary. The intrinsic component accounts for the loss of energy due to friction and  
42 heat loss as the wave propagates through the mantle with different viscous properties caused by the motion of  
43 defects in the crystalline lattice structure of silicates or by the motion of melt at grain boundaries or in pores.  
44 Intrinsic attenuation manifests itself in body waves by amplitude decay, pulse broadening, and velocity dispersion.  
45 The scattering attenuation accounts for the energy loss that is scattered into different directions as elastic  
46 heterogeneities are encountered along the path of a body wave. In addition to amplitude decay and pulse broadening  
47 of the main phase, scattering generates increased levels of coda energy comprised of redistributed energy arriving  
48 later than the main phase. Many past studies calculating the apparent attenuation of multiple ScS use spectral  
49 amplitude ratios (Kovach and Anderson, 1964, Yoshida and Tsujiura, 1975, Sipkin and Jordan, 1980, Lay and  
50 Wallace, 1983) and time domain amplitude ratios (Kanamori and Riviera, 2015) of adjacent ScS waveforms. An  
51 alternative analysis technique seeks the attenuation operator that converts an ScS<sub>n-1</sub> waveform into an ScS<sub>n</sub>  
52 waveform (Jordan and Sipkin, 1977, Revenaugh and Jordan, 1989). Sipkin and Revenaugh (1994) concluded that a  
53 frequency domain approach works better for  $Q_{ScS}$  measurements, especially in continental regions that tend to have  
54 lower shear  $Q$  values compared to oceanic regions. Lee et al. (2003) compared observations and numerical  
55 simulations of coda envelope offsets before and after ScS synthesized with two-layer scattering models  
56 superimposed on a PREM reference model to calculate the scattering contribution to total attenuation measurements.  
57 They concluded that scattering loss dominates intrinsic loss in the lower mantle.  
58 Our effort employs an estimate for a ScS<sub>n</sub> attenuation operator to evaluate the relative percentages of scattering and  
59 intrinsic attenuation contributing to the apparent attenuation observed from simulated mantle heterogeneity models.  
60 Observations of scattered body waves together with geodynamic modeling have established that heterogeneities of  
61 scale lengths as small as 4 to 10 km with RMS (root mean square) velocity perturbations of 1 to 8 % can persist  
62 throughout the mantle, even in the presence of constant convective stirring (Hedlin et al., 1997, Shearer and Earl,  
63 2008, Kaneshima and Helffrich, 2010). Our investigation considers the effects of similar dimensions and  
64 perturbation strengths for heterogeneity models. We also consider the effects of a maximum plausible depth-  
65 dependent model of mantle heterogeneity power from thermodynamically constrained estimates of mantle chemistry  
66 and phase. Such models predict significantly higher heterogeneity than the models of global tomography (Stixrude  
67 and Lithgow-Bertelloni, 2007, Stixrude and Lithgow-Bertelloni, 2012). We have recently validated (Cormier et al.,  
68 2019) a thermodynamic model of mantle heterogeneity by applying stochastic tomography (Zheng and Wu, 2008) to  
69 the upper 1000 km of the mantle to invert for amplitude and phase fluctuations observed by the US transportable  
70 array. Assessing the scattering attenuation induced by thermodynamic models, which predict heterogeneity to be



71 concentrated in mantle phase transition zones, can assist in quantifying mantle heterogeneity and testing for the  
72 existence of additional phase transitions.

73

## 74 **2 Method**

75

### 76 **2.1 Models**

77

78 Apparent attenuations are measured from ScSn waveforms observed in synthetic seismograms for 4 different models  
79 of mantle heterogeneity. All of these assume PREM as the one dimensional background velocity and density model,  
80 with the PREM shear wave attenuations providing the purely intrinsic component of attenuation. Model 1 does not  
81 perturb PREM with any lateral heterogeneities. Therefore, the apparent attenuation measured for this case will be  
82 purely intrinsic. Model 2 (Fig.1) applies a depth-dependent shear velocity perturbation to the PREM mantle similar  
83 to those determined from many seismic tomographic studies (Megnin and Romanowicz, 2000, Ritsema et al., 2004).  
84 Model 3 (Fig. 2) applies a shear velocity perturbation to the PREM mantle similar to the predictions of  
85 thermodynamic studies for the upper 1000 km of the mantle (Stixrude and Lithgow-Bertelloni, 2012). Model 4 (Fig.  
86 3) is the same as Model 3 in the upper 1000 km of the mantle but includes an additional peak in heterogeneity power  
87 in the lowermost mantle predicted from the effect of the post-perovskite phase transition. In Model 5, the intrinsic  
88 attenuations are turned off while still applying the thermodynamic model of mantle heterogeneity to shear velocity  
89 perturbations. Hence the synthetic seismograms for this model will exhibit purely scattering effects in any  
90 attenuation measurement. In all models, heterogeneities are represented as stochastic random media with an  
91 exponential autocorrelation having a corner scale equal to 10 km. In Models 2, 3, 4, and 5 we assume a relation  
92 between density and shear velocity perturbations such that  $\ln \rho / \rho = 0.8 \ln V_s / V_s$ . This value for density  
93 perturbation in a mantle close to neutral buoyancy is relatively large, but is commonly assumed in studies of crustal  
94 and upper mantle scattering based on Birch's law (Birch, 1952).

95

### 96 **2.2 Apparent attenuation measurements**

97

98 All simulations are performed by a numerical pseudospectral method in 2-D (Cormier, 2000), assuming an SH line  
99 source at 500 km depth with a Gaussian-shaped source-time function having a half-width of 1.2 seconds. Wave  
100 propagation uses a 2D staggered grid of radial step size 3.0 km and lateral step size 5.427 km, with time sampling  
101 set to 0.025 seconds ensuring stability and negligible grid dispersion. Intrinsic attenuation, taken to be  
102 approximately constant across a broad frequency band, is introduced by three memory functions using the methods  
103 described by Robertson et al., (1994). Waveforms are computed at a great circle distance of 18° in order to avoid  
104 contamination of ScSn phases with depth phases or other nearby arrivals. These are corrected for 3D geometric  
105 spreading, and a line-to-point source conversion is made. For each of the 5 models a 2-parameter attenuation  
106 operator (Eq. 1) is determined that converts the ScS waveform into an ScSScS waveform. Each attenuation operator



107 depends on  $Q_{ScS}$  and the high frequency corner ( $1/\tau_m$ ) of a relaxation spectrum, where attenuation is constant for 5  
 108 decades of frequency.

109 In the inversion procedure, the predicted ScSScS velocity waveform is generated by convolving the ScS waveform  
 110 with an attenuation operator corresponding to a peak attenuation  $1/Q_{ScS}$  and a high frequency corner  $1/\tau_m$ . A least  
 111 squares norm is calculated (Eq. 2) for the difference between observed and predicted ScSScS velocity waveforms,  
 112 which are aligned by the arrival times of first maximum and normalized by the peak-to-trough amplitudes (Fig. 4).  
 113 A search over the two attenuation parameters is then performed to minimize an L2 norm difference to maximize a  
 114 Gaussian probability density constructed using the L2 norm difference (Cormier et al., 1998). Half-widths of the  
 115 probability density functions are used to infer errors.

116

117 An operator to convert an ScS waveform into an ScSScS waveform is defined in the frequency domain by

118

$$119 \quad O(\omega, Q, \tau) = \exp(-i\omega[\int_{ScSScS} \frac{ds}{\hat{v}(\omega)} - \int_{ScS} \frac{ds}{\hat{v}(\omega)}]) \quad (1)$$

120 where

$$121 \quad \hat{V}(\omega, Q, \tau) = \frac{\sqrt{1 + \frac{2}{\pi Q_{ScS}^{-1} \ln\left(\frac{-i\omega + 1/\tau_l}{-i\omega + 1/\tau_m}\right)}}}{\sqrt{1 + \frac{2}{\pi Q_{ScS}^{-1} \ln\left(\frac{-i2\pi + 1/\tau_l}{-i2\pi + 1/\tau_m}\right)}}$$

122 and where

123  $\tau_l$  is the period of the low frequency corner in relaxation spectrum and  $\frac{\tau_l}{\tau_m} = 10^5$

124 The least squares norm difference between observed and predicted waveforms is calculated from

125

$$126 \quad L2N(Obs, Pred) = \sqrt{\sum_t \frac{(Amp_{obs}(t) - Amp_{pred}(t))^2}{\sigma^2}} \quad (2)$$

127 where  $\sigma$  is a  $\frac{noise}{signal}$  measurement from a 100 second time window preceding the ScSScS observation.

128

129 Our goal was to simply estimate an apparent attenuation parameter  $Q_{ScS}$  for the whole of the mantle when the effects  
 130 of scattering are included rather than to seek a best fitting depth and frequency dependent attenuation model. Our  
 131 estimates for the high frequency corner parameter  $1/\tau_m$ , were within the range bounded by estimates for the upper  
 132 and lower mantle found in the study by Choy and Cormier (1986).

133

134 **3. Results**

135



136 We found MODEL 1, which has pure intrinsic attenuation and no small-scale heterogeneity, to have an apparent  
137 attenuation value of 0.004167 corresponding to a  $Q_{ScS} = 240$ . This estimated  $Q_{ScS}$  value differs by only 2.2 % from  
138 the theoretical estimate of the depth averaged  $Q_{ScS}$  obtained for PREM with the relation  $Q_{ScS} = (\int_{x_{ScSScS}} dt -$   
139  $\int_{x_{ScS}} dt) / (\int_{x_{ScSScS}} dt / Q_s(x) - \int_{x_{ScS}} dt / Q_s(x))$ . Here  $x_{ScSScS}$  and  $x_{ScS}$  denote points along the path of  
140 ScSScS and ScS respectively,  $Q_s(x)$  denote the  $Q_s$  values at those points read from 1D PREM. This result verifies  
141 the accuracy of the waveform L2 norm method for estimating  $Q_{ScS}$ .

142

143 With MODEL 2, which has a conventional tomographic estimate of mantle heterogeneity, we find that the apparent  
144 attenuation is increased to 0.005 ( $Q_{ScS}$  decreased to 200). Together with the knowledge of the purely intrinsic  
145 contribution ( $\frac{1}{Q_{intr}}$ ) calculated in MODEL 1, the scattering component of attenuation ( $\frac{1}{Q_{scat}}$ ) in MODEL 2 is  
146 estimated to be 0.000833. Hence the scattering caused by small-scale ( $\sim 10$  km) heterogeneities with a  $dVs/Vs$  depth  
147 profile similar to S20RTS (Ritsema et al., 2004), would account for 16.7 % of the measured ScS apparent  
148 attenuation. MODEL 3, which has a higher amount of heterogeneity due to increased  $Vs$  perturbations associated  
149 with predicted lateral variations in phase changes in the upper mantle, results in a higher apparent attenuation of  
150 0.005747 ( $Q_{ScS} = 174$ ). MODEL 4, which includes additional heterogeneity predicted for the effects of a post-  
151 perovskite phase transition results in an even higher apparent attenuation of 0.007100 ( $Q_{ScS} = 140$ ). We calculate  
152 that the scattering attenuation in the lower mantle (below 1000 km) and upper mantle (above 1000 km) of MODEL  
153 4 to be 0.0014 and 0.0016 with their percent contributions to the total apparent attenuation being 19.6 % and 22.4 %  
154 respectively. The overall scattering attenuation of MODEL 4 is 0.002933 with the scattering component accounting  
155 for 41.3 % of the measured ScS total apparent attenuation.

156 Finally, in MODEL 5 the intrinsic attenuation in the mantle is turned off while applying the mantle heterogeneity of  
157 MODEL 4. The apparent attenuation (now purely due to scattering) is measured to be 0.0029 ( $Q_{ScS} = 340$ ). This high  
158  $Q$  value lies towards the upper bound of regional estimates ( $\sim 360$ ) of  $Q_{ScS}$  (Nakanishi, 1979, Sipkin & Revenaugh  
159 1994, Gomer & Okal, 2003). It is also found that apparent attenuation measurements of MODEL 5 and MODEL 1  
160 add up to be exactly equal to MODEL 4, validating the attenuation estimation method in conjunction with the  
161 assumption of  $\frac{1}{Q_{apparent}} = \frac{1}{Q_{intr+scat}} = \frac{1}{Q_{intr}} - \frac{1}{Q_{scat}}$ .

162

163 Figure 6 compares the levels of scattered coda energy arriving in the vicinity ( $\sim \pm 150$  s) of the ScSScS main arrival  
164 generated by different models of mantle heterogeneity models to the synthetic ScSScS predicted by MODEL 1  
165 having no scattering. Observing the envelopes of squared velocity for MODEL 2 versus MODEL 4, it is apparent  
166 that the levels of energy arriving in the coda and before the main phase significantly increase and the ScSScS pulse  
167 width increases due to the presence of increased small-scale heterogeneity in the regions associated with mantle  
168 phase changes. It also is important to recognize that intrinsic attenuation can affect the ratio of coda energy to the  
169 main pulse. The results for MODEL 5, which omits intrinsic attenuation, demonstrate the importance of intrinsic  
170 attenuation for the coda as well as the direct phases. In this case the coda, unaffected by intrinsic attenuation,  
171 approaches the amplitude of the direct ScSScS phase.



172

#### 173 4. Discussion

174

##### 175 4.1 Comparison with regional variations

176

177 Regional variations measured for  $Q_{ScS}$  generally fall in the range of 140 – 360 (Nakanishi, 1979, Sipkin &  
178 Revenaugh, 1994, Gomer & Okal 2003). Variations on this order are confirmed when we apply our inversion  
179 method to two example multiple ScS observations observed from deep focus earthquakes (Fig. 7). We obtain  $Q_{ScS} =$   
180 153 for an earthquake beneath Papua New Guinea region observed at a station located at Charters Towers in  
181 Australia, and  $Q_{ScS} = 200$  for an earthquake beneath the eastern China-Russia border region observed at a station  
182 located at Yakutsk in eastern Siberia. In Fig. 8 we overlay synthetic seismograms computed from several of our  
183 models to determine of how scattering in combination with intrinsic attenuation can affect the relative amplitudes of  
184 the direct ScSScS phase and its coda. The heterogeneity power of MODEL 2 inferred from global tomography is too  
185 weak to match the excitation of coda relative to ScSScS in both our data examples. MODEL 4, having PREM  
186 attenuation and heterogeneity predicted for a thermodynamic model of the mantle, best matches the relative coda  
187 and direct phase excitations for both events. The match can be improved by either a small decrease in intrinsic  
188 attenuation or a small increase in heterogeneity power for the eastern China-Russia border region to Yakutsk. ScSn  
189 paths from both earthquakes traverse a region of the mantle on the back-arc side of dipping slabs, a southwest  
190 dipping slab toward the Australian craton in the case of the New Guinea event (Tregoning and Gorbatov, 2004), and  
191 a western dipping Kuril-Kamchatka slab (Koulakov et al., 2011) toward the Siberian craton in the case of the eastern  
192 China-Russia border event. The multiple ScSn paths for the eastern China-Russia border event are more slab  
193 parallel and distant from the descending slab and more strongly sample the cratonic upper mantle compared to the  
194 New Guinea event. Hence, it is likely that the intrinsic attenuation of PREM overestimates the effects of mantle  
195 attenuation on ScSn's. Finally, a comparison of observations with the prediction of Model 5, having no intrinsic  
196 attenuation, over-predicts coda excitation relative to ScSScS for both events. This confirms that some intrinsic  
197 attenuation in the mantle is necessary to dampen the coda generated by the most extreme plausible suggestions of  
198 heterogeneity power.

199

##### 200 4.2 Upper and lower mantle scattering and intrinsic attenuation

201

202 Strong depth dependence of mantle attenuation, both intrinsic and scattering, has long been documented. Intrinsic  
203 attenuation has been found to be relatively low in the mid and deep mantle compared to the upper mantle. Evidence  
204 of some scattering in the mid and deep mantle has been confirmed in studies of PKIKP precursors in the 120° to  
205 140° great circle range (e.g., Hedlin et al., 1997), including strong regional and depth variations that may be  
206 consistent with the effects of either remnant subducted oceanic crust or with a peak in heterogeneity power  
207 associated with a post-perovskite phase change. From a study of S and ScS coda, Lee et al. (2003) estimated that  
208 scattering attenuation dominates intrinsic attenuation in the lower mantle, reporting their results in terms of the



209 scattering coefficients for a two-layered model of mantle heterogeneity. The scattering coefficients  $g$  are related to  
210 scattering attenuation by  $g = \omega / (Q_{\text{scat}} V_s)$ . Our results for MODEL 3 and MODEL 4 show that seismic albedo,  
211 the ratio of scattering loss to total attenuation, below 1000 km depth in the mantle is 30 % while above 1000 km it is  
212 27 %. This is assuming the PREM average intrinsic shear  $Q$  of 225 and 312 for the two depth regions. Hence, we do  
213 not observe the scattering to dominate over intrinsic effects in either lower or upper mantle, although regional  
214 exceptions can be expected. Additionally, considering the estimated scattering attenuations for MODEL 3 and  
215 MODEL 4, we can deduce the scattering coefficients to be  $6.25 \times 10^{-5} \text{ km}^{-1}$  for the mantle below 1000 km and  $1.256$   
216  $\times 10^{-4} \text{ km}^{-1}$  for mantle above 1000 km in MODEL 4. These scattering coefficients, calculated for a dominant  
217 frequency of 0.05 Hz, are comparable to the low frequency estimates of Lee et al. (2003). This result implies a  
218 relatively lower scattering coefficient (i.e. slightly lower scattering attenuation) in the lower mantle compared to the  
219 upper mantle in MODEL 4, which agrees with the Lee et al. estimates of scattering coefficients.

220

#### 221 **4.3 Origins of heterogeneity and scale length anisotropy**

222

223 In suggesting that scattering attenuation may dominate intrinsic attenuation throughout the mantle Ricard et al.  
224 (2014) considered the effects of heterogeneity distributed primarily in the form of horizontal layers based on  
225 geodynamic numerical experiments that predict folding and horizontal stretching of chemical heterogeneity (e.g.,  
226 Manga, 1996) whose origin primarily originates from the convective cycling of oceanic crust. The attenuative  
227 effects of horizontally layered structure have been well known since the classic paper by O'Doherty and Anstey  
228 (1971) and are simply calculated. In this paper, we have instead considered the effects of scale lengths predicted by  
229 thermodynamic models in which variations in temperature and chemistry dictate the stability of silicate mineral  
230 phases. These variations in temperature and chemistry can also be connected to the convective cycling of oceanic  
231 crust, but instead predict that peaks in heterogeneity power will be concentrated near phase transitions. Such models  
232 have not yet fully considered the effects of mechanical mixing on the anisotropy of scale lengths within these  
233 relatively narrow regions of depth. Nonetheless, thermodynamic models, when verified by observations of scattering  
234 effects that supplement tomographic imaging, may at least provide a more reliable estimate of the upper bound to  
235 velocity and density fluctuations in the mantle. Experiments similar to ours may be extended to include the effects  
236 of anisotropy of scale lengths. Our results indicate that some intrinsic attenuation will always be required to explain  
237 the attenuation of body waves, regardless of the state of isotropy of scale lengths.

238

#### 239 **5. Conclusions**

240

241 An inversion algorithm for apparent mantle attenuation based on L2 norm differences between observed and  
242 predicted ScSScS velocity waveforms has been verified by inversion of synthetic seismograms and applied to  
243 estimate the relative contributions of intrinsic and scattering attenuation to the total apparent attenuation.  
244 Thermodynamic models of mantle heterogeneity predict significantly higher heterogeneity power than the  
245 predictions from global tomography, and a correspondingly higher relative contribution to apparent attenuation



246 measured from body waves. Taking the depth-dependent heterogeneity power of thermodynamic models of mantle  
247 heterogeneity as the maximum plausible heterogeneity we estimate that scattering may explain up to 41.3 % of  
248 apparent mantle attenuation with up to 3 % RMS shear velocity perturbations concentrated near mantle phase  
249 transitions and 1 % everywhere else. We estimate the scattering contribution to the apparent attenuation from  
250 heterogeneity in the upper and lower mantle to be roughly equal in global averages, but regional variations between  
251 upper and lower mantle scattering contributions are likely. These estimates agree well with the excitation of coda  
252 surrounding ScSn waves observed from deep focus earthquakes. These codas can only be matched by the existence  
253 of both intrinsic and scattering attenuation.

254

255 **Data Availability.** The data set of SH component synthetic seismograms can be found at  
256 <https://doi.org/10.5281/zenodo.3460694> (Desilva and Cormier, 2019).

257

258 **Acknowledgements.** This work was supported by grants EAR 14-46509 from the National Science Foundation.

259

## 260 References

261

262 Birch F.: Elasticity and constitution of the Earth's interior, *J. Geophys. Res.* 57, 227–286, 1952..

263 Choy, G., and Cormier, V.F.: Direct measurement of the mantle attenuation operator from broadband P and S  
264 waves, *J. Geophys. Res.* 91, 7326-7342, 1986.

265 Cormier, V., Li, X., and Choy, G.: Seismic attenuation of the inner core: Viscoelastic or stratigraphic?, *Geophysical*  
266 *Research Letters*, 25 (21), 4019-4022, 1998.

267 Cormier, V. F.:  $D''$  as a transition in the heterogeneity spectrum of the lowermost mantle, *Journal of Geophysical*  
268 *Research: Solid Earth*, 105(B7), 16193-16205, 2000.

269 Cormier, V.,F., Tian, Y., and Zheng, Y.: Heterogeneity spectrum of Earth's upper mantle obtained from the  
270 coherence of teleseismic P waves, *Communications in Computational Physics*, 26(5), 1-27, doi: 10.4208/cicp.OA-  
271 2018-079, 2019.

272 Desilva, S. and Cormier. V.: SH component synthetic seismograms (SE\_Supplementary\_data) (Version v1.0.0)  
273 [Data set]. Zenodo. <http://doi.org/10.5281/zenodo.3460695>, 2019.

274 Dziewonski, A. D. and Anderson, D. L.: Preliminary reference earth Model, *Phys. Earth Planet. Inter.*, 25, 297-356,  
275 1981.

276 Gomer, B., and Okal, E.: Multiple-Scs probing of the Ontong-Java Plateau, *Physics of the Earth and Planetary*  
277 *Interiors*, 138(3-4), 317-331, 2003.

278 Hedlin M A, Shearer P, and Earle P.: Seismic evidence for small-scale heterogeneity throughout the Earth's mantle,  
279 *Nature*, 387(6629), 145-150, 1997.





- 280 Jordan, T. H., and Sipkin S. A.: Estimates of the attenuation operator for multiple ScS Waves, Geophysical Research  
281 Letters, 4(4), 167-170, 1977.
- 282 Kanamori, H., and Rivera, L.: Nearvertical multiple ScS phases and vertically averaged mantle properties,  
283 interdisciplinary Earth: A Volume in Honor of Don L. Anderson: Geological Society of America Special Paper, 514  
284 and American Geophysical Union Special Publication, 71, 9–31, 2015.
- 285 Kaneshima, S., and Helffrich, G.: Small scale heterogeneity in the mid-lower mantle beneath the circum-Pacific  
286 area, Physics of the Earth and Planetary Interiors, 183(1-2), 91-103, 2010.
- 287 Kovach, R. L., and Anderson, D. L.: Attenuation of shear waves in the upper and lower mantle, Bulletin of the  
288 Seismological Society of America, 54(6A), 1855-1864, 1964.
- 289 Koulakov, I.Y., Dobretsov, N.L., Bushenkova, N.A., and Yakovlev, A.V.: Slab shape in subduction zones beneath  
290 the Kurile-Kamchatka and Aleutian arcs based on regional tomography results, Russ. Geol. and Geophys., 52, 650-  
291 667, 2011.
- 292 Lay, T., and Wallace, C.: Multiple scs travel times and attenuation beneath Mexico and central America, Geophysical  
293 Research Letters, 10(4), 301-304, 1983.
- 294 Lee, W., Sato, H., and Lee, K.: Estimation of S-wave scattering coefficient in the mantle from envelope  
295 characteristics before and after the ScS arrival, Geophysical Research Letters, 30(24), 1-5, 2003.
- 296 Manga, M.: Mixing of heterogeneities in the mantle: Effect of viscosity differences, Geophysical Research Letters,  
297 23(4), 403-406, doi: 10.1029/96GL00242, 1996.
- 298 Megnin, C., and Romanowicz, B.: The three-dimensional shear velocity structure of the mantle from the inversion of  
299 body, surface and higher-mode waveforms, Geophysical Journal International, 143, 709-728, 2000.
- 300 Nakanishi, I.: Attenuation of multiple ScS waves beneath the Japanese arc, Physics of the Earth and Planetary  
301 Interiors, 19(4), 337-347, 1979.
- 302 O'Doherty, R. F. and Anstey, N. A.: Reflections on amplitudes, Geophys. Prosp., 19, 430- 458, 1971.
- 303 Revenaugh, J., and Jordan, T.: A study of mantle layering beneath the western Pacific, Journal of Geophysical  
304 Research, 94(B5), 5787-5813, 1989.
- 305 Ricard, Y., Durand, S., Montagner, J., and Chambat, F.: Is there seismic attenuation in the mantle?, Earth and  
306 Planetary Science Letters, 388, 257-264, 2014.
- 307 Ritsema, J., van Heijst, H., and Woodhouse, J.: Global transition zone tomography, Journal of Geophysical  
308 Research: Solid Earth, 109(B2), 2004.
- 309 Robertson, J.O.A., Blanch, J.O., and Symes, W.W.: Viscoelastic finite-difference modeling, Geophysics, 58, 1444-  
310 1456, 1994.
- 311 Sato, H.: Power spectra of random heterogeneities in the solid earth, Solid Earth, 10, 275-292, 2019.
- 312 Shearer, P.: Seismic scattering in the deep Earth, Treatise on Geophysics Second Edition, 1, 759-787, 2015.



313 Shearer, P., and Earle, P.: Observing and Modeling Elastic Scattering in the Deep Earth, *Advances in Geophysics*,  
314 50(08), 167-193, 2008.

315 Sipkin, S., and Jordan, T.: Regional variation of Qscs, *Bulletin of the Seismological Society of America*, 70 (4),  
316 1071-1102, 1980.

317 Sipkin, S., and Revenaugh, J.: Regional variation of attenuation and travel time in China from analysis of multiple-  
318 ScS phases, *Journal of Geophysical Research*, 99(B2), 2687-2699, 1994.

319 Stixrude, L., and Lithgow-Bertelloni, C.: Influence of phase transformations on lateral heterogeneities and dynamics  
320 in Earth's mantle, *Earth Planet. Sci. Lett.*, 263, 45-55, 2007.

321 Stixrude, L., and Lithgow-Bertelloni, C.: Geophysics of Chemical Heterogeneity in the Mantle, *Annual Review of*  
322 *Earth and Planetary Sciences*, 40(1), 569-595, 2012.

323 Tregoning, P., and Gorbatov, A.: Evidence for active subduction at the New Guinea Trench, *Geophys. Res. Lett.*,  
324 doi: 10.1029/2004GL020190, 2004.

325 Yoshida, M., and Tsujiura, M.: Spectrum and attenuation of multiply reflected core phases, *Journal of Physics of the*  
326 *Earth*, 23(1), 31-42, 1975.

327 Zheng, Y., and Wu, R.: Theory of transmission fluctuations in random media with a depth-dependent background  
328 velocity structure, *Advances in Geophysics*, 50, 21-41, 2008.

329

330

331

332

333

334

335

336

337

338

339

340

341



342  
343  
344  
345  
346  
347  
348  
349  
350  
351  
352  
353  
354  
355  
356  
357  
358  
359  
360  
361  
362  
363  
364  
365  
366  
367  
368  
369  
370  
371  
372  
373  
374  
375  
376  
377  
378  
379  
380  
381

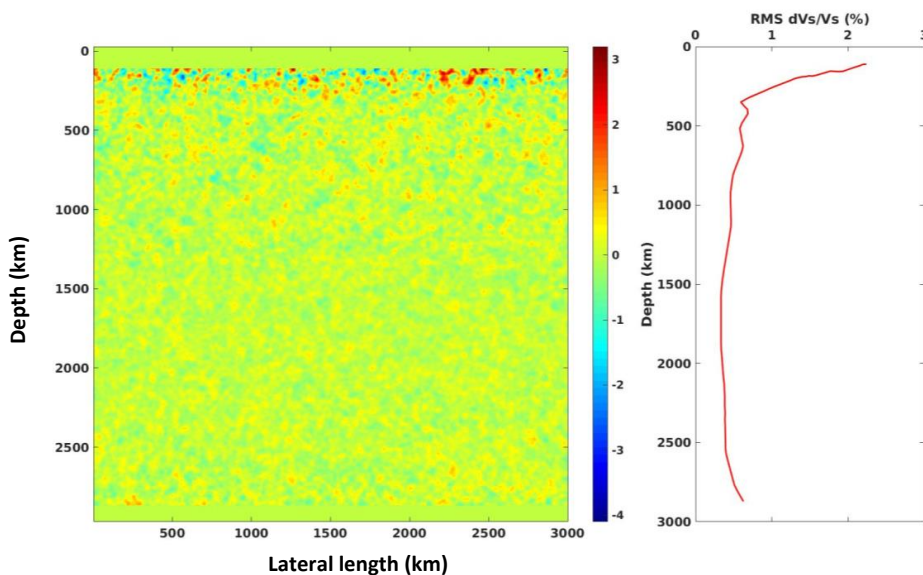


Figure 1: Right: depth dependent RMS shear velocity perturbation profile applied in Model 2. This is extracted from S20RTS. Left: 2D representation of the same depth dependent profile. Heterogeneous media is for an exponential autocorrelation (corner scale  $a = 10$  km) function. Note the increase in heterogeneity power near the top and bottom of the mantle.



382  
383  
384  
385  
386  
387  
388  
389  
390  
391  
392  
393  
394  
395  
396  
397  
398  
399  
400  
401  
402  
403  
404  
405  
406  
407  
408  
409  
410  
411  
412  
413  
414  
415  
416  
417  
418  
419  
420  
421  
422  
423

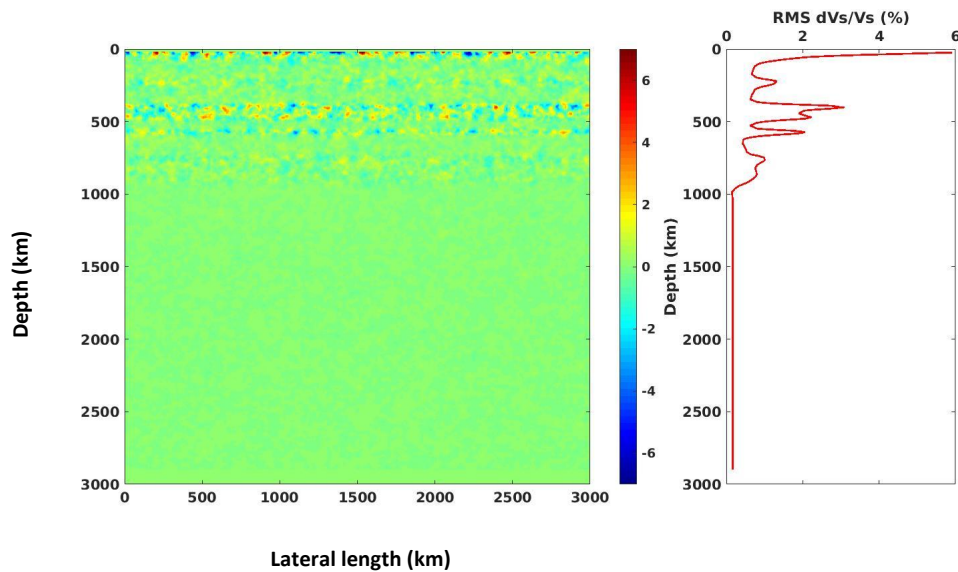


Figure 2: Right: Depth dependent RMS shear velocity perturbation profile applied in Model 3 vs. perturbation values from crust to 1000 km depth is extracted from the stochastic tomography result of Cormier et al. (2019). Left: 2D representation of the same depth dependent profile. Compared to Model 2 note the additional peaks in heterogeneity power associated with phase transitions in the upper mantle.



424  
425  
426  
427  
428  
429  
430  
431  
432  
433  
434  
435  
436  
437  
438  
439  
440  
441  
442  
443  
444  
445  
446  
447  
448  
449  
450  
451  
452  
453  
454  
455  
456  
457  
458  
459  
460  
461  
462  
463  
464  
465

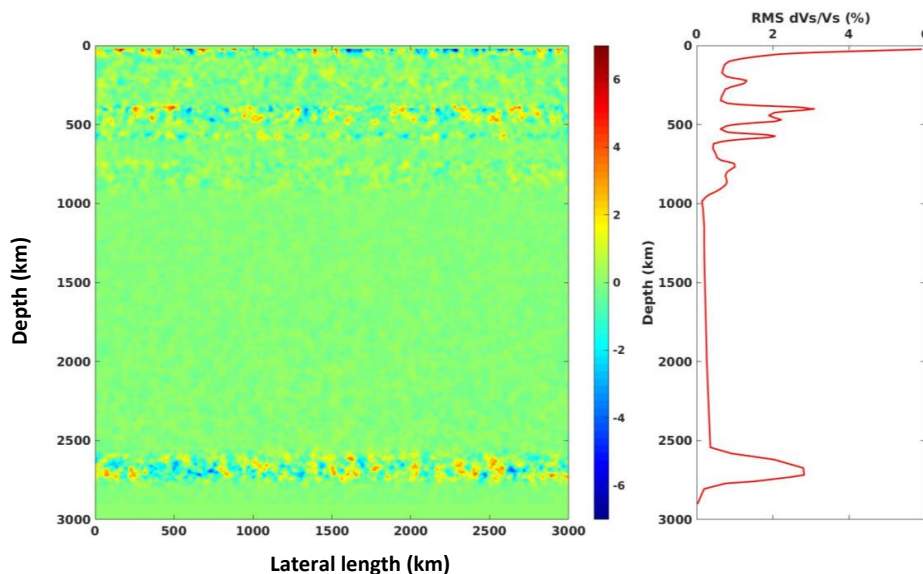


Figure 3: Right: Depth dependent RMS shear velocity perturbation profile applied in Model 4 vs. perturbation values from crust to 1000 km depth is extracted from the stochastic tomography result of Cormier et al. (2019). Compared to Model 3 an additional peak is added near the core mantle boundary to incorporate the increased lower mantle associated with the post-perovskite phase change (Stixrude & Lithgow-Bertelloni, 2012) Left: 2D representation of the same depth dependent profile.



466  
467  
468  
469  
470  
471  
472  
473  
474  
475  
476  
477  
478  
479  
480  
481  
482  
483  
484  
485  
486  
487  
488  
489  
490  
491  
492  
493  
494  
495  
496  
497  
498  
499  
500  
501  
502  
503  
504  
505  
506  
507

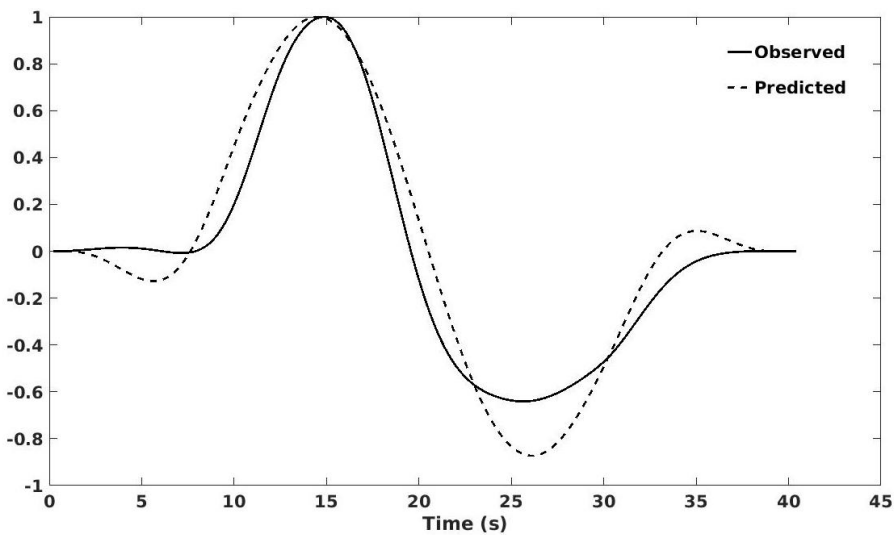


Figure 4: Observed and predicted ScSScS velocity waveform aligned by the arrival time of first extremum and normalized by the peak to trough amplitude. The least squares norm difference between these two waveforms is obtained using a summation of amplitude differences over time.



508  
509  
510  
511  
512  
513  
514  
515  
516  
517  
518  
519  
520  
521  
522  
523  
524  
525  
526  
527  
528  
529  
530  
531  
532  
533  
534  
535  
536  
537  
538  
539  
540  
541  
542  
543  
544  
545  
546  
547  
548  
549

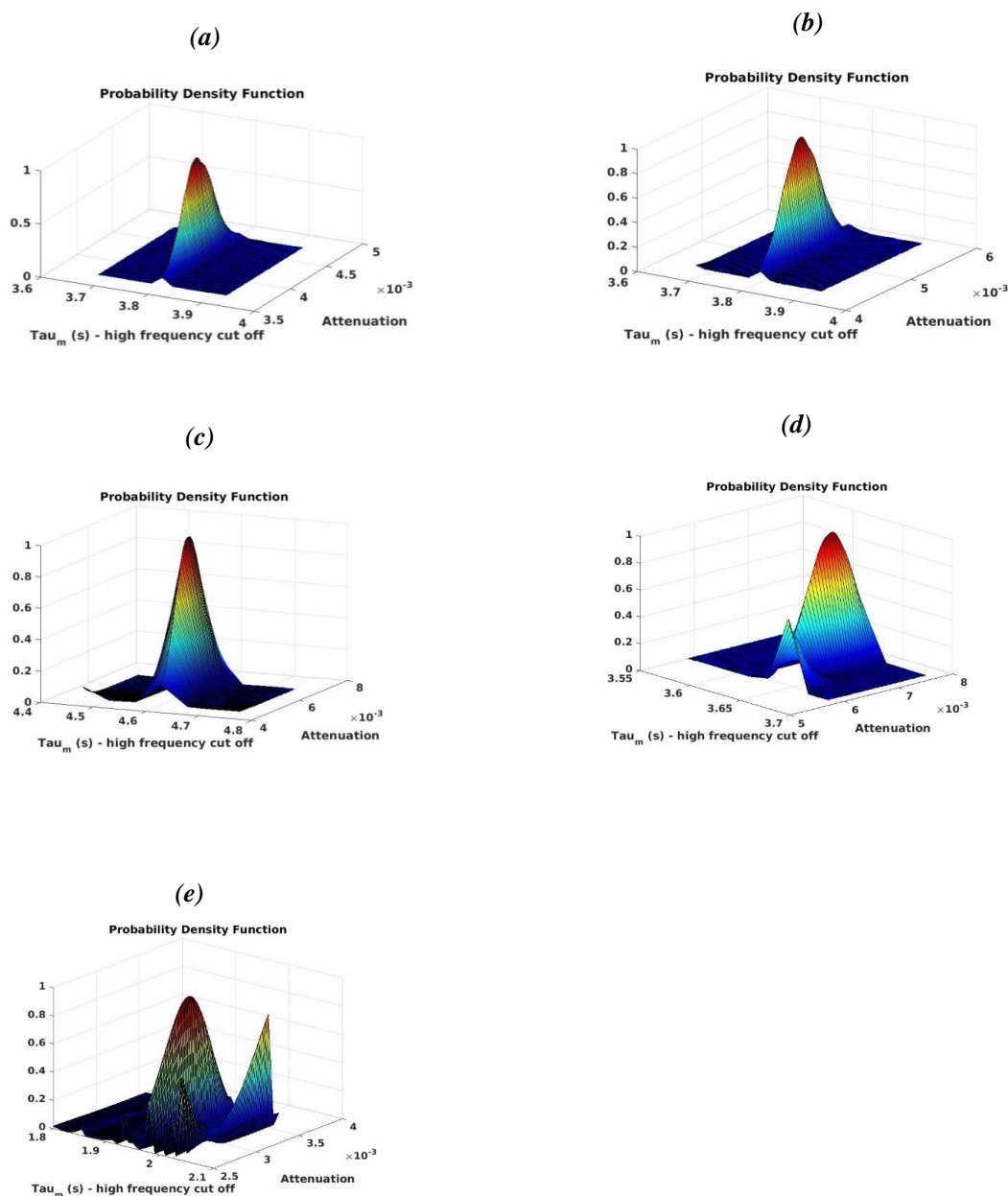


Figure 5: Gaussian probability density function constructed with the least squares norm difference between predictions and simulated observations for (a) MODEL 1, (b) MODEL 2, (c) MODEL 3, (d) MODEL 4 and (e) MODEL 5.



550  
551  
552  
553  
554  
555  
556  
557  
558  
559  
560  
561  
562  
563  
564  
565  
566  
567  
568  
569  
570  
571  
572  
573  
574  
575  
576  
577  
578  
579  
580  
581  
582  
583  
584  
585  
586  
587  
588  
589  
590  
591

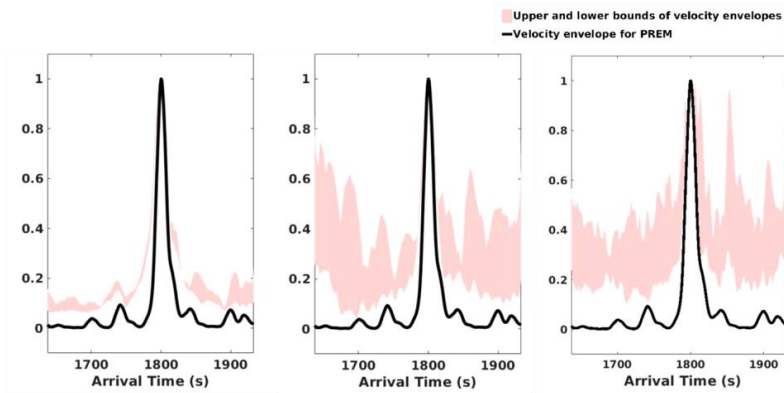
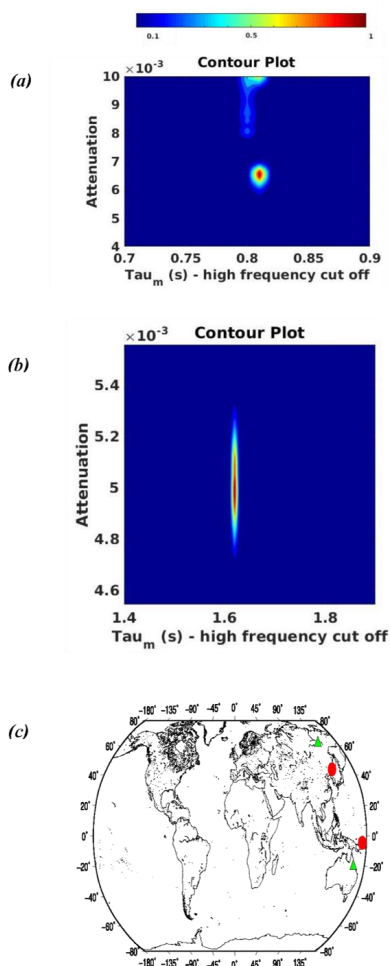


Figure 6: Upper-lower bounds of coda envelopes (shaded area) calculated from 5 random heterogeneity realizations of each MODEL 2 (left), MODEL 4 (middle) and MODEL 5 (right), compared to PREM (black line).





592  
593  
594  
595  
596  
597  
598  
599  
600  
601  
602  
603  
604  
605  
606  
607  
608  
609  
610  
611  
612  
613  
614  
615  
616  
617  
618  
619  
620  
621  
622  
623  
624  
625



626 **Figure 7: Contour plots of probability density functions obtained with multiple ScS observations in two regions. Event**  
627 **locations for the two regions described below are shown in panel (c).**

- 628 (a) **Mantle beneath Papua New Guinea region : Observations are recorded by station CTAO (146.25° E, 20.08° S)**  
629 **for a 490 km deep, mw 6.6 event (154.88° E, 45.43° S) which occurred on May 02 1998, 13:34:28 UTC. Event-**  
630 **station distance is 17.6°.**
- 631 (b) **Mantle beneath Eastern China-Russia border region : Observations are recorded by station YAK (129.68° E,**  
632 **62.03° N) for a 568 km deep, mw 7.3 event (130.66° E, 43.76° N) which occurred on June 28 2002, 17:19:30**  
633 **UTC. Event-station distance is 18.3°.**



634  
635  
636  
637  
638  
639  
640  
641  
642  
643  
644  
645  
646  
647  
648  
649  
650  
651  
652  
653  
654  
655  
656  
657  
658  
659  
660  
661  
662  
663  
664  
665  
666  
667  
668  
669  
670  
671  
672  
673  
674  
675

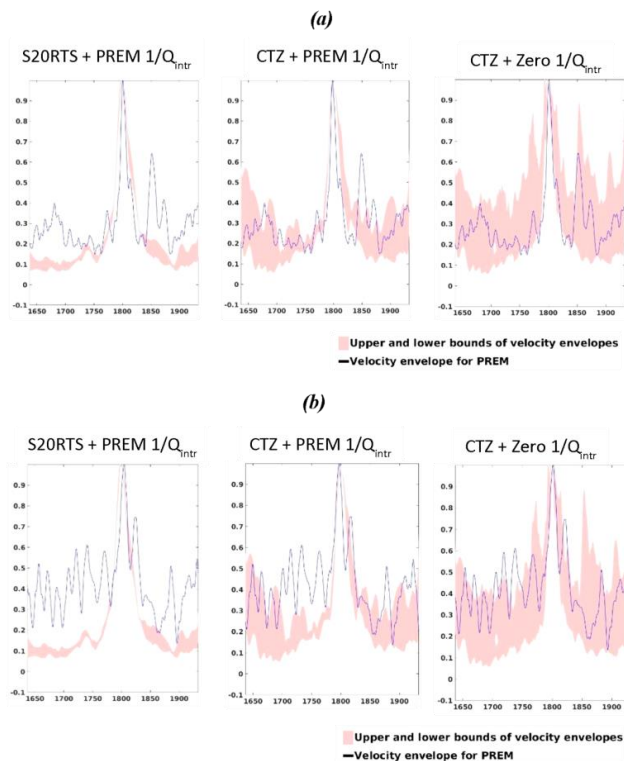


Figure 8: Upper-lower bounds of coda envelopes (shaded area) calculated from 5 random heterogeneity realizations of each MODEL 2 (left), MODEL 4 (middle) and MODEL 5 (right), compared to the squared velocity envelopes of data traces (solid blue lines) from (a) Papua New Guinea data and (b) Eastern China-Russia border region.



676  
677  
678  
679  
680  
681  
682  
683  
684  
685  
686  
687  
688  
689  
690  
691  
692  
693  
694  
695  
696  
697  
698  
699  
700  
701  
702  
703  
704  
705  
706  
707  
708  
709  
710  
711  
712  
713  
714  
715  
716  
717

	$Q_{ses} \pm \delta Q_{ses}$	$\tau_m \pm \delta \tau_m$ (sec)
MODEL 1	$0.004167 \pm 0.00028$	$3.800 \pm 0.004$
MODEL 2	$0.005000 \pm 0.00034$	$3.790 \pm 0.004$
MODEL 3	$0.005747 \pm 0.00066$	$4.600 \pm 0.010$
MODEL 4	$0.007100 \pm 0.0005$	$3.630 \pm 0.007$
MODEL 5	$0.002900 \pm 0.0003$	$1.980 \pm 0.005$

**Table 1 : Apparent attenuation parameters and their errors estimated for the five simulated models using probability density functions shown on Fig. 5..**



718

	$Q_{ses}$	<i>Scattering Attenuation</i> <i>Apparent Attenuation</i>	<i>Intrinsic Attenuation</i> <i>Apparent Attenuation</i>
MODEL 1 (PREM)	240		100 %
MODEL 2 (Tomographic dVs/Vs model (exponential ACF, a = 10km) + PREM )	200	16.7 %	83.3 %
MODEL 3 (Thermodynamic dVs/Vs model for UM only (exponential ACF, a= 10 km) + PREM )	174	27.5%	72.5%
MODEL4 (Thermodynamic dVs/Vs model for both UM and LM (exponential ACF, a = 10 km) + PREM)	140	41.3 %	58.7 %
MODEL 5 (Thermodynamic heterogeneity + no intrinsic attenuation + PREM velocities and densities)	340	100 %	

719  
 720  
 721  
 722  
 723  
 724  
 725

**Table 2 : Estimated relative contributions to apparent  $1/Q_{ses}$ .**