

## ***Interactive comment on “Pragmatic Solvers for 3D Stokes and Elasticity Problems with Heterogeneous Coefficients: Evaluating Modern Incomplete LDL<sup>T</sup> Preconditioners” by Patrick Sanan et al.***

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### General comments:

This paper presents the benchmark experiment with the direct and iterative solvers for the Stokes flow and elastic problems targeted by the solid earth simulation. The authors especially focus on the ILDL factorization which is not yet commonly used in the numerical solid earth community. Their performance test showed the tradeoff relations among the robustness, time to solution, and memory cost. This paper is well

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organized and presented results may motivate the computational geoscientist to utilize ILDL in their own geodynamics and seismic applications. Thus, this paper essentially fits the scope of the method paper of Solid Earth (SE).

On the other hand, there is some room for improvement in presentation and experimental design. The author claims that the robustness of ILDL is the advantage over the iterative ABF solver, but supporting experimental data is found only in the extreme case which solves 10 inclusions of  $10^6$  viscosity contrast within  $32^3$  elements simulation. In other cases, iterative ABF solver shows better results in time-to-solution, memory usage, parallel performance. On the other hand, ILDL shows practical advantages against direct solver in memory cost. Thus, in conclusion, the ILDL solver is found to be the potentially good alternative of direct solver rather than an iterative solver. So, the expected reader would be the user of direct solver. However, their performance analysis is presented mainly for ILDL vs iterative ABF solver rather than vs direct solver, especially in the parallel performance section. I encourage the author to continue this work, but the presentation should be improved and more detailed performance analysis should be addressed before I recommend this for publication in SE.

### Detail comments:

1. In introduction: Several sentences sound your opinion rather than the objective view (e.g. “This is unfortunate” in line 102). Such phrases are not appropriate for the research paper.
2. In introduction: Please more review the progress and difficulty in direct solvers, although the author mainly reviews the recent progress of iterative solver.
3. In line 45: The hierarchical grid system with such as AMR [Rudi et.al. 2015] worked well as the solution of highly variable viscosity problem with controlling the coefficient.
4. In line 95: Since expected readers of this journal are not specialists in linear alge-

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bra, a more comprehensive review is needed. For example, how much memory was saved against direct solver with increasing/decreasing the time-to-solution in the past successful application?

5. In line 211: Delete the space after “( ”

6. In line 253: The spectral analysis for scaled pressure mass matrix can cite [1]

7. In Numerical experiment: I think that the experiment starts with  $x=0$ . This problem setting is suitable for steady-state solution. But in practice, we solve the time-stepping/nonlinear problems. Thus, it is interesting if ILDL largely outperforms the direct solver from the second step. The solution of the previous step will be a good initial gauss for reducing the iteration of ABF and ILDL.

8. In line 264: In practice, direct solver is mainly used in 2D problems. Also, in memory capacity, the difference in maximum element size in 3D ( $40^3$  for PARADISO  $< 48^3$  for ILDL) seems to be trivial but that in 2D ( $252^2 < 332^2$ ) is significant in scientific application. Then, the experiment in 2D should worth considering in SE.

9. In line 317: It is confusing that ABF does not fail to converge in Figure1. Why not plot the case with contrast =  $10^6$  with 8 inclusions?

10. In line 318: Do we really need to solve the problem with over 10 inclusions in  $32^3$ ? The accuracy of such a setting seems to be a useless solution in physics. In addition, to check the robustness, SINKER box test of [May and Moresi, 2008] is better than this setting.

11. In Figure 1: Sample glyphs are difficult to see.

12. In Figure 1: What is the message from the peak memory foot point? Why memory size in Table2 is not enough?

13. In Table2: For a fair performance comparison, it should be noted that the number of iterations independent from the DOF for ABF.

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14. In line 320: Since your ABF is based on Jacobi smoother and Arnoldi type Krylov method, more smoothing iteration or avoiding rounding error of GMRES are promising to gain the convergence even with  $10^6$  problem. It is interesting to see the performance of ABF with increasing the number of inner smoothing iterations to converge  $10^6$  problem (I argue that such simple tuning is out of the expertise.). Whether such robust ABF can solve the  $10^6$  problem faster than the ILDL method or not, is the matter for ILDL to be the alternative of ABF.

15. In line 297: Please write Eqs. (5), (11), and the norm should be a consistent form.

16. In line 353: Additive Schwarz Method (ASM) should be noted.

17. In “Using ILDL within a parallel preconditioner”: Since ILDL is worth investigating as an alternative of direct solver PRADISO rather than ABF solver, the performance on SMP system (openMP) is more interesting than distributed memory parallelization (MPI). Please reconsider the way of presentation. Since ABF is inherently suitable for the distributed memory parallelization, Table3 did not show any advantage of ILDL.

18. In lines in 400-404: These lines seem to be a jump in the context. Please introduce them in more detail if you want to address them. By the way, “incomplete LDL” should be ILDL

[1] P. P. Grinevich and M. A. Olshanskii, An iterative method for the Stokes-type problem with variable viscosity, SIAM Journal on Scientific Computing, 31 (2009), pp. 3959–3978

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