Referee 1 (E. M. Dunham)

This study examines the dependence of dissipated energy on rupture history using simulations of earthquake ruptures with thermal pressurization as the dominant fault weakening mechanism. Dissipated energy can be divided into work done by sliding against the residual strength of the fault (referred to in this study as dynamic strength) and breakdown energy. Breakdown energy is one of the few earthquake source properties that can be indirectly estimated from far-field seismic radiation, and numerous observational studies have explored the dependence of breakdown energy on earthquake magnitude, local slip, etc. Likewise, theoretical studies of proposed fault weakening processes, like thermal pressurization, provide predictions of how breakdown energy depends on slip and various parameters (like thermal and fluid transport properties of the fault zone). However, in order to obtain closed form analytical solutions, these theoretical studies often make assumptions like constant slip velocity, that are unlikely to be to be met in reality. The current study utilizes more complex earthquake simulations with thermal pressurization that provide more realistic rupture and slip histories. The authors calculate breakdown energy, both locally at each point on the fault and in a suitably averaged sense, and compare to both theoretical predictions and observational constraints.

The main conclusions are that local breakdown energy can exhibit large spatial variations across the fault, due the complex rupture history, and can be quite different from the average breakdown energy that is estimated from far-field seismic observations. The study thus provides an important caveat for researchers who hope to infer fault weakening mechanisms from seismic observations. The study is well designed and the manuscript is clearly written; I recommend publication after addressing the following minor comments:

Thank you for the positive assessment of our work and comments that helped us improve the manuscript. Please find our responses to your comments below.

1. Line 45. It is stated that peak and dynamic strengths are expected to be material properties of a fault, but I disagree that this is how people usually think about it. It is more common to regard static and dynamic friction coefficients as material properties, recognizing that shear strength depends on both friction and effective normal stress. I think it is widely understood that the ambient effective normal stress is not a material property of fault, but depends on tectonic loading and fluid state. I recommend explaining this in more detail, pointing out that for the set-up in your 2D simulations, the ambient effective normal stress is a prescribed quantity that is unaltered by fault slip (unlike in a dipping fault configuration, etc.).

We agree that the statement is imprecise and it is not essential to the manuscript so we have removed it. We have added the discussion about potential variations in ambient effective normal stress.

2. Line 53. The LEFM relationship between rupture velocity and breakdown energy requires that the small-scale yielding criterion is met. You later explain this, but it might be helpful to mention small-scale yielding here. (Optional)

We have added this specific example where small-scale yielding is introduced on line 84 (now 98).

3. Line 60. I think you mean greater than 1 m/s, not 10^3 m/s!

We have corrected the text with the correct value 10^{-3} m/s or 1 mm/s, which generally represents the initiation of enhanced weakening in the lab. Thank you for noting this typographical error.

4. Line 136. “or” should be “of”

Thank you.

5. Equation (11). Should the integral be over Sigma, not Omega? In any case, please make sure to define Sigma and/or Omega.

We have updated the notation to consistently use Omega to denote the ruptured domain and clarified this in the text.

6. Line 194, lines 258-260, line 339, and elsewhere. (Depending on how you choose to respond to this suggestion, this could warrant a major revision.) You compare your simulation results to the two closed-form thermal pressurization solutions in Rice (2006), both of which utilize the constant slip velocity assumption. However, this was improved upon by Viesca and Garagash (2015) to account for a more realistic slip velocity history that accounts, in the context of a steadily propagating rupture, for elastodynamic relations between slip and stress change. Viesca and Garagash along provide solutions for thermal pressurization and the dependence of breakdown energy on slip. I think your paper would be substantially strengthened by comparing your simulation results to their theoretical predictions, in addition to the Rice (2006) predictions. This might provide insight into the validity of a steady state solution for describing more complex ruptures that accelerate, decelerate, interact with arrest waves, etc. Perhaps there are situations where the steady state solution is a good approximation, or maybe not. It would be very useful to know this since it will help guide the field to either invest more time in developing steady state solutions for other weakening mechanisms or to instead shift toward fully numerical rupture simulations like you have done.

This is an excellent suggestion that we have implemented in the manuscript. The approximation of a steadily propagating rupture does not capture the variability in local G vs. slip as observed in our simulated dynamic ruptures. We have added the end-member curves for the drained and undrained cases from Viesca and Garagash (2015) to Fig. 9 (new Fig. 11), which exhibit marginal differences on the log-log plot from the solutions of Rice (2006). Both sets of solutions from Viesca & Garagash (2015) and Rice (2006) provide qualitative insight into the increase in G with slip, but do not provide much detail in the variability for individual points throughout ruptures.

7. Figure 1a. What is the small white triangle? Should this region be shaded blue?

The white section of the dashed red trapezoid in Figure 1a corresponds to the portion of the strain energy change per unit area that corresponds to the breakdown energy outside of the red trapezoid (that arises when the initial and peak shear stress do not coincide). This additional dissipated energy outside of the red trapezoid comes at the expense of the radiated energy. We have added this comment to the caption.

8. Figure 3 and elsewhere. You utilize a 2D model, but then make a comparison to observationally interred breakdown energy from real earthquakes in 3D. It would be useful to add a few sentences or a paragraph discussing whether or not the 2D idealization alters the predicted scaling behavior. Many people familiar with wave propagation understand that there are substantial differences between 2D and 3D, but for various reasons (discussed, for example, in Freund's Dynamic Fracture Mechanics textbook) this is far less the case for fracture problems. Please comment on this to avoid confusion and to give readers more confidence that the comparison you've made is relevant.

We have added a paragraph in section 5 discussing that the exact scaling relationship between breakdown energy and slip should be examined in 3D simulations. However, the main conclusions of this work that breakdown energy is a rupture-dependent quantity should be the same in 3D models.

9. Figure 5. It appears that there is weakening that is confined to very small slip, prior to the main effects of thermal pressurization. Is this due to the drop in friction coefficient from standard rate-and-state effects? If so, it might be possible to capture this (small) contribution to breakdown energy through a typical LEFM fracture energy idealization, as was done by Brantut and Rice (GRL, 2011). Consider commenting on this.

The initial weakening at very small slip in Fig. 5 (new Fig. 6) indeed mostly comes from the drop in friction coefficient due to the standard rate-and-state friction. The dynamic resistance level for the rate-and-state component at 1-10 m/s slip rate would be tau_{ss}(V) = 13.3-13.0 MPa, comparable to where the rapid drop transitions to more gradual effect due to thermal pressurization, although, for some points, rapid weakening due to adiabatic thermal pressurization seems to start within that small slip as well. It would be difficult to accurately estimate the breakdown energy involved apriori, as we know only the slope of that weakening - b*sigma/D_rs - but not the peak stress (which depends on the pre-rupture state variable), the very small slips involved, or how much thermal pressurization is mixed in. However, assuming that the thermal pressurization is not yet involved, and taking typical inter-event times 10 years as an estimate for the pre-rupture state variable, we can get the upper bound on the breakdown energy associated with the rate-and-state processes to be 0.15 MJ/m^2, much smaller than the overall breakdown energies we obtain. We have added a comment on this to the text.