Response to reviewer 3

Thank you for your comments. Please find our detailed responses to your comments below in red font.

The authors present a novel approach for Bayesian full-waveform Inversion of VSP tube waves with the aim of estimating fracture aperture and compliance. The manuscript is well written and interesting to read. The choice of the research methodology is appropriate and it supports the research objective. The authors validate the proposed approach using both synthetic and experimental data.

I have a couple of comments and questions:

- Could you please elaborate more on the choice of inversion parameters and their influence on the results, e.g. number of Markov chains, number of runs.
  The number of Markov chains was chosen as a trade-off between exploring the solution space as comprehensively as possible and keeping the computational costs manageable. More Markov chains allow for a more exhaustive exploration of the solution space in search of the posterior probability density function, but require more computational resources. If vast computational resources are available, many Markov chains can be run in parallel. As this was not the case for us, we chose the minimal number of Markov chains that still allows for an adequate exploration of the solution space.
  The number of runs was chosen based on a similar reasoning. For infinite chains, our Bayesian inversion should always find the posterior probability density function. However, as the chains are finite, there is a certain risk that the algorithm ends up in a local minimum. If the latter is the case, several different runs would lead to different results. So, to ensure that the posterior probability density function has been found, multiple runs are necessary, at least two, ideally more. Three runs was the maximum that could be done within a reasonable amount of time with the computational resources at hand.
  The section presenting the real-data experiment has been updated to explain our choice of these parameters.

- I understand that the number of fractures is a known parameter. How do you define the total number of fractures for inversion and what is the smallest fracture aperture that you can consider?
  To determine how many fractures are included in the inversion, the measured data are inspected. For each clearly discernible tube-wave event, a fracture is included in the inversion. The origin of the tube wave thereby defines the depth of the corresponding fracture. If available, as in our study, an optical televiewer log can be consulted to avoid associating non-fracture-related tube-wave events with fractures or identifying cases where multiple, closely spaced fractures create one big tube-wave event. The manuscript has been adapted to state that the fractures have been identified through visual inspection of the seismic data while also considering the available televiewer data.
  As our forward solver is semi-analytical, there is no limit to fracture apertures that we can consider. However, there is a spatial limitation with respect to the depth sampling. In our case, we sample along the borehole with a spacing of 0.1 m. As fractures have to be located at these sampling points, two adjacent fractures cannot be closer together than 0.1 m. In order to model smaller distances between fractures, the depth sampling needs to be densified, thus, slowing down the algorithm significantly, unless the considered borehole section is shortened. A statement explaining the link between depth sampling and minimal distance between fractures has been added to the methods section.

- It is known that the tube wave reflection and transmission is not only generated through a fracture intersecting a borehole, but it can be also caused by borehole diameter changes (i.e. washouts). Can your inversion algorithm account for this effect?
  Our forward solver does not take these secondary sources of tube waves into account (except if fractures act as secondary sources). We assume that borehole diameter changes are spatially separated from fractures. In that case, such an event will only increase the data misfit, but will not affect the estimate of fracture-related parameters. In contrast, if a fracture is close to a borehole diameter change, the tube wave caused by the fracture will be affected by the borehole diameter change and, consequently, also the fracture-parameter estimates will be affected. We have added some text in the methods section clarifying that other sources for tube waves, besides fractures, are neglected in our forward solver.

- I'd suggest to include the synthetic example in the main body rather than in the Appendix. This will improve the readability and understanding of the method.
  We have followed your suggestion and adapted the manuscript accordingly.

- The authors state that the inversion results are consistent with the televiewer data and refer to Krietsch et al. (2018) many times. Which figure in Krietsch et al. (2018) is showing the interval selected for the Bayesian inversion? Could you please include it in your manuscript for clarity?
  The paper by Krietsch et al. (2018) is a data description for a dataset that is freely accessible here:
  `https://www.research-collection.ethz.ch/handle/20.500.11850/243199`
  The compressed folder "3D static geological Model.zip" contains the following file:
  `/3D static geological Model/05_GeophysicalBoreholeLogs/INJ2/INJ2_structures.txt`
  which lists the estimated fracture apertures from televiewer data for each fracture. We compared our results with these data. As there is no figure in the paper by Krietsch et al. (2018) that shows these data, we cannot reproduce it in our paper.

- "The inferred apertures are consistent with televiewer data and the inferred compliances are roughly in the same range as those derived from sonic logs at the same site." Please be more precise, what are the apertures and compliances values derived from the televiewer data and sonic logs. What is the vertical resolution of the televiewer data?
  The first paragraph of the discussion section has been extended to state the values explicitly and how we interpret them. The vertical resolution of the televiewer data is 0.21 mm. This information has also been added to the discussion section of the manuscript.