

# ***Interactive comment on “Azimuth-, angle- and frequency-dependent seismic velocities of cracked rocks due to squirt flow” by Yury Alkhimenkov et al.***

**Yury Alkhimenkov et al.**

yury.alkhimenkov@unil.ch

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## Referee's comment 1

The paper is focused on a numerical model aiming at calculating the frequency and anisotropic response of a saturated cracked rock for a passing seismic wave. The model provides a numerical calculation following the previous work of B. Quintal. The model considers two orthogonal thin cracks embedded in a homogenous background. Two situations are examined: the two cracks are either connected or disconnected. The numerical method has been previously used by Quintal (2016, 2019). It consists in applying relaxation tests to a viscoelastic medium. The CIJ constants are obtained

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by averaging.

#### # Author's reply 1

Dear Reviewer,

Thank you very much for the time you have dedicated to review and comment our manuscript. We believe that your comments have helped us to improve significantly the quality of the work. Please find below the responses to your comments.

Kind regards, Yury Alkhimenkov, on behalf of the authors

#### # Changes in the manuscript 1

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#### ## Referee's comment 2

Several remarks are the following. First the frequency dependent curves extend over a broad frequency range, for a unique crack aspect ratio. This implies that squirt flow would never be focused on a narrow frequency range (unless the system size plays a dominant role in the calculation). This remark is important for the geophysical implications: Fig. 4a shows that the width of the attenuation peak (at half amplitude) is about one order and half magnitude.

#### # Author's reply 2

That's indeed true. Although several analytical solutions predict a narrow frequency range, for example, Gurevich et al., (2010) and Collet and Gurevich (2016), we published results showing a broad frequency range for squirt flow in a different paper [Alkhimenkov et. al., 2020] considering a different pore-space geometry. For the pore-space geometry presented in the current manuscript we again consistently observe that squirt flow is not focused on a narrow frequency range.

#### # Changes in the manuscript 2

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page 10, line 207-209:

"Note that the width of the inverse quality factor peak (at half amplitude) for the components c22 and c33 is about one order and half magnitude (Figures 4a and 4c). It means that attenuation and dispersion due to squirt flow play a significant role over a broad frequency range even for cracks with a single aspect ratio."

## Referee's comment 3

Second, the model considers two cracks of 0.1m in a cube of 0.24 m size. In terms of crack density, this means a very high crack density (close to 1). This is consistent with the large decrease of C22 and C33 in the dry case, compared to the original values of the intact rock (table 1). But such high values are not realistic.

# Author's reply 3

Yes, indeed, the crack density is quite high. This is a synthetic study and all physical effects will be the same for a medium with a smaller crack density (the magnitude will be smaller). According to our calculations, our crack density is closer to 0.1.  $p = 1 / 0.24^3 * 0.1^3 * 2 = 0.1447$ , where 0.24m – a cuboid size, 0.1m - the radius of the crack, 2 - the number of the cracks. This is a correct formula for cracks with random orientations [Bristow, J. R., 1960; Kachanov, M., & Mishakin, V. V. 2019]. For our model geometry, this definition can be considered as a rough approximation. A better definition for our geometry requires the usage of the crack density tensor (second-order) plus another fourth order tensor [Kachanov, M., & Mishakin, V. V. 2019].

# Changes in the manuscript 3

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## Referee's comment 4

Third, Fig. 4c shows a negative 1/Q and a very high dispersion for C23. Probably, the negative sign (which is unphysical) is an error of convention. But the high value (almost

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0.4 for  $1/Q$ ) should be related to the very high crack density (i.e. the size of the system) and the low value of  $C_{23}$ .

#### # Author's reply 4

Yes, indeed the dispersion as well as “attenuation” for the  $C_{23}$  component is strong. First, the negative sign for “attenuation” is correct. We are not the first who observe this negative ratio of  $\text{Im}(C_{23})/\text{Re}(C_{23})$ . For example, Guo et. al. (2017) already reported a negative sign in their results of 2D numerical simulations. Let us first discuss the negative sign and, then, why the value of  $1/Q$  is high.

The  $C_{23}$  component is the coupling component between  $C_{22}$  and  $C_{33}$  components. In the stiffness tensor, we do have high and low limits for components which are based on the energy constraints. When we are talking about wave dispersion and attenuation in anisotropic media, we should consider all stiffness components which are needed to calculate wave speeds and attenuations. The  $C_{23}$  component does not enter any wave mode in any direction alone, the  $C_{23}$  component is always used together with  $C_{22}$  or/and  $C_{33}$ , which have high positive values of attenuation. Therefore, no wave will gain energy. In other words, different components of the stiffness tensor might have positive or negative values of “attenuation” but when we calculate the velocity and attenuation of a wave, the cumulative effect of all  $C_{ij}$  components must be physical (it can be seen in Figures 5-7) and no negative attenuation will be observed. This negative “attenuation” sign for  $C_{23}$  was also verified using Kramers-Kronig relations. We think that there is a terminology issue. In fact, we have a negative value of the ratio  $\text{Im}(C_{23})/\text{Re}(C_{23})$ , but it might be misleading to call that attenuation, which is associated with the characteristic of a wave following a definition based on energy-related considerations.

The high negative value of  $\text{Im}(C_{23})/\text{Re}(C_{23})$  is due to the low values of  $C_{23}$  in figure 4c (17 GPa or 7 GPa). It is related to the high crack density in our model geometry but for a model with lower crack density, we think, that the negative value of  $\text{Im}(C_{23})/\text{Re}(C_{23})$

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will be also high.

Now in the manuscript, we refrain from using the terminology “attenuation” for the ratio  $\text{Im}(C_{23})/\text{Re}(C_{23})$ . We now simply refer to it as a ratio and we reserve the terminology attenuation for wave modes. Furthermore, the wave modes as explained above won't ever show a negative attenuation.

#### # Changes in the manuscript 4

Page 4, line 118-120: "Note that usually the inverse quality factor is used as a measure of attenuation (O'connell and Budiansky, 1978). In this study, we show the inverse quality factor for each component of the stiffness tensor, even though the ratio  $\text{Im}(c_{ij}(\omega))/\text{Re}(c_{ij}(\omega))$  does not represent attenuation of any corresponding wave mode for some components."

Page 9, lines 199-206: "The  $c_{12}$  and  $c_{13}$  components are non-dispersive, the  $c_{23}$  component exhibits strong negative dispersion and a negative inverse quality factor peak shifted towards high frequencies compared to that of the  $c_{22}$ ,  $c_{33}$  components. A similar phenomenon has been reported by Guo et al. (2017) in the context of two-dimensional simulations. The  $c_{23}$  component does not correspond to a wave mode alone, it is always used together with  $c_{22}$  or/and  $c_{33}$  components. Therefore, no wave will gain energy. This negative inverse quality factor sign for the  $c_{23}$  component was also verified using Kramers-Kronig relations. In other words, different components of the stiffness tensor might have positive or negative values of the ratio  $\text{Im}(c_{23})/\text{Re}(c_{23})$  but when we calculate the velocity and the inverse quality factor of a wave, the cumulative effect of all  $c_{ij}$  components must be physical and no negative attenuation will be observed."

#### ## Referee's comment 5

Four, although a precise comparison is impossible, it would be of interest to discuss these results against (effective medium) calculations published some years ago

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(Guéguen, Y. , and Sarout,J., 2009. Crack-induced anisotropy in crustal rocks: predicted dry and fluid-saturated Thomsen's parameters. *Physics of the Earth and Planetary Interiors*, 172, 116-124; and Guéguen, Y. , and Sarout, J., 2011. Characteristics of anisotropy and dispersion in cracked medium. *Tectonophysics*, 503, 1-2, 165-172.) In both cases, the goal is similar but the methods differ. In terms of anisotropic compliances dispersion, it seems (from a first check) that the predictions of GS agree with the present results. They give a prediction of dispersion for Sijkl in terms of the two crack density tensors alpha and beta. Non-zero values are predicted only if the i,j,k,l index is 2 or 3 (given the cracks orientations in the present case).

#### # Author's reply 5

We compared the results of our numerical solver against an analytical solution for squirt flow in a recently published manuscript [Alkhimenkov et. al., 2020]. There is no analytical solution for the presented model geometry, only the low and high-frequency limits can be described analytically. Guéguen, and Sarout, (2009, 2011) presented analytical models considering poroelastic and squirt flow effects at low and high-frequency limits. They observe that the anisotropy (described by Thompson's parameters) is, in general, more pronounced at high frequencies than at low frequencies.

#### # Changes in the manuscript 5

page 2, line 47-50: "There are several analytical solutions for squirt flow (O'Connell and Budiansky, 1977; Dvorkin et al., 1995; Chapman et al., 2002; Guéguen and Sarout, 2009, 2011; Gurevich et al., 2010) which are based on simplified pore geometries and many physical assumptions."

Page 17, line 352-365 (new subsection):

"A qualitative comparison against analytical models

Numerical simulations are useful but analytical models are especially attractive since they help us to better understand the physics and do not require sophisticated numeri-

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cal simulations. The limitations of the analytical solutions are restricted to simple pore space geometry and some assumptions related to physics are needed to derive the closed form analytical formulas. Such a comparison of the numerical results against an analytical solution has been performed by Alkhimenkov et al. (2020) for a different pore space geometry. Unfortunately, there is no analytical solution for the present study considering a periodic distribution of intersecting cracks in three-dimensions. But the qualitative comparison of the low and high-frequency limits (which correspond to relaxed and unrelaxed states) is possible (Mavko and Jizba, 1991). Several analytical studies show that the anisotropy (described by Thompson's parameters) is, in general, more pronounced at high frequencies than at low frequencies (Guéguen, and Sarout, 2009, 2011). In the relaxed state, one can calculate the effective dry elastic moduli and use Gassmann's equations to obtain the effective moduli of the saturated medium. In the unrelaxed state, one can calculate the effective elastic moduli by restricting fluid flow (by using zero displacement boundary conditions in the cracks intersections). The low and high-frequency limits for elastic moduli have been calculated using these semi-analytical approaches and numerical results have been reproduced."

## Referee's comment 6

In conclusion, this is an interesting paper.

25 January 2020 Yves Guéguen.

# Author's reply 6

Thank you for your comments, which helped us to improve significantly the manuscript.

# Changes in the manuscript 6

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