A response to the interactive comment on "Regularization methods for the combination of heterogeneous observations using spherical radial basis functions" by Qing Liu et al.

Anonymous Referee #1

Dear Reviewer,

Thank you very much for taking the time to review the manuscript and give comments that help us to improve it. Please find below our point-by-point response to your comments. The original comments are in black, and our answers are in blue. The revised manuscript with tracked changes is also attached.

The manuscript is a good scientific writing, with clearly defined question, i.e. how to choose the optimal Tikhonov regularization parameter in the combination of heterogeneous observation for regional gravity field modeling. Numerical experiments are properly designed to investigate this problem and the results are fully discussed to get a reasonable conclusion. However, I couldn't see much scientific significance from the manuscript.

As you said, this manuscript is trying to answer the question "how to choose the optimal Tikhonov regularization parameter in the combination of heterogeneous observation for regional gravity field modeling." Xu et al. (2006) pointed out that when heterogeneous measurements are involved, the conventional regularization parameter choice method, including the L-curve method and GCV may be not proper to apply. The reason is that the L-curve method or GCV cannot determine the relative weighting between different observation types. On the other hand, VCE can estimate the variance components of different observation techniques as well as the regularization parameter (interpreted as the ratio between the variance factors of the observations and the prior information) simultaneously. However, Naeimi (2013), Liang (2017) and Lieb (2017) all showed that VCE sometimes could not provide a reliable regularization result by incorporating prior information. Because in VCE, the prior information is interpreted as an additional observation technique, which is required to be stochastic. In regional gravity field modeling, usually a background model serves as the prior information, which is deterministic.

Based on these facts, the three commonly used regularization parameter choice methods seem all have pitfalls when heterogeneous observations are combined. Thus, we proposed two methods which combine VCE for determining the relative weights between the observation types and the L-curve method for determining the regularization parameter. The idea of combining VCE for weighting different data sets and a method for determining the regularization parameter was introduced in the Section 'future work' of both Naeimi (2013, p.121) and Liang (2017, p.134). The study in this manuscript is also inspired by Wang et al. (2018), who combine two methods successively for determining the regularization parameter and relative weights for GPS and InSAR. However, to the best of our knowledge, there are still no publications applying this idea for combining heterogeneous observations in regional gravity field modeling.

Our experiment results prove that the original L-curve method or the VCE cannot always give a satisfying result (please refer to Table 6 in the revised manuscript), and our proposed 'VCE + L-curve method' provides the best results which means the smallest RMS error and the largest correlation for all our study cases. A further work based on the proposed 'VCE + L-curve method' for combining real observation data to model the regional gravity field in Colorado, USA, is under investigation. Moreover, this new approach could be used not only for regional gravity field modeling but also in many other fields such as atmospheric science.

We have rewritten parts of Section 1 and added two paragraphs explaining the limitations of the existing methods as well as the reasons for proposing the new methods. We have also highlighted the scientific significance of the new methods in the discussion part as well as the conclusion section.

Among the five candidate regularization parameter choice methods, the first two methods that based on CM 1, i.e. equal weighting between different observation types, are naturally anticipated to be worse than the other three methods that based on CM 2 even without numerical experiments. The proposed two new approaches, VCE+L-curve and L-curve+VCE, differs in fixing which relative weighting first, between different observation types or between observation and a-prior information, and seems there is not much scientific rationale in these two approaches. Moreover, the differences in residual RMS of the two approaches are not significant, just around 1-2%, the differences in correlations are even smaller.

We have removed the method 'VCE based on CM 1' since it is expected to perform worse than 'VCE based on CM 2'. However, due to the aforementioned reason, VCE cannot generate a reliable regularization parameter. In this case, VCE based on CM 2 sometimes gives even worse results than the L-curve method based on CM 1. Please refer to the results of study cases B and D in Table 6 and Table 7 of the revised manuscript.
We have rewritten the result section, and the comparison between 'VCE based on CM 1' and 'VCE based on CM 2' is removed. Now we focus on the comparison of the two proposed methods to the original L-curve method and VCE.

We have added the scientific rationale for the proposed two new approaches, which is to use VCE for determining the relative weights between different observation types and the L-curve method for determining the regularization parameter. The two methods differ in the order of the applied procedures, i.e., applying VCE or the L-curve method first. The improvements in the RMS values of 'VCE + L-curve method' compare to 'L-curve method + VCE' with respect to the validation data are generally not significant when the Shannon function is used, except for case study B where the difference reaches 6.6%. However, when the CuP function is used, the RMS errors of 'VCE + L-curve' decrease 8.3%, 1.7%, 11.4%, 4.2%, 6.8%, 3.6% (see Table 8) compared to 'L-curve + VCE' in the six study cases, respectively. Thus, we conclude that both newly proposed approaches are performing well, but 'VCE + L-curve' is more stable regarding different type of SRBFs (see Page 14, Line 5-7). More important, 'VCE + L-curve method' outperforms the original VCE in all the study cases no matter using a smoothing or non-smoothing SRBF; 'L-curve method + VCE' outperforms VCE when the Shannon function is used, but does not show significant improvements compared to the VCE when the CuP is used.

Instead of the proposed two approaches, I prefer the third approach, i.e. VCE based on CM2, but with iteration. Since the relative weighting between different observations (weighting factors) and relative weighting between observations and a-prior information (regularization parameter) are adjusted simultaneously. Give the same initial variance components in the VCE+L-curve method, apply VCE based on CM2, then iterate this process with newly estimated variance components. I didn't find description on whether iteration is applied on the 'VCE based on CM2' approach in this manuscript, if not, I highly suggest the authors to test this iterated VCE method.

We would like to clarify that the 'VCE based on CM 2' approach presented in the manuscript has already included the iteration procedure, and the same initial values were given to both the 'VCE based on CM 2' and 'VCE + L-curve method'. We have mentioned in Page 9, Line 9 that we estimate the variance components within an iterative process.
We have added more explanations to Section 4.2 showing why VCE could lead to unreliable regularization results.

Also the 'regularization methods' in the title seems to be too 'big' for this manuscript, actually only one regularization method, Tikhonov regularization, is applied in this manuscript, something like 'regularization parameter choice methods' may be more appropriate.

We have changed the title to 'Regularization parameter choice methods for the combination of heterogeneous observations using spherical radial basis functions'.

Then there are some minor problems and typos:
Line 28, page 1, the fact that they are fulfilling the Laplacian ..., change to 'they fulfill...'.

We have changed this sentence as suggested.

Line 5, page 2, remove the acronym 'SRBF' since it's already defined in the abstract.

We have removed it.

Line 6, page 3, ... the aforementioned two combination models, actually the CM1 and CM2 are not mentioned yet.

The two combination models have been mentioned and introduced on Page 2, Line 18-20. However, this whole paragraph has been changed in the revised version corresponding to the first comment.

Line 3, page 5, ... the tensor ΔV of the gravity gradients V_{ab}, change to '... the tensor of the gravity potential ...'.

We have changed the sentence to '…observed the gravity gradients V_{ab} with $a, b \in \{x, y, z\}$, i.e. all second-order derivatives of the gravitational potential V…'

Equation 10, usually we use the symbol Δ to denote Laplacian operator in physical geodesy, suggest to use $\nabla^2 V$ for tensor of gravity potential, and change $\text{trace}\,(\Delta V) = 0$ to $\Delta V = 0$.

To keep the symbol consistent, we have changed Eq. (10) to $V_{ab} = \begin{bmatrix} V_{xx} & V_{xy} & V_{xz} \\ V_{yx} & V_{yy} & V_{yz} \\ V_{zx} & V_{zy} & V_{zz} \end{bmatrix}$, and changed $\text{trace}\,(\Delta V) = 0$ to $\text{trace}\,(V_{ab}) = 0$.

Line 7 page 5, $V_{zz} \approx V_{rr}$, this doesn't stand universally, actually they are only approximately equal around the z-axis.

We have changed this sentence to 'The observation data of GOCE used in this study are simulated as the radial component $V_{rr} = \frac{\partial^2 V}{\partial r^2}$, and the observation equation reads…'.

Typo in equation 25, the variances are all the same $\sigma_1^2 \boldsymbol{P}_1^{-1}$, which should not.

This is a typo, we have changed it to $D\left(\begin{bmatrix} \boldsymbol{f}_1 \\ \boldsymbol{f}_2 \\ \vdots \\ \boldsymbol{f}_p \\ \boldsymbol{\mu}_d \end{bmatrix}\right) = \begin{bmatrix} \sigma_1^2 \boldsymbol{P}_1^{-1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \sigma_2^2 \boldsymbol{P}_2^{-1} & \vdots & \vdots & \vdots \\ \vdots & \mathbf{0} & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \sigma_p^2 \boldsymbol{P}_p^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \sigma_d^2 \boldsymbol{P}_d^{-1} \end{bmatrix}$

Line 2, page 9, plotting the norm of ..., change to 'plotting the log of the norm of ...'.

Done.

Line 11, page 12, ... is chosen to 4 degree, change to ’... is chosen to be 4 degree’, same problem in line 1 page 13.

We have changed both of them.

Figure 5, according to the legend, the blue dots represent terrestrial I data, but how can there be terrestrial data even over the Mediterranean sea? Moreover, I can’t identify the validation area in the figure.

Parts of the terrestrial I data are over the Mediterranean sea because these data are not real observations but simulated from the EGM2008.
For clarification we have added a sentence to Line 23, page 10, “The two validation areas are presented in Figure 5 with black rectangles.”

References:
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