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Interactive comment on "Bilinear pressure diffusion and termination of bilinear flow in a vertically fractured well injecting at constant pressure" *by* Patricio-Ignacio Pérez D. et al.

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Dear Editor:

We are grateful to the Anonymous Referee #2 for his devoted time and his feedback, which allows us to enhance the quality of our manuscript and clarify some observations made by the Referee #2. In particular, we want to thank the Anonymous Referee #2 also for his relatively negative opinion, which may have originated from a series of misunderstandings while reading the manuscript. That motivated and encouraged us to make some points in the manuscript even clearer so that such possible misunderstandings are avoided. In the following we want to clear up those misunderstandings.

A. Referee #2 statement "the paper seeks for a numerical solution to be compared with an analytical solution already existed".

Authors response: The comparison between the numerical simulation result with the semi-analytical (not analytical) solution proposed by Guppy et al. (1981b) was performed solely with the objective of verifying that our numerical model was well set. Subsequently, by using (i) the validated numerical solution and (ii) different methodologies, we were able to produce novel results that are presented and documented for the first time in our work for the case of constant pressure in the well. As explained below, it is important to strengthen the point that the numerical experiments conducted in this study and the extent of the analyses performed go far beyond the study of the transient flow rate done by Guppy et al. (1981b).

B. Referee #2 statement "to me the paper does not have a novelty and as written does not add additional value".

Authors answer: Since it may be the case that the novelty of our present work has not been clearly highlighted in the manuscript, we present the new findings of this work for the case of injecting/producing at constant pressure in the well:

i. In this work, we present for the first time for the case of injection/production at constant pressure in the well the equation describing the spatiotemporal evolution of the isobars along the fracture during the bilinear flow regime (Eq. 16).

ii. In this study, expressions are presented for the first time that quantitatively identify the termination time of bilinear flow when injecting/producing at constant pressure into/from the fracture. The criteria used to quantitatively identify the termination of bilinear flow are explained in detail in sections 3.2.1, 3.2.2, 3.2.3, and 3.2.4. In this work two methodologies are employed to detect the termination of bilinear flow under constant pressure conditions in the well: (a) considering the transition of flow rate in the well and (b) considering the propagation of isobars P_N along the fracture (highlighted in section 3.2 "Termination of bilinear flow").

iii. In this investigation, a new methodology is exposed to constrain the fracture length, based on the end time of the bilinear flow and using the Eq. 16 that describes the spatiotemporal evolution of the isobars along the fracture during the bilinear flow regime (see section 4.1).

iv. In this manuscript, an expression is presented for the first time that allows to determine the time at which a specific isobar arrives at the fracture tip. In terms of dimensionless parameters, this expression is dependent only on T_D (see section 3.2.3 and τ_a in Fig. 7 of the manuscript version read by the Referee #2).

v. A study is conducted for the first time in this work with the purpose of analyzing the velocity of the isobars along the fracture, aiming at distinguishing that the isobars experience an acceleration shortly before they arrive at the fracture tip, which differs from their previous behavior (see end of section 3.1).

The comments made by the Referee #2 encouraged us to carry out an extensive and detailed revision of the manuscript. We acknowledge that the novelty of the results and the key points in our work might have not been highlighted enough in the old version of the manuscript. Therefore, we highlighted the most significant findings of this work in the conclusion section, making it clearer what the novelty of this work is. The conclusion section has been restructured and reformulated in the new version of the manuscript as follows:

"Numerical results obtained in this work corroborated the relation of proportionality previously presented by Guppy et al. (1981b) between the reciprocal of dimensionless flow rate 1/q_wD and the fourth root of dimensionless time τ during the bilinear flow regime for the case of injection/production at constant pressure in the well. Guppy et al. (1981b) obtained the proportionality factor A = 2.722 (Eq. 10), which is slightly greater than the factor obtained here A = 2.60 (Eq. 12). This discrepancy may be attributed to our finer spatial and temporal discretization in comparison with the discretization used by Guppy et al. (1981b). The most significant findings of this work are:

i) During the bilinear flow regime, the migration of isobars along the fracture is described as: x_i (t)=α_b*(D_b*t)^(1/4), where D_b=(T_F^2)/(k_m η_f s_m) (m4 s-1) is the effective hydraulic diffusivity of fracture during the bilinear flow regime. In addition, the migration of isobars in the matrix is given by: y_i (t)=α_m*(D_m*t)^(1/2), where D_m=k_m/(η_f s_m) (m2 s-1) denotes the hydraulic diffusivity of matrix. This simulation results are in line with the study conducted by Ortiz R. et al. (2013) for the case of wells injecting/producing at constant flow rate.

ii) The termination of bilinear flow obtained from transient flow rate analysis is given by (a) the transition time τ_t (circumferences in Fig. 8 and Eq. 20), valid for low T_D and (b) the reflection time τ_r (squares in Fig. 8 and Eq. 21), valid for high T_D.

iii) From the physical point of view, it is of interest to study the propagation of isobars along the fracture, for which the termination of bilinear flow has been found in this work to be given by (a) the fracture time τ_F (filled circles in Fig. 8 and Eq. 22), valid for low T_D and (b) the arrival time τ_a (triangles in Fig. 8), valid for high T_D. However, this methodology may encounter technological obstacles in real field situations.

iv) A new methodology is presented to constrain the fracture length (section 4.1), based on the end time of the bilinear flow and using Eq. (16) that describes the spatiotemporal evolution of the isobars along the fracture during the bilinear flow regime.

v) In terms of dimensionless parameters, the time at which a specific isobar arrives at the fracture tip is dependent only on T_D (see section 3.2.3 and τ_a in Fig. 8).

Similarly as in Ortiz R. et al. (2013), it is observed that the isobars exhibit a peak of acceleration shortly before they arrive at the fracture tip (Figs. 4 and 6). This acceleration was verified by studying the velocity of isobars using the graphs v_iD vs. τ and v_iD vs. x_iD (Fig. 7). It was concluded that for a fixed dimensionless position in the fracture x_iD, the velocity v_iD is higher for lower values of normalized isobars p_N as well as for higher dimensionless fracture conductivities T_D (see Figs. 7b and 7d).

In a follow-up study, it would be interesting to include the effect of fracture storativity and investigate, utilizing an analogue method to that discussed in this work, the behavior of a fracture with conductivity high enough to lead to fracture and formation linear flow."

C. Referee #2 statement "the authors have previously published a similar paper on the subject: "Two-dimensional numerical investigations on the termination of bilinear flow in fractures" by Ortiz and Renner 2013".

Authors answer: Although, the present work uses some of the methodologies presented by Ortiz R. et al. (2013), the present work considers, among other aspects, a different study case. Ortiz R. et al. (2013) studied the behavior of the bilinear flow regime in a fracture and matrix formation injecting/producing at constant flow rate in the well, whereas we investigate in the present work the case of injecting/producing at constant pressure in the well. In addition, we want to clarify that only one of the present authors published the article cited by the Referee #2.

Despite the fact that we refer to the previous work conducted by Ortiz R. et al. (2013) in the introduction section while addressing the state of the art in the topic in question, we additionally refer to the study of Ortiz R. et al. (2013) in the new version of this manuscript by adding the following sentence in the introduction, in the line 106 of the manuscript version read by the Referee #2: "Some of the methodologies used in this work are inspired by the study conducted by Ortiz R. et al. (2013) for wells operating at constant flow rate (pressure transient analysis)".

Our work constitutes a complement and a further development of the work previously published by Ortiz R. et al. (2013).

Here the responses to the comments:

1.1. Referee #2 statement "the problem statement is very simplified".

Authors answer: This observation may have derived from the Referees #2's assumption that the authors did not use a dual-porosity dual-permeability model. As we explain

later, since the general formulation of the physical problem in question is performed using a dual-porosity dual-permeability approach (see few lines below). Further, we use a fit-for-purpose model and with the aim of investigating the behavior of isobars along a fracture with finite conductivity, the model captures the main physical processes and reliably represents reservoir structure and property distribution (dual-porosity dual-permeability).

1.2. Referee # statement "numerical solutions already exist"

Authors answer: To the best of our knowledge, only a semi-analytical solution for the transient well flow rate exists when imposing a constant pressure in the well, which has been presented by Guppy et al. (1981b). No numerical investigation has been documented for (i) the study of the advancement of isobars along the fracture and (ii) the termination time of bilinear flow, when operating with constant pressure in the well. We want to emphasize that finding a numerical solution to be compared with a semi-analytical solution documented by Guppy et al. (1981b) does not constitute the main purpose of our investigation. This comparison was performed only with the purpose of corroborating that our numerical experimental design was well posed. Subsequently, by using the validated numerical solution we were able to produce the novel results mentioned previously in this letter.

We carefully revised each publication mentioned by the Referee #2. It is correct that all these investigations seek for analytical or semi-analytical solutions, however, with the exception of Guppy et al. (1981b), none of them consider the problem statement with a constant pressure in the well. As mentioned earlier, the semi-analytical solution documented by Guppy et al. (1981b) was used in our work solely to validate the numerical solution obtained using the simulation software COMSOL Multiphysics.

We kindly ask the Referee #2 to have a look at the further remarks 1-2 included in the supplement.

2. Authors answer: We want to clarify that we did not use "Comsol Porous Media

Flow Module". The Comsol Multiphysics module we used is "Subsurface Flow Module", which includes groundwater flow in porous and fractured geologic media. This is clearly stated in our manuscript (see section 2.4 of the new version of the manuscript). For further details we kindly ask the Referee #2 to have a look at:

i. COMSOL Multiphysics Reference Manual, version 5.4, COMSOL, Inc, www.comsol.com

ii. Subsurface Flow Module User's Guide, version 5.4, COMSOL, Inc, www.comsol.com. Chapter 3 Porous Media and Subsurface Flow Interfaces, and specially Subchapter The Darcy's Law Interface and Subchapter The Fracture Flow Interface.

It is worth noting that dual-porosity dual-permeability models set up in COMSOL Multiphysics have been successfully tested, validated and benchmarked in numerous published works (e.g. Shao et al. 2014).

We agree with the Referee #2 that for the question at hand one must use a dual-porosity dual-permeability model and so we did indeed in our work. We kindly encourage the Referee #2 to carefully read the Eqs. (1) and (2) (see k_m (m2) and T_F (m3)), where we explicitly consider two permeabilities, one for the matrix formation and one for the fracture.

As for porosity of the matrix rock and the fracture, these are considered in the respective diffusivity equations (Eqs. 1 and 2, respectively). In our work, porosity is implicitly included in the respective specific storage capacity for the matrix and the fracture (s_m (Pa-1) and s_F (Pa-1), see Eqs. 1 and 2, respectively). The value of s_m (Pa-1) used in our work is documented in section 2.4 and the value of s_F (Pa-1) is neglected since the fracture is considered nondeformable and the amount of fluid in the fracture is considered small enough to consider its compressibility as negligible. In addition, the porosity of the fracture is negligible in comparison to the porosity of the matrix. The pressure in the fracture is dictated by an inhomogeneous diffusivity equation, which

contains a time-dependent source term q_F (x,t) but it does not involve an intrinsic transient term. We added in the new version of the manuscript the latter clarifying statement right before Eq. (3). The storativity or more precisely the storage coefficient depends on porosity of rock and compressibility of fluid and rock. We kindly ask the Referee #2 to have a look at:

- Singhal and Gupta 2010, Chapter 8 Hydraulic Properties of Rocks, and specifically Eqs. 8.11 and 8.12, as well as Subchapter 8.2.1 Relationship of Hydraulic Conductivity with Fracture Aperture and Spacing, Eq. 8.15.

- Maliva 2016, Chapter 1 Aquifer Characterization and Properties, and specially Subchapter 1.4.3 Storativity.

- Bear 2007, Chapter 5 Mathematical Statement of the Groundwater Forecasting Problem, and more precisely Subchapter 5.1 Aquifer Storativity and Subchapter 5.2 Basic Continuity Equation, Eqs. 5.20 – 5.29.

- Bear and Cheng 2010, Chapter 5.1 Mass Balance Equations, and particularly Subchapter 5.1.2 Deformable Porous Medium and Subchapter 5.1.3 Specific Storativity, Eqs. 5.1.30 – 5.1.32 and Eqs. 5.1.47 – 5.1.50; to mention a few.

In particular, we kindly ask the Referee #2 to have a look at the following link for the physical and numerical formulation of dual-porosity dual-permeability model in COMSOL Multiphysics:

- https://www.comsol.com/model/discrete-fracture-691

For the formulation of the groundwater flow equation (so-called "diffusivity equation"), which is a result of the combination of the impulse (Darcy equation) and mass (continuity equation) balance equations, in terms of the storage coefficient we kindly ask the Referee #2 to have a look at:

- Singhal and Gupta 2010, Subchapter 7.1.3 General Equation of Flow, Eqs. 7.19-7.25, Subchapter 19.5 Modeling of Homogeneous Porous Aquifer, Eqs. 19.1, 19.3 and 19.4.

- Bear 2018, Chapter 5 Modeling Single-Phase Mass Transport, and specifically Subchapter 5.3.1 Deriving 2-D Balance Equations by Integration, A. Confined Aquifer, Eqs. 5.3.26 – 5.3.33.

- Bear 2007, Chapter 5 Mathematical Statement of the Groundwater Forecasting Problem, and more precisely Subchapter 5.1 Aquifer Storativity and Subchapter 5.2 Basic Continuity Equation, Eqs. 5.20 – 5.29.

- Bear and Cheng 2010, Chapter 5.1 Mass Balance Equations, and in particular Subchapter 5.1.4 Flow equations, Eqs. 5.1.73 – 5.1.76; to mention a few.

In reservoir engineering, it is more typical for the transient diffusivity equation to be given explicitly in terms of porosity of the formation and compressibility of fluid and rock. In groundwater hydraulics and hydrogeology, it is more common to express the transient diffusivity equation in terms of the storage coefficient as we did in our work.

That said, we want to point out that dual-porosity dual-permeability models have been successfully mathematically modelled and simulated using the simulation software COMSOL Multiphysics (e.g. Shao et al. 2014 and references therein). As for the mathematical physics, as described in section 2.1, in our work the dual-porosity dual-permeability model, implemented in COMSOL Multiphysics, is examined by considering the diffusivity equation for the rock matrix and for the fracture (Eqs. 1 and 2, respectively), each containing their respective permeability and porosity (read above) parameters (k_m (m2), T_F (m3) and s_m (Pa-1), s_F (Pa-1)). The coupling between the two equations is given by the term q_F (x,t) (Eq.4), which expresses the mass exchange between fracture and matrix. We hope that it is now clear that the general formulation of our numerical model is expressed in terms of a dual-porosity dual-permeability approach.

To avoid possible misunderstandings and make this point even clearer in the new version of the manuscript, we rephrased the second sentence of the beginning of section 2.1 "governing equations and parameters". Now we write in the revised manuscript "In

a general formulation of a dual-porosity dual-permeability model, the equation utilized to describe the hydraulics of single-phase compressible Newtonian fluid in a reservoir matrix is given by:". Additionally, right after the presentation of Eq. (1) and when referring to the storage coefficient, we write the following "It is worth noting that the storage coefficient depends on porosity of rock and compressibility of fluid and rock". The Referee #2 will be able to see these clarifications in the revised version of the manuscript. We hope now that this fundamental misunderstanding is cleared up.

Please note that all the references of this answer are included in the supplement.

We kindly ask the Referee #2 to have a look at the further remarks 3-7 included in the supplement.

3. Authors answer: We agree with the Referee #2 that the effect of boundary condition must be investigated. There are different ways to conduct such study. As explained in the manuscript version read by the Referee #2, we performed such a study of the effect of boundary condition on the simulation results. For the concrete model described in the present work, we chose the method of enlarging the modeling domain size until a boundary-condition-independent simulation outcome was observed. That is, the boundaries of the model were set far enough that the chosen boundary condition (no-flow) had no impact on the simulation results.

The Referee #2 proposed to study the effect of boundary conditions by changing them from close to open reservoir. We followed Referee #2's suggestion since it represents another way of proving that in our model the boundary conditions do not affect the results for the interested simulation time. We see here an opportunity to make this point clearer and avoid misunderstandings.

In Fig. 1 of this reply we show a similar graph to that used in the manuscript to study the behavior of the reciprocal of flow rate in the well vs. time (Fig. 2 in the manuscript). We find this graph appropriate to investigate the influence of changing the boundary condition from no-flow to constant pressure (p=100 kPa, equal to the initial condition in the fracture-matrix system). In this work, we studied a range of T_D from 0.1 up to 100, therefore we accordingly chose the following three representative examples: T_D=0.3, 6.3 and 50. We can see in Fig. 1 that we obtain the same simulation results when considering no-flow or constant pressure boundary condition for the simulation time considered in our investigation. Thus, boundary condition-independency of the simulation results is guaranteed for the simulation time considered in our numerical experiments. In Fig. 1 the termination time of bilinear flow, which is the time window of most interest from the entire simulation time for this work, for the case of T_D=50, 6.3 and 0.3 is τ_r=3.86$\times$10ˆ(-8), τ_r=1.69$\times$10ˆ(-4) and τ_t=1.78$\times$10ˆ(-2), respectively. It is worth noting that the termination time of bilinear flow regime is identified by the deviation of the respective type-curves from the bilinear-fit-curve.

Alternatively, for the case of no-flow boundary condition considered in the present work, monitoring the pressure at the boundaries of the model constitutes another way of studying the effects of the imposed boundary condition on the simulation results. If the pressure at the boundaries does not change during the entire simulation time considered, this means more evidently that the boundary condition does not affect the modeling outcomes. We additionally conducted such a study for three selected points at the boundaries of the modeling domain (see Figs. 2 and 3 of this answer). We were able to observe that the pressure does not change, representing this a strong indication that the no-flow boundary condition set does not affect the simulation results.

We now hope that the study of the effect of different boundary conditions on the simulation results is clarified. We offer to include these additional studies in the supplement of the online version of the paper. Furthermore, we offer to upload the data related to the model setup and simulation results obtained with COMSOL Multiphysics to provide the interested reader with the possibility of testing and verifying the model.

We hope that all the questions raised by the Anonymous Referee #2 have been addressed.

Sincerely,

The authors (Patricio-Ignacio Pérez D., Adrián-Enrique Ortiz R., Ernesto Meneses Rioseco).

Please also note the supplement to this comment:
https://www.solid-earth-discuss.net/se-2019-170/se-2019-170-AC3-supplement.pdf

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Interactive comment on Solid Earth Discuss., https://doi.org/10.5194/se-2019-170, 2020.

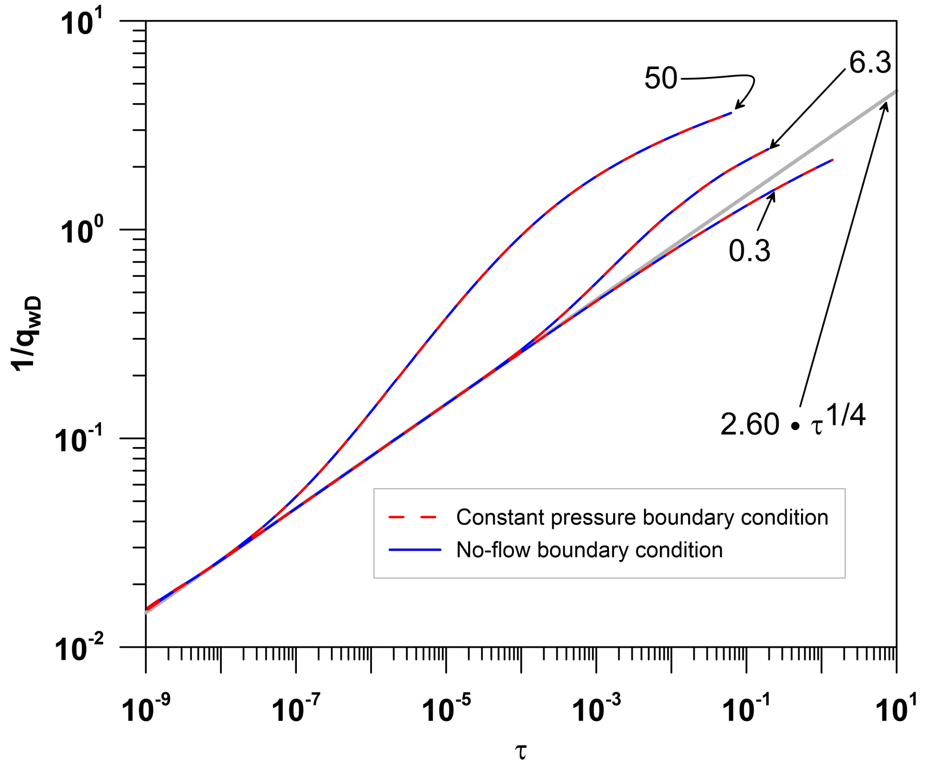

Fig. 1. Model results displayed as 1/q_wD vs. τ in log-log scale. Bilinear-fit-curve (grey line) and type-curves for different boundary conditions: no-flow and constant pressure.

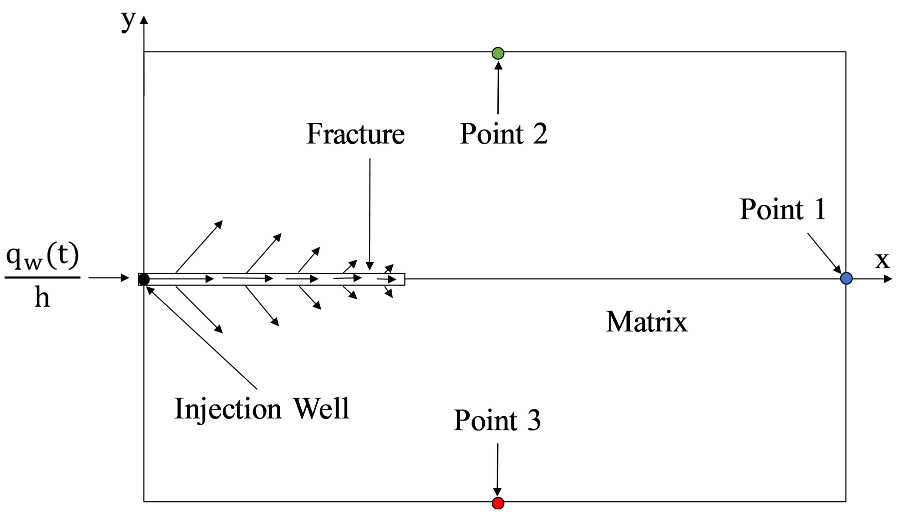

Fig. 2. Representation of the main features of the model (not scaled). Note the 3 points at the boundaries of the model where the pressure was monitored during the simulation time (see Fig. 3 in this reply).

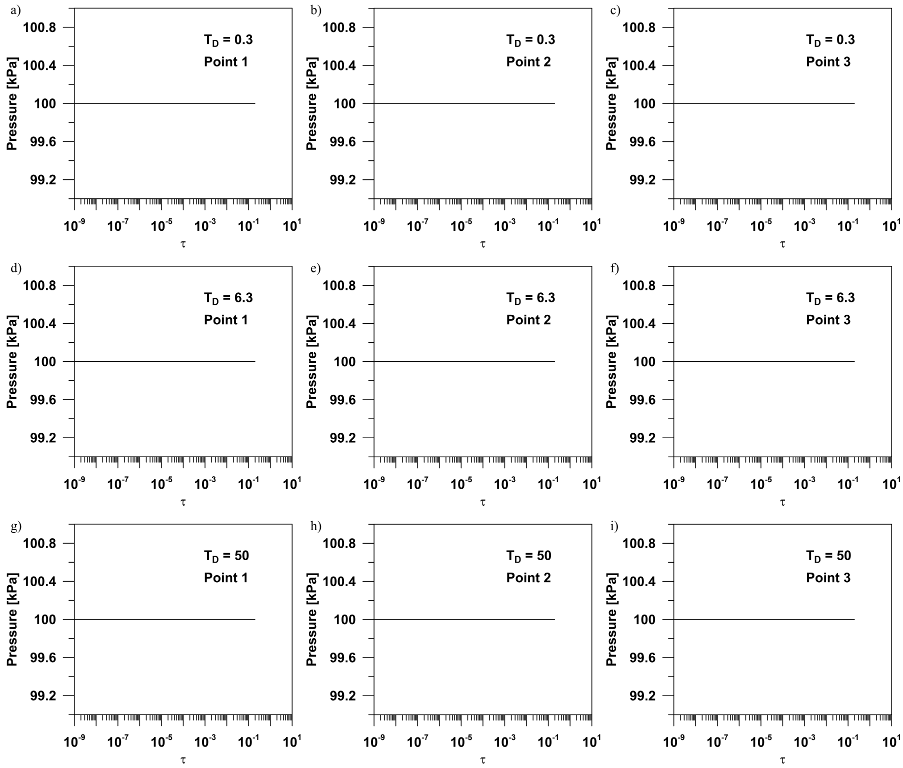

Fig. 3. Monitored pressure at three different points located at the boundaries of the model (see Fig. 2) for three representative cases of T_D displayed in Fig. 1, when imposing no-flow boundary condition.