



1 **Matrix gas flow through ‘impermeable’ rocks - shales and tight** 2 **sandstone**

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8 **Abstract.** The effective pressure sensitivity of gas flow through two shales (Bowland and Haynesville shales) and
9 a tight gas sandstone (Pennant sandstone) was measured over the typical range of reservoir pressure conditions.
10 These are low permeability rocks such as can be exploited as caprocks above reservoirs that might be developed to
11 store compressed air, methane, hydrogen or to bury waste carbon dioxide, all of which may become important
12 components of the forthcoming major changes in methods of energy generation and storage. Knowledge of the
13 petrophysical properties of such tight rocks will be of great importance in such developments. All three rocks
14 display only a small range in \log_{10} permeability at low pressures, but these decrease at dramatically different rates
15 with increasing effective pressure, and the rate of decrease itself decreases with pressure, as the rocks stiffen. The
16 pressure sensitivity of the bulk moduli of each of these rocks was also measured, and used to formulate a
17 description of the permeability decrease in terms of the progressive closure of narrow, crack-like pores with
18 increasing pressure. In the case of the shales in particular, only a very small proportion of the total porosity takes
19 part in the flow of gases, particularly along the bedding layering.

20 Key words: Permeability, shales, sandstone, bulk modulus, pressure sensitivity, gas porous flow

21 Supplementary data file: DF1.csv at <https://zenodo.org/record/5675601>

22 **1. Introduction**

23 Shales (laminated mudstones) are of particular importance because their fine grain size and tight pore structure
24 gives them a particularly low matrix permeability and hence makes them excellent cap rocks for the containment
25 of oil, water and gases. This includes their future use as a sealant for the storage containment of fuel gases
26 hydrogen and methane, compressed air storage and for the disposal deep underground of waste liquids and gases,
27 including waste carbon dioxide. Organic shales are source rocks for petroleum and become source, reservoir and
28 seal for unconventional natural gas (shale gas). The enormous economic importance of shales cannot be
29 overstated, and this demands an ever-increasing understanding of their petrophysical properties.

30 Compared to conventional reservoir rock materials (sandstones, limestones), shales are particularly difficult to
31 work with. Their commonly laminated nature makes them often highly fissile, with a tendency to split along the
32 layering. Thus coring and cutting operations for sample preparation are often difficult, and their physical
33 properties (elasticity, mechanical strength, permeability, elastic wave velocities) are generally anisotropic.
34 Determination of properties that involve working with elevated pore pressures become time-dependent, according
35 to the slow rates of fluid permeation through the microstructure in response to applied effective pressure changes,
36 and the rock itself may display time-dependent deformation (creep) under load. Mineralogically, shales can be
37 highly variable, particularly with respect to the relative proportions of the major mineral components: framework
38 silicates, clays and other phyllosilicates, and carbonates (Lazar et al. 2015; Diaz et al. 2013; Dowe and Taylor
39 2020), and this can be expected to be reflected in the spectrum of petrophysical properties of shales.



40 In contrast to shales, tight gas sandstones (e.g. Zee Ma et al., 2016)) may display similarly low permeabilities
41 and porosities, but lack extreme fissility and typically possess a matrix of coarser-grained framework silicate
42 minerals (quartz and feldspar) but with primary pore spaces filled with some detrital micas but also authigenic
43 growths of clay minerals and hydrated oxide phases. Thus their properties tend to form an upper (more permeable
44 and less anisotropic) bound to the range of properties displayed by shales. For this reason, we have included for
45 comparison in this study such a rock type. Here also we present a study of the matrix permeability of two, rather
46 different shales. Permeability and storativity were measured parallel to the layering under hydrostatic loading
47 conditions as a function of total confining pressure and pore pressure of argon gas, and normal to layering at one
48 pore pressure only. Results were fitted to a simple physical model. The spectrum of behaviours observed provides
49 insight into the physical controls on the matrix permeabilities of these rocks.

50 2. Sample materials and characterization

51 Two shale samples recovered from depth in boreholes were used. The samples are strikingly mineralogically
52 and microstructurally different. They were characterized mineralogically by quantified X-ray diffraction analysis,
53 which was also used to estimate grain density using published mineral densities. All samples were oven dried at
54 60 °C until constant weight (at least one week), and then maintained at that temperature until use. All experiments
55 were carried out in this oven dried state. Other than with the degree of water saturation in the as-supplied state, it
56 can be very difficult to test shales with varying degrees of controlled or with total water saturation. The sandstone
57 studied was from a surface exposure but was treated in the same way as the shales.

58 2.1 Pennant sandstone.

59 This is a hard, grey marine sandstone (Fig. 1a and b) of upper Carboniferous age (Kelling 2017), that outcrops
60 in south Wales, Great Britain. We have previously reported rock mechanics studies on this rock in Hackston and
61 Rutter (2016) and Rutter and Hackston (2017). All measurements reported were made normal to bedding. Bedding
62 planes are not apparent in hand specimen.

63 Modal proportions (vol% solids): Quartz + Feldspar 73.73 ; Phyllosilicates 9.81 ; (estimated uncertainties \pm
64 4% of cited percentages)

65 Grain density $2661 \pm 120 \text{ kg/m}^3$; Bulk density $2558 \pm 35 \text{ kg/m}^3$: Total porosity $3.89 \pm 0.04 \%$ from XRD,
66 $4.60\% \pm 0.01$ using a helium porosimeter.

67 **2.2 Bowland Shale.** This is a phyllosilicate-rich, carbonate-poor siliceous mudstone (Fig. 1c), very pyrite-rich,
68 (8.3 wt%), of Lower Carboniferous age. It was the target formation for exploitation of shale gas in Northern
69 England.

70 Depth 2060.55 m. Provider sample identifier IG 5-8W. Location: west Manchester, UK.

71 Modal proportions (vol% solids): Quartz + Pyrite 38.4 ; Phyllosilicates 61.6 ; Carbonates 0 (estimated
72 uncertainties $\pm 4\%$ of cited percentages)

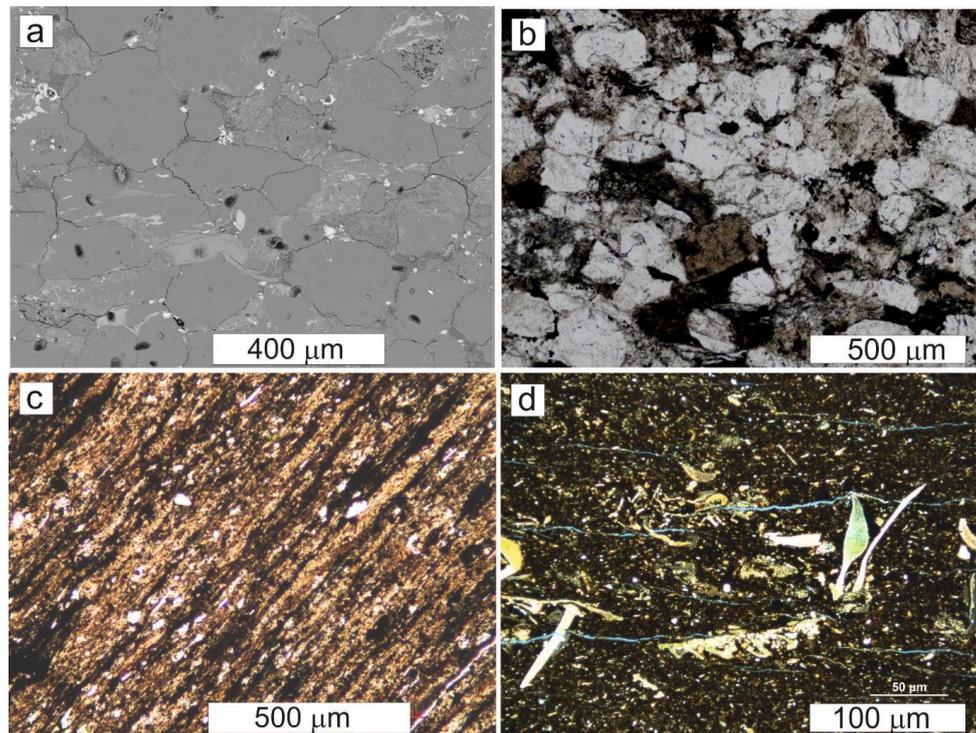
73 Grain density $2842 \pm 120 \text{ kg/m}^3$: Bulk density $2714 \pm 38 \text{ kg/m}^3$: Total porosity $4.50 \pm 0.02\%$ from XRD; $4.6\% \pm 0.1$
74 using a helium porosimeter.

75 Total organic carbon $1.14 \pm 0.2 \text{ wt}\%$; Water loss from drying $0.74 \pm 0.15 \text{ vol}\%$, hence initial water saturation =
76 13%.

77 2.3 Haynesville Shale



78 This is a phyllosilicate-poor, carbonate-rich siliceous mudstone (Fig. 1d). Pyrite-poor (0.7 wt%), of Upper
79 Jurassic age (Hammes et al. 2011), successfully exploited for shale gas in the southern United States.
80 Depth 3730.6 m. (Sample identifier). Location: Hewitt Land LLC well, Caspian Field, de Soto parish, Louisiana,
81 USA.
82 Modal proportions (vol% solids): Quartz + Feldspar + Pyrite 34.5; Phyllosilicates 13.4; Carbonates 52.1;
83 (estimated uncertainties $\pm 4\%$ of cited percentages)
84 Grain density $2703 \pm 120 \text{ kg/m}^3$; Bulk density $2453 \pm 35 \text{ kg/m}^3$; Total porosity $9.26 \pm 0.04 \%$ from XRD, $7.6\% \pm 0.1$
85 using a helium porosimeter. Total organic carbon $1.3 \pm 0.2 \text{ wt}\%$.



86

87 **Figure 1. Microstructures of the rocks tested.**

88 (a) Back-scattered electron (BSE) image and (b) optical image (PPL) of Pennant sandstone, bedding
89 horizontal, showing large quartz grains (mid-grey in (a)) with sutured contacts caused by pressure solution
90 and remaining pore spaces largely filled by iron hydroxide (white in (a)) and authigenic clay minerals (light
91 grey in (a)), reducing the overall porosity to 4.6%

92 (c) Microstructure (Plane-polarized light (PPL) image of polished thin section) of Bowland shale, finely and
93 homogeneously banded with elongate clusters of organic material and pyrite (black) and silt-sized grains of
94 quartz in a matrix of elongate clusters of phyllosilicate (clay + detrital micas) grains.

95 (d) Microstructure of Haynesville shale. (PPL image of polished thin section, horizontal dimension is
96 parallel to layering). Bioturbation destroys continuity of layering. Rock is only weakly banded but
97 nevertheless fissile; bedding-parallel cracks can be seen, opened during thin-section preparation.
98 Calcareous fossil fragments and authigenic calcite-filled voids, in matrix of finer grained phyllosilicate
99 (clays + detrital micas) and fine silt-sized framework silicates.



100 Defining velocity anisotropy as $2(V_{max} - V_{min}) / (V_{max} + V_{min})$, the anisotropies of Bowland and Haynesville shales
 101 are respectively 30.7% and 32.2% at 100 MPa total confining pressure. The velocity anisotropy of Pennant
 102 sandstone at elevated pressure was not determined. It is 15.5% axial and 3.1% radial at room pressure, but will be
 103 less at elevated pressure.

104 The wt% values for the mineralogical composition of all rock types were converted to vol% using tabulated
 105 densities from the literature (Mavko et al., 2009; Mondol et al., 2008), and together with averaged mineral elastic
 106 properties the bulk elastic properties of the rocks estimated were as Voigt-Reuss-Hill (VRH) averages assuming
 107 zero porosity. These are listed in Table 1.

108 Some comparisons of behaviour are made with previously published (Mckernan et al., 2017) data on Whitby
 109 shale. This is a well-foliated, silt-bearing, clay-rich, carbonate-poor mudstone of Liassic age, with 8.1% total
 110 porosity and 1.5% volume amorphous organic matter.

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113 **Table 1. Phase fractions, mineral densities and Voigt-averaged bulk and shear moduli K_v and G_v (from**
 114 **literature) and calculated zero porosity elastic moduli as Voigt-Reuss-Hill (VRH) averages (GPa) for**
 115 **Bowland and Haynesville shales and for Pennant sandstone. Organic fraction not included. Mineral phase**
 116 **Reuss-average elastic moduli can be calculated from the other values supplied. K_0 = bulk modulus, G_0 =**
 117 **shear modulus, E_0 = Young's modulus (VRH-averaged whole-rock values assuming isotropy). Modal**
 118 **volume percent is % of the solids.**

119

Bowland Shale IG5-8WC						
Phase	Wt%	±Error%	Density kg m ⁻³	Vol%	Kv GPa	Gv GPa
Quartz	30.98	1.42	2648	33.64	12.73	14.90
Pyrite	8.32	0.44	5020	4.77	6.63	5.36
Muscovite 2M	60.44	2.04	2844	61.11	35.55	21.61
Kaolinite	0.26	2.60	1580	0.48	.0072	.0067
Total	100.0			100.0		
Zero porosity moduli (GPa):			VRH(K_0)	VRH(G_0)	VRH(E_0)	
			52.79	40.69	97.13	

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Haynesville Shale YB03						
Phase	Wt%	±Error%	Density kg m ⁻³	Vol%	Kv GPa	Gv GPa
Albite	10.49	0.505	2610	11.01	5.59	3.22
Ankerite Fe0.55	4.65	0.36	3050	4.17	4.80	2.46
Calcite	47.22	1.25	2712	47.69	32.94	15.24
Clinocllore 1lb-24.11	0.41	2.90	3880	2.26	1.37	
Muscovite 1M	9.97	1.46	2844	9.50	5.27	3.36
Pyrite	1.27	0.10	5020	0.69	.958	.775
Quartz	18.71	0.74	2648	19.35	7.32	8.57
Siderite	0.47	0.07	3960	0.33	.408	.168
Orthoclase	3.20	0.46	2540	3.45	1.61	.815
Total	100			100.1		
Zero porosity moduli (GPa)				VRH(G_0)	VRH(E_0)	VRH(K_0)
				60.57	34.91	87.86

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Pennant Sandstone Pe2						
Phase	Wt%	±Error%	Density kg m ⁻³	Vol%	Kv GPa	Gv GPa
Albite	16.14	0.70	2610	16.46	8.20	4.72
Phyllosilicates	10.48	1.5	2840	9.81	6.10	3.71
Quartz	73.37	2.8	26480	73.73	27.77	32.50
Total	99.99			100.0		
Zero porosity moduli (GPa):				VRH(K ₀)	VRH(G ₀)	VRH(E ₀)
				41.55	40.42	91.57

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124 **3. Experimental Methods**

125 **3.1 Permeability measurements**

126 Permeability measurements were made on cylindrical samples of either 25.4 or 20 mm nominal diameter, cut to
 127 lengths of the same order or shorter. The latter is generally necessary for very low permeability rocks, but quite
 128 apart from this it was not possible to obtain long cores from slabbed drill cores of the shales. Problems were also
 129 encountered during shale specimen preparation owing to the friable nature of these materials. Porous sintered
 130 stainless steel (316L) filter plates (17% porosity) were placed at either end of the sample to spread the pore fluid
 131 uniformly over the ends of the rock samples. The assembly was jacketed in a heat-shrinkable polymer jacket, so
 132 that pore fluid pressures less than the confining pressure could be applied. Confining pressures (hydraulic oil, a
 133 synthetic ester, di-octyl sebacate, trade name Reolube DOS®) ranging up to a little over 100 MPa were used. This
 134 fluid has the advantage of a relatively small rate of change of viscosity with pressure (see Rutter and
 135 Mecklenburgh 2017 and 2018 for further details). In all experiments argon gas was used as the pore fluid, at
 136 pressures ranging up to 80 MPa. The higher viscosity of a liquid pore fluid would have led to very long
 137 experimental durations. The confining and pore pressures ranges cover the full extent of likely pressures to be
 138 encountered in engineering operations to depths of *ca* 4 km.

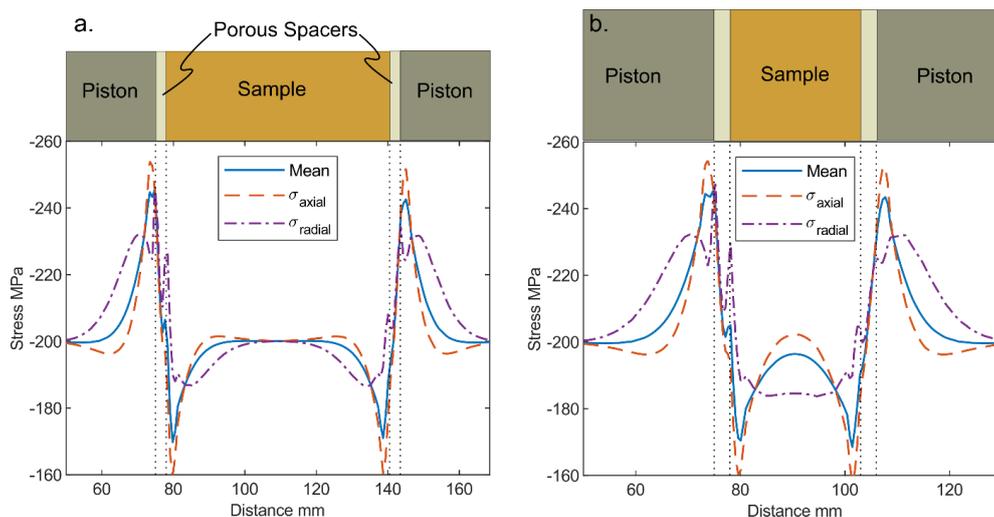
139 Although it was intended that experiments would be carried out under hydrostatic confinement conditions, the
 140 presence of a contrast in elastic properties of the specimen against the porous end plates and the steel loading
 141 pistons induces a shear stress along these interfaces. This in turn causes the stress state in the specimen to deviate
 142 from hydrostatic and to reduce the average mean stress. Deviations from hydrostatic loading are most severe when
 143 the length of the specimen becomes less than twice the diameter. For this reason, mechanical testing of rocks is
 144 usually carried out on specimens with a length:diameter ratio of 2.5:1 or more. Finite element analysis (FEA) of
 145 the stress state in rocks confined between steel end plates were carried out to assess the expected departures from
 146 hydrostatic loading, and the effects predicted must be borne in mind when interpreting the permeability data.

147 **Table 2. Elastic constants of the components in the finite element models.**

	Young's Modulus <i>E</i> GPa	Poisson's Ratio
Sample	60	0.250
Piston	190	0.265
Spacer (17% Porosity)	108.6	0.260

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151 **Figure 2. Results of finite element analyses showing stress profiles of mean stress, axial normal stress and**
152 **radial normal stress along the axes of samples respectively of length:diameter ratios (a) 2.5:1 and (b) 1:1,**
153 **each with a diameter of 25.4 mm. At the top of each figure is a scaled schematic of the assembly; notice the**
154 **aspect ratio of the sample in each case. Externally applied hydrostatic stress was 200 MPa. For the longer**
155 **sample the stress state in the greater part of the sample was near homogeneous, but for the shorter one a**
156 **differential stress on the order of 7% of the applied hydrostatic stress was induced.**

157 Figure 2 presents the results of finite element analyses showing stress profiles along the axes of samples
158 respectively of length:diameter ratios (a) 2.5:1 and (b) 1:1, with a hydrostatic pressure of 200 MPa applied to the
159 outer cylindrical surfaces. At each end of the sample a 3 mm thick, porous sintered steel disk was placed.
160 Positions of boundaries between the solid steel pistons, the porous disks and the sample material are indicated. In
161 both cases the sample diameter was 25.0 mm. Along-axis stress component variations were more varied than
162 across the radius. Most of the stress heterogeneity (departure from the applied 200 MPa hydrostatic pressure)
163 resides in and immediately adjacent to these disks, and for each stress component is of similar magnitude for both
164 specimen lengths. Within the greater part of the sample volume in each case the axial normal stress is higher than
165 the radial normal stress, and these components are similar to the principal stress values. For the longer sample, the
166 stress state is near hydrostatic over 0.8 of the specimen length, but in the case of the shorter sample the stress
167 components are notably non-hydrostatic over most of the specimen length, with maximum differential stress
168 reaching 15 MPa (7% of the applied hydrostatic stress) in the central part of the sample.

169 A small number of permeability measurements were made using the pulse-transient-decay method of Brace et al.
170 (1968), as modified by Cui et al. (2009). However, most experimental results were obtained using the oscillating
171 pore pressure method (Kranz et al., 1990; Fischer and Paterson, 1992; Faulkner and Rutter, 2000; Bernabé et al.,
172 2006; Mckernan et al., 2017). Whilst keeping the confining pressure constant and after establishing a constant
173 pore pressure in the sample, a sinusoidal oscillation of pore pressure, of known period and of amplitude about 1
174 MPa, is applied at one end of the sample (upstream). As the pressure wave propagates through the sample it is
175 phase-shifted and loses amplitude. The amplitude ratio (gain) and phase shift angle are measured. The solution to



176 the transport equation for these measured parameters is given by Bernabé et al., (2006) in terms of two
177 dimensionless numbers, η and ξ , from which permeability and sample storativity can be calculated using

187
$$\xi = \frac{SL\beta}{\beta_D}, \quad \eta = \frac{STk}{\pi L\mu\beta_D} \quad (1)$$

178 Here, S is cross sectional area of the sample (normal to flow path), L is specimen length, β_D is downstream
179 volume storativity and β is specimen storativity, T is the period of the pore pressure oscillation, k is specimen
180 permeability, and μ is viscosity of the pore fluid. Argon gas viscosity as a function of pressure data was reported
181 by Michels et al., (1954). Storativity is the product of the volume of the void space concerned, with the pore fluid
182 (isothermal) gas compressibility. Argon compressibility is non-linear over the pore pressure range used (Gosman
183 et al., 1969) and substantially non-ideal above about 20 MPa. $\xi \approx \phi V_s/V_d$ where ϕ is specimen effective porosity,
184 V_s is total specimen volume and V_d is downstream reservoir volume. It cannot be assumed that effective
185 (conductive) porosity estimated from permeability measurements will necessarily be equal to total porosity
186 measured independently.

188 The apparatus used was the same as used for experiments reported by Rutter and Mecklenburgh (2017; 2018).
189 Pressure transducers with a resolution of 0.02 MPa were used for pore pressure measurements, and confining
190 pressure was measured to an accuracy better than 0.3 MPa. The minimum pore pressure used was 10.0 MPa. This
191 is sufficiently high to avoid exsorption of gas from mineral surfaces and to avoid slip flow of gas through pore
192 spaces (Knudsen/Klinkenberg effect, Mckernan et al. 2017). It was determined that the experimental assembly
193 shows no detectable gas flow when a rock sample is replaced by an impermeable steel plug.

194 3.2 Error, uncertainty and reproducibility

195 Accuracy of reported permeability depends on uncertainties of the parameters in Eq. (1). η and ξ can be
196 measured to within about 2% of the true value, and S , TL and μ to within 1%. The least certainly known
197 parameter is the downstream volume, which is determined as the difference between the total volume of the pore
198 pressure pipework measured with and without the downstream pipework connected, each measured by the pore
199 pressure change produced by a known volumometer piston displacement. The downstream reservoir volume V_d
200 was measured to be $445 \pm 30 \text{ mm}^3$, including the volume of the downstream porous steel filter. These
201 uncertainties translate to an accuracy of \log_{10} permeability of ± 0.1 log units. This is small, given that permeability
202 varies with pressure by 1 to 3 orders of magnitude.

203 The largest apparent uncertainties in reported permeability data arise from hysteretic changes in the behaviour of
204 the rock itself as effective pressure is cycled and will be discussed when the data are presented.

205 3.3 Bulk modulus measurements

206 Bulk modulus measurements as a function of confining and pore pressures were made as far as possible on
207 physically the same samples that were used for the permeability measurements, to avoid any influence of
208 mineralogical or microstructural differences. Measurements were made over a range of total confining pressures
209 up to 200 MPa, after the permeability measurements were made, with constant pore pressures of argon gas,
210 typically at nominally 10, 35, 67 and 100 MPa. The method involved measuring volume of pore fluid (argon gas)
211 progressively expelled as the total confining pressure was increased at constant pore pressure. This measures the



212 compressibility of the pore spaces. P-wave acoustic velocity measurements were made at the same time, although
213 these data are not reported here.

214 Unlike for permeability measurements, porous steel plates were not used at the ends of the specimens for pore
215 fluid displacement measurements. For the relatively porous and permeable Haynesville shale and Pennant
216 sandstone, a short hole, normally 15 mm long and 1.5 mm diameter, was drilled into the end of the specimen
217 facing the pore pressure inlet pipe, to facilitate flow of gas into and out of the specimen. This was thought to be
218 unlikely to be adequate for the lower porosity and permeability Bowland shale, therefore samples were cut in half
219 parallel to the long axis so that a 2mm thick, porous steel plate could be inserted, to facilitate gas flow over a wide
220 surface area of the rock, yet without affecting the P-wave velocity along the length of the specimen.

221 When considering the results, the procedure for pressure application is of importance. For the tests with pore
222 pressure, the application of a confining pressure slightly greater than the eventual pore pressure was made,
223 followed by application of the pore pressure. Then the total confining pressure was increased stepwise away from
224 the constant pore pressure. Thus tests at high pore pressure have been exposed to much higher effective pressures
225 before application of pore pressure, than when the test pore pressure is to be low.

226 When pore pressure was made non-zero, constant pore pressure was maintained using a servo-controlled pore
227 volumometer. Each applied increment of the confining pressure caused a small elastic contraction of the pore
228 volume that attempts to raise the pore pressure. The servo-controller backs off the moveable piston in the pore
229 volumometer in order to keep the pore pressure constant. The distance swept by the volumometer piston at
230 constant pore pressure allows the volume of gas expelled to be measured to a resolution of 0.4 mm³. In this way
231 the history of *pore volume* change at constant pore pressure during progressive loading by the confining pressure
232 can be determined. The compressibility of the pore space C_{pc} is given by the fractional change in pore volume V_p
233 in response to a change in confining pressure P_c at constant pore pressure P_p (Zimmerman, 1991), and is the
234 reciprocal of the dry pore space bulk modulus K_ϕ :

$$238 \quad C_{pc} = \frac{1}{K_\phi} = \frac{1}{V_p} \left(\frac{\partial V_p}{\partial P_c} \right)_{P_p} \quad (2)$$

235 Note $V_p = \phi V_b$, where V_b is the total sample volume. K_{dry} is the bulk modulus of the porous aggregate. Its
236 reciprocal, compressibility C_{bc} , the bulk volume change in response to a change in confining pressure at constant
237 pore pressure, is defined by

$$239 \quad C_{bc} = \frac{1}{K_{dry}} = \frac{1}{V_b} \left(\frac{\partial V_b}{\partial P_c} \right)_{P_p} \quad (3)$$

240 where V_b is the bulk volume, including the pore space. The zero-porosity bulk modulus of the constituent mineral
241 aggregate is defined as K_o (Table 1), then the dry bulk modulus K_{dry} ($= K_{bc}$) is given (Mavko et al., 2009) by

$$242 \quad \frac{1}{K_{dry}} = \frac{1}{K_o} + \frac{\phi}{K_\phi} \quad (4)$$

243 Decrease in permeability with increasing Terzaghi effective pressure ($P_c - P_p$) (Terzaghi, 1923) is primarily due to
244 the pressure dependence of K_{dry} , leading to progressive closure of pore space. Thus the independent determination of
245 K_{dry} from pore volumetry measurements provides a basis for the interpretation of the pressure sensitivity of
246 permeability.



247 Note that we have no means of measuring directly the influence of pore pressure change on bulk deformation of
248 the sample, characterized by the compressibility C_{bp} , or

$$249 \quad C_{bp} = \frac{1}{K_{bp}} = \frac{1}{V_b} \left(\frac{\partial V_b}{\partial P_p} \right)_{P_c} \quad (5)$$

250 This would require strain gauges or equivalent to be mounted on the outer surface of the rock sample (e.g.
251 Hasanov et al., 2019, 2020). However, it can be obtained from

$$252 \quad \frac{1}{K_{bp}} = \frac{1}{K_{bc}} - \frac{1}{K_o} \quad (6)$$

(Mavko et al., 2009).

254 Biot and Willis (1957), Skempton (1960) and Nur and Byerlee (1971) obtained a theoretical expression for the
255 effective pressure coefficient (Biot coefficient) m for elastic *deformations* (including deformations of pore spaces)
256 of a mechanically linear, homogeneous and isotropic rock, so that effective pressure $P_{eff} = (P_c - mP_p)$, and

$$257 \quad m = 1 - \frac{K_{dry}}{K_o} \quad (7)$$

258 Note that this effective pressure coefficient is not necessarily the same as that describing empirically the influence
259 of pore pressure on permeability, nor on elastic wave velocities nor the failure characteristics of rocks (whether
260 frictional sliding or intact rock failure).

261 m is also given by

$$262 \quad m = \frac{K_o}{K_{bp} + K_o} = \frac{K_o - K_{bc}}{K_o} = 1 - \frac{K_\phi}{K_\phi + K_o \phi} \quad (8)$$

263 Sample storativity is related to these stiffness parameters by

$$264 \quad \beta = \frac{1}{K_{bp}} + \phi \left(\frac{1}{K_f} - \frac{1}{K_o} \right) \quad (9)$$

265 where K_f is pore fluid bulk modulus (Hasanov et al., 2019).

266 In all calculations we assume K_o is negligibly sensitive to effective pressure, compared to porous rock stiffnesses
267 such as K_{dry} , following data for K_o for minerals such as quartz via ultrasonic measurements (e.g. Calderón et al.,
268 2007, who give $K_o = 37.5(\text{GPa}) + 4.7*P(\text{GPa})$).

269 4. Experimental results

270 A full tabulation of experimental results is given in the supplementary data file DF1.

271 4.1 Permeability results

272 Experimental conditions and results are presented graphically in Figs. 3 through 8. The first pressure cycle
273 applied to most rocks results in higher permeabilities and a relatively rapid rate of decrease of permeability with
274 pressure, as inelastic cracks become progressively and permanently closed. Subsequent pressure cycles up to the
275 maximum pressure previously attained are more nearly elastic and reproducible, although there can be a small
276 tendency to reduce permeability slightly with subsequent pressure cycles. The first stage in a suite of permeability
277 measurements covering a wide range of confining and pore pressures therefore must be to take the sample to the



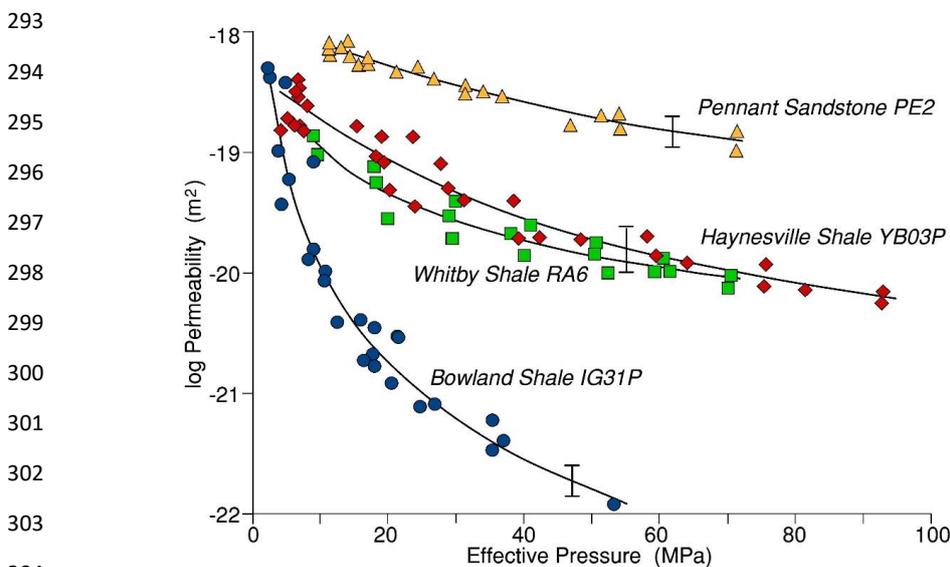
278 maximum effective pressure to which it is to be exposed, to ensure closure of these inelastic cracks and pores up
279 to that pressure.

280 **4.1.1 Form of data and reproducibility**

281 In the regime of elastic behaviour permeability (as $\log k$) is not usually linear neither on a k vs P_c plot nor even
282 on a $\log k$ vs P_c plot but is concave upwards (Fig. 3). The decrease of permeability with effective pressure is due
283 to elastic closure of conductive cracks and pores, and this is expected to become more difficult as the porous
284 material stiffens at higher pressure. Thus although it is common, and useful for the purpose of modelling reservoir
285 behaviour (e.g. Kwon et al., 2001; Bustin et al., 2008; Cui et al., 2009; Heller et al., 2014; Mckernan et al., 2017)
286 to describe quantitatively the relationship between $\log_{10} k$ and P_c by making a least-squares linear fit to the data, a
287 better description would take into account the curvature.

288 In order to estimate the reproducibility of the permeability data, a determination of the standard error was made
289 about a polynomial fit to the 10 MPa pore pressure data (after the first pressure cycle) for each rock type. For
290 Bowland shale it is $\pm 0.22 \log_{10} k$ units, for Haynesville shale it is $\pm 0.19 \log_{10} k$ units and for Pennant sandstone it is
291 $\pm 0.10 \log_{10} k$ units.

292 **4.1.2 Influence of confining (P_c) and pore pressures (P_p) on permeability**



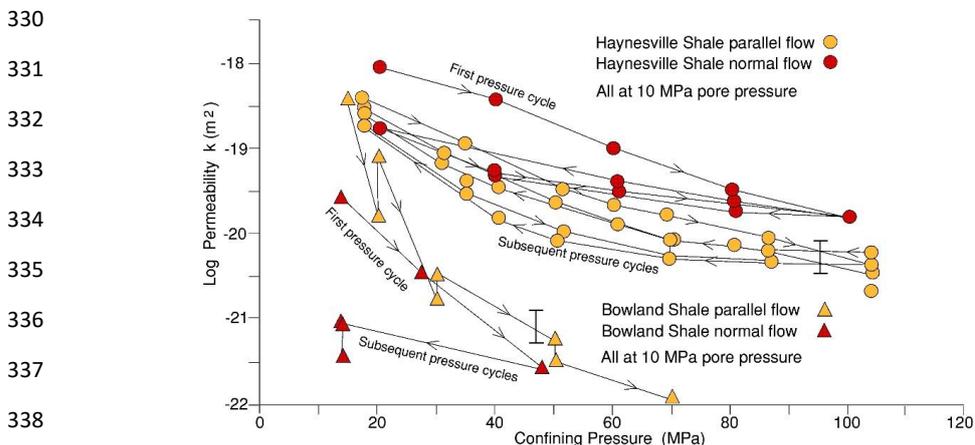
305 **Figure 3. Matrix permeability of Pennant sandstone for flow normal to bedding, and for Bowland and**
306 **Haynesville shales for flow parallel to layering, as a function of Terzaghi effective pressure ($P_c - nP_p$) over a**
307 **wide range of pore pressures of argon gas. Data of Mckernan et al. (2017) for Whitby shale sample RA6 at**
308 **a constant argon gas pore pressure of 25 MPa are also shown for comparison. In each case data from the**
309 **first pressure cycle up to the maximum effective pressure attained has been excluded. All rocks show**
310 **permeability decreasing more slowly with effective pressure at higher effective pressures. Error bars are**
311 **shown as estimated for the 10 MPa pore pressure data.**

312 Figure 3 shows the influence of Terzaghi effective pressure on matrix permeability over a range of pore
313 pressures, for Haynesville and Bowland shales for flow parallel to layering and for Pennant sandstone normal to
314 bedding after the first pressure cycle. They are expressed as $\log_{10} k$ versus effective pressure ($P_c - nP_p$), where

315 the pore pressure parameter $n = 0.86$ for Pennant sandstone and is 0.99 for Haynesville shale. For the Bowland
316 shale the data showed that permeability varied over almost four orders of magnitude, much greater than for the
317 other two rock types, and as a result it was evident that parameter n tended to increase with the value of Terzaghi
318 effective pressure, varying from unity at low pressures to 1.6 at high effective pressures. The least squares best-
319 fit curve to each of these data sets is shown in Fig. 3. For all three rocks the form of the behaviour is similar, each
320 showing a decreasing slope at higher effective pressures, as would be expected from pressure-induced
321 constriction of pore spaces. The Pennant sandstone showed the least sensitivity to effective pressure variations,
322 whilst the Bowland shale displays a far greater sensitivity to effective pressure. The Haynesville shale takes an
323 intermediate position that is closely comparable to the data for Whitby shale (sample RA6 taken from the data
324 reported by Mckernan et al. 2017 for pressure cycles 2, 3, 4 and 5).

325 Whilst these rocks display relatively small differences in permeability at low effective pressures, increase in
326 pressure results in markedly divergent trends, resulting in large differences in permeability developing over the
327 range of effective pressures expected to encountered under reservoir conditions. This observation emphasises the
328 importance of understanding the pressure sensitivity of shales that are to be exploited for engineering purposes.

329 4.1.3 Influence of flow direction at constant pore pressure.



339 **Figure 4. Comparison of data at 10 MPa pore pressure for flow parallel and normal to layering in the two**
340 **shales. Parallel flow data are shown without the first pressure cycle, during which some pores become**
341 **permanently closed. Normal-to-layering flow data are shown including the first pressure cycle. For**
342 **Bowland shale, flow normal to layering is slower, but for Haynesville shale there is little effect, except that**
343 **pressure sensitivity is less for flow normal to layering.**

344 Flow normal to layering in shales is often much slower than flow parallel to layering, but not always. Layer-
345 normal flow was therefore measured for these rocks using shorter samples than for flow along the layering, and
346 only at 10 MPa argon pore pressure (Fig. 4). However, for Haynesville shale the direction of flow makes little
347 difference, except that pressure sensitivity is reduced for layer-normal flow, as would be expected if flow parallel
348 to the layering is dominated by low aspect ratio, crack-like pores that are relatively compressible. The different
349 pressure sensitivities of permeability mean that (after the first pressure cycle) flow along the layering becomes
350 faster at low effective pressures, but slower at higher effective pressures. Bowland shale shows a small reduction
351 in permeability for flow normal to layering relative to parallel to layering (post the first pressure cycle), and there
352 is also some indication of a reduced pressure sensitivity, although the dataset is small.

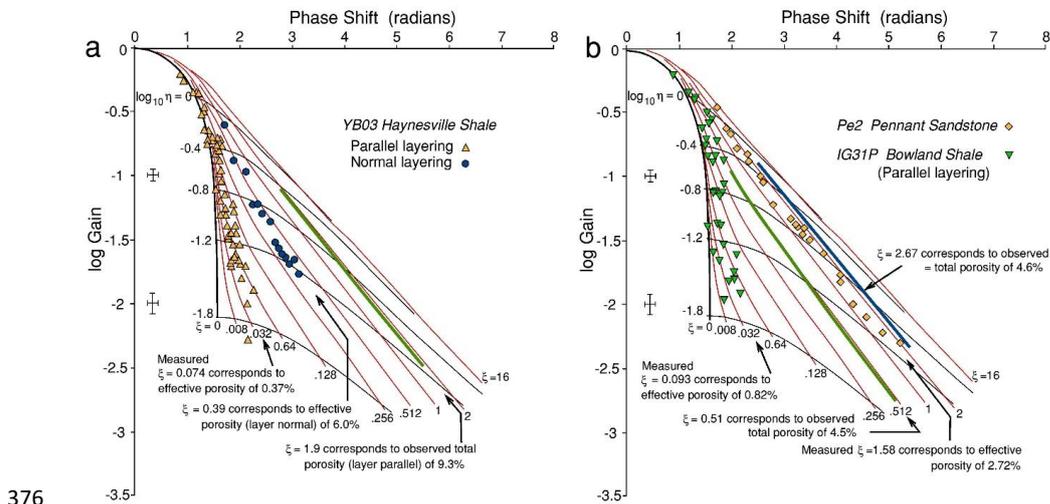
353 **4.1.4 Storativity of the rocks**

354 Oscillating pore pressure permeametry yields a dimensionless permeability parameter η and a dimensionless
 355 storativity parameter ξ (Eq. (1)), which is the ratio of the accessible pore volume in the rock to the downstream
 356 reservoir volume. A plot of experimentally measured log gain vs signal phase shift angle lies along a line of constant ξ
 357 if the sample storativity is constant (Fig. 5). Thus the effective (conductive) porosity of the sample during the course
 358 of the experiment can be calculated. The conductive porosity of many rocks is smaller than the total porosity.

359 The total porosity also corresponds to a particular value of ξ . If all of the porosity were to be involved in the flow,
 360 these ξ values will be equal. Note that a value of $\xi = 1$ corresponds to the downstream volume of the apparatus being
 361 equal to the pore volume of the rock sample. A storativity can also be calculated from data from elastic pore
 362 compressibility measurements. Hasanov et al. (2019) calculated storativity in these two ways.

363 Figure 5a shows log gain vs phase angle data for Haynesville shale for flows both parallel and normal to layering.
 364 Figure 5b shows corresponding data for Bowland shale and Pennant sandstone, but insufficient data was obtained for
 365 Bowland shale normal to layering, given its much lower permeability. For flow along the layering, both of the shale
 366 types show $\xi < 0.1$, corresponding to the conductive porosity being much smaller ($< 1\%$) than the total porosity of the
 367 rocks (respectively 4.5% and 9.3%). Thus whilst the bulk of the pore space can contribute to gas storage, only a very
 368 small fraction of well-connected porosity contributes to gas flow along the layering in the shales.

369 The log gain vs phase angle data was non-linear least-squares fitted to obtain an average value for ξ for each rock
 370 type, subject to the constraint that ξ is constant. For Haynesville shale for flow across the layering ξ lies along the
 371 trend $\xi = 0.39$, corresponding to a conductive porosity of $\sim 6.0\%$. Thus flow across the layering ‘sees’ more of the
 372 total porosity than flow along the layering, though still substantially less than the amount of total porosity. Whitby
 373 shale (Mckernan et al., 2017) displays the same effect. In marked contrast, for the Pennant sandstone $\xi = 2.72$. This
 374 is close to the value of $\xi = 2.67$ corresponding to the total porosity (4.6%) of the rock, implying a high degree of
 375 connectivity between the pore spaces in Pennant sandstone.



378 **Figure 5. Log gain vs phase angle data from oscillating pore pressure measurements on :**

379 (a) Haynesville shale. $\xi = 1.9$ would correspond to total porosity 9.3% for flow in the sample parallel to
380 layering if all porosity participates in the flow. Observed $\xi = 0.39$ normal to layering is much greater
381 than parallel to layering $\xi = 0.074$, but both are substantially less than that corresponding to total
382 porosity. Flow parallel to layering only ‘sees’ or ‘uses’ about 4% of the total pore space, and normal to
383 layering about 42% of the total pore space.

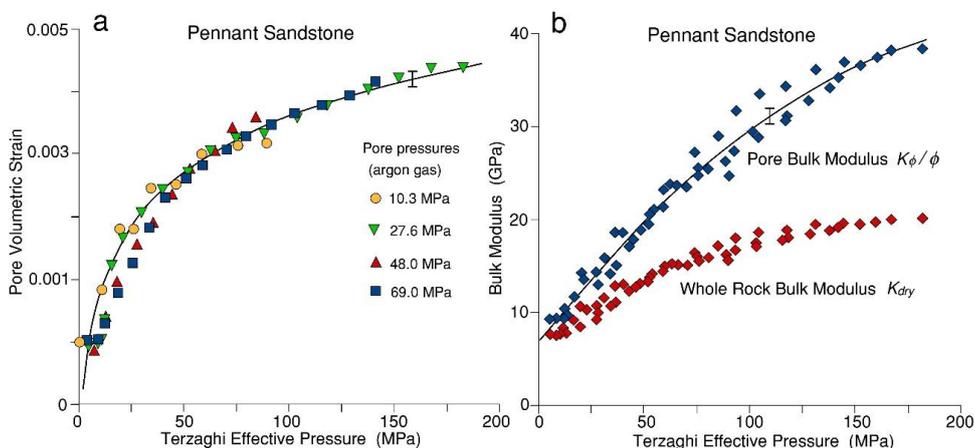
384 (b) Bowland shale. $\xi = 0.51$ would correspond to total porosity 9.3% for flow in the sample parallel to
385 layering if all porosity participates in the flow. Observed $\xi = 0.093$ for flow parallel to the layering
386 corresponds to a conductive porosity (0.82%) much less than total porosity. In contrast, data for
387 Pennant sandstone show observed $\xi = 1.58$ to be closer to that $\xi = 2.67$ which corresponds to the
388 total porosity of the rock.

389 4.1.5 Bulk moduli of compressibility for Pennant sandstone

390 Bulk modulus of porosity K_ϕ (defined in Eq. (2)) and its effective pressure sensitivity can be measured from the
391 volume of argon expelled from the rock during increments of confining pressure at constant pore pressure, and
392 K_{dry} can be calculated using Eq. (4) (Fig. 6a). K_o is the mineral bulk compressibility estimated as the VRH
393 average at zero porosity (given for these rocks in Table 1).

394 K_ϕ/ϕ is the value of the pore bulk modulus referred to the total volume of the rock, rather than to the pore space
395 volume. K_ϕ/ϕ and K_{dry} versus Terzaghi effective confining pressure are shown in Fig. 6 for Pennant Sandstone.
396 K_{dry} is asymptotic to K_o (41.5 GPa) at high pressure.

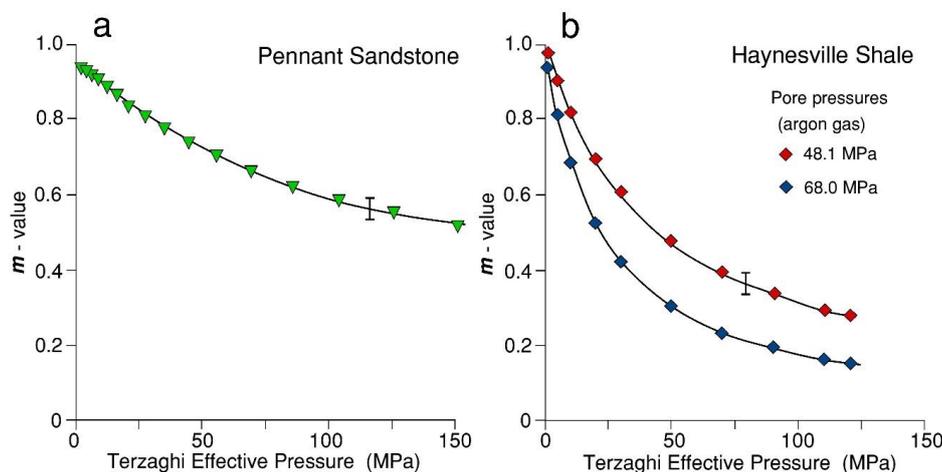
397 The pore pressure coefficient m , describing the effects of pore pressures on elastic distortions of a porous rock,
398 and defined in $P_{eff} = P_c(1 - m P_p)$ is given in terms of the bulk moduli K_{dry} and K_o in Eq. (7). In Fig. 7 the resultant
399 m versus effective pressure curves are shown for both Pennant sandstone and Haynesville shale. Bulk moduli are
400 isotropic properties with values unaffected even when the aggregate displays preferred orientation (shape and
401 crystallographic) of constituent grains (Andrews, 1978; Mendelson, 1981).



402

403 **Figure 6.**

- 404 (a) Volumetric strain (with respect to whole sample volume) for Pennant sandstone at four different
405 constant gas pore pressures. There is no significant effect of magnitude of pore pressure. About
406 20% of the total pore volume is elastically reduced over a range of 200 MPa effective pressure.
407 (b) Pore bulk modulus K_{ϕ}/ϕ from gas expulsion data in (a) for Pennant sandstone, and whole rock
408 bulk modulus calculated from K_{ϕ}/ϕ and K_o (41.5 GPa). Pore spaces become rapidly less compliant
409 as effective pressure increases.



410

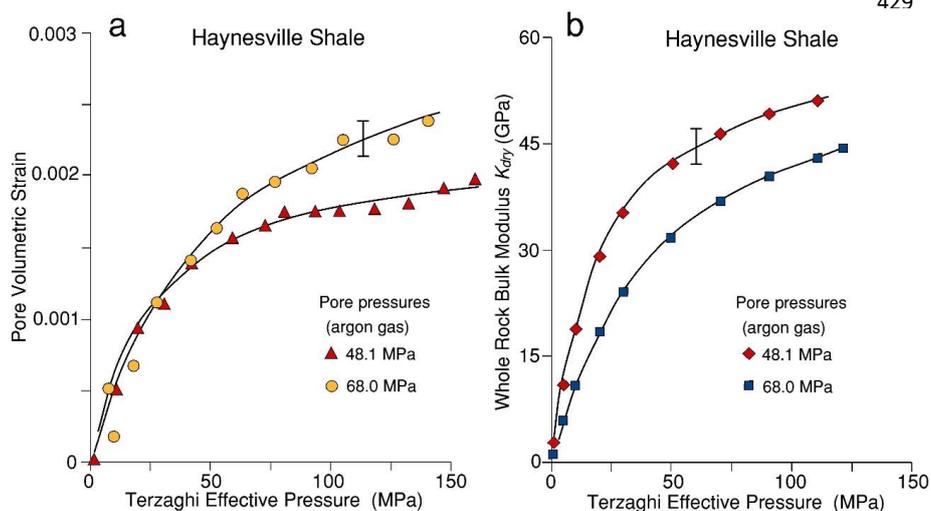
411 **Figure 7.**

412 *m* from bulk modulus data and Eq. (7) for (a) Pennant sandstone and (b) Haynesville shale. The
413 decrease of *m* with P_{eff} arises from the stiffening of the pore spaces with effective pressure, and the
414 effect is greater for the shale than for the sandstone.

415 At low pressure K_{dry} is much less than K_o , hence *m* approaches 1. As K_{dry} increases with pressure it approaches
416 K_o , hence *m* decreases with pressure, and will eventually reach zero when all pore space has collapsed. Any small
417 increase of K_o with pressure has been ignored (e.g. Calderón et al., 2007). The variation of *m* with pressure forms
418 the basis for describing the decrease in permeability observed as effective pressure increases.

419 4.1.6 Bulk moduli of compressibility for Haynesville shale

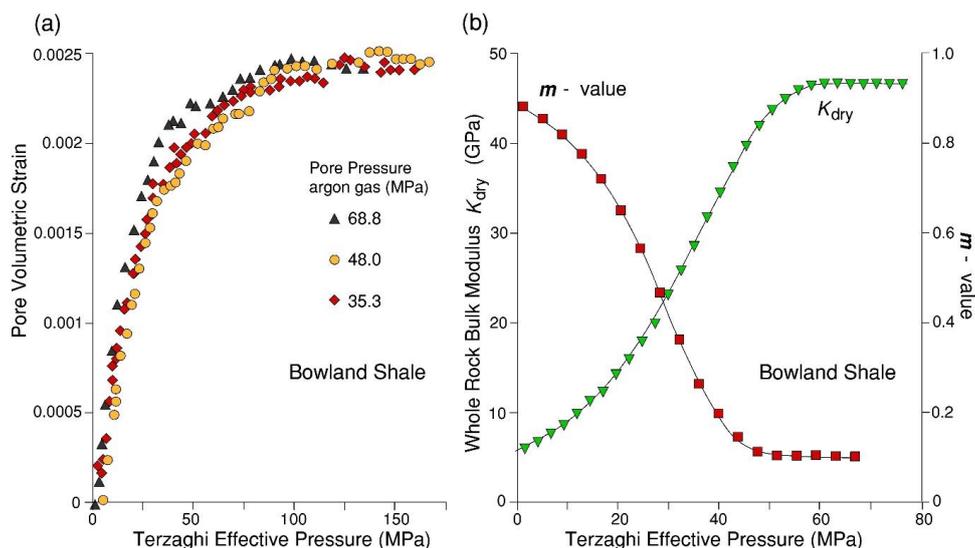
420 Pore volumetry by the expelled gas volume method during progressive increase in confining pressure was
421 carried out on the two shale samples used (Fig 8). The resolution of the pore volume change data is poor because
422 the specimen size was rather small (1.9 cm long). The rapid increase in slope translates to a rapid rise of calculated
423 K_{dry} compared to Pennant sandstone, until it is a substantial fraction of K_o (61 GPa). However, the total amount of
424 gas expelled corresponds to a closure of about 2% of the initial porosity (0.15% of the whole sample volume).
425 Figure 7b shows pore pressure coefficient *m* calculated from the pore volumetry. *m* decreases rapidly because
426 the K_{dry} value rises rapidly to become a substantial fraction of K_o . It is not clear why the measurements at two
427 different pore pressures are so different, but it is thought to be attributable to different degrees of gas trapping in
428 poorly connected pore spaces.



440 **Figure 8.**

- 441 (a) Pore volumetric strain (as fraction of total specimen volume) vs Terzaghi effective pressure for
 442 Haynesville shale at the pore pressures indicated. Pore volume loss is approx. only 2% of the
 443 initial pore volume of the rock. Logarithmic fits to two of the data sets are shown.
 444 (b) The gradients of the fitted lines in (a) correspond to the pore compressibility, and were used to
 445 obtain K_{dry} vs P_{eff} , as shown in (b) for the two pore pressures used. $K_0 = 61$ GPa.

446 **4.1.7 Bulk moduli of compressibility for Bowland shale.**



447 **Figure 9.**

- 448 (a) Pore volumetry of Bowland shale at 34.5, 48 and 69 MPa gas pore pressure. There is no significant
 449 difference at the three pore pressures, so that a single polynomial function can be fitted to all the data. The
 450 slope of the curves corresponds to the pore compressibility, which decreases markedly with increasing
 451 effective pressure.

452 (b) Shows calculated K_{dry} bulk modulus of the sample (pore pressure = 69 MPa) from pore volumetry
453 measurements (inverted triangle symbols). $K_0 = 52.8$ MPa. Also plotted is the m value from pore
454 volumetry (square symbols) for Bowland shale at 69 MPa pore pressure.

455 A large specimen (25 mm diameter and 50 mm long) was used for these measurements on Bowland shale, cored
456 parallel to the layering. Because this is a low permeability rock, a 2 mm thick longitudinal slab of porous sintered
457 stainless steel was deployed as described earlier, to facilitate gas flow between the rock pores and the pore
458 pressure system. During pressure cycling it was necessary to correct data for the storativity of this plate. Figure 9
459 shows pore volumetry at 34.5, 48.1 and 69 MPa MPa argon gas pore pressure and K_{dry} data for Bowland shale.
460 Measurements were very reproducible and, unlike the Haynesville shale sample, there was no significant effect of
461 the magnitude of the pore pressure used. The amount of gas expelled during an effective confining pressure cycle
462 of 150 MPa corresponds to closure of ~8.4% of the initial (4.5% porosity) pore space, or about 0.04% of the total
463 rock volume. As also observed for Haynesville shale, this represents a very small fraction of the total porosity.

464 The poroelastic coefficient m calculated from the volumetry data is shown in Fig. 9b. Like the Haynesville
465 shale, the poroelastic coefficient obtained from pore volumetry decreases substantially with Terzaghi effective
466 pressure but does so at a similar rate to the Haynesville shale.

467

468 5. Discussion

469 5.1 Generation of pore pressure during undrained loading

470 If drainage channelways become constricted during application of increments of P_c whilst P_p is also high, the
471 rock might become effectively undrained and hence pore pressure increments can arise. The magnitude of an
472 induced pore pressure under undrained conditions can be estimated from the Skempton parameter B , where

$$473 \quad dP_p(\text{induced}) = BdP_c = \frac{C_{pp} + C_0}{C_{pp} + C_f} dP_c \quad (10)$$

474 B is the Skempton B parameter of soil mechanics (Lockner and Stanchits, 2002). C_{pp} is the compressibility of the
475 pore space arising from a change in pore pressure, and is usually much less than the compressibility of the pore
476 fluid C_f . Thus B will lie between 0 and 1.0. Because usually $C_{pp} \gg C_0$ (where $C_0 = 1/K_0$),

$$477 \quad B \approx \frac{C_{pp}}{C_{pp} + C_f} = \frac{1}{1 + \frac{C_f}{C_{pp}}} \quad (11)$$

478 For a gas saturated rock $C_f > C_{pp}$, hence $B \rightarrow 0$, and a gas-saturated rock will therefore never develop appreciable
479 pore pressures, especially at high porosities and from low initial gas pressures even when undrained, hence was
480 not considered to be an issue in the present experiments.

481 For a liquid-saturated rock however, this will not be true. B will approach 1 when $C_{pp} \gg C_f$. For liquid-
482 saturated porous sandstones under hydrostatic loading, Green and Wang (1986) found that under undrained
483 conditions, induced pore pressures were close to the applied confining pressures over a range of 60 MPa confining
484 pressure, thus the mean externally applied stress is almost totally transferred to the pore fluid via the
485 compressibility of the pore spaces.



486 The time constant for the dissipation of excess pore pressure in a region of characteristic dimension L in a
487 material of permeability k is on the order of

$$488 \quad t = \frac{\phi\mu(C_f + C_{pp})L^2}{k} \quad (12)$$

489 t is the time required for pressure to decay by factor $1/e$ at distance L . The ratio $k / \mu(C_f + C_{pp})$ is the hydraulic
490 diffusivity κ (dimensions m^2/s). For water, viscosity μ is 0.001 Pa s. Taking the bulk modulus K_f ($= 1/\text{fluid}$
491 compressibility, C_f) to be 2 GPa, and the permeability to be $10^{-18.5} m^2$ for Haynesville shale at about 5 MPa
492 effective pressure (this is the highest permeability measured, which would apply after an excess fluid pressure had
493 been generated by compaction), $\kappa \sim 2 \times 10^{-6.5} m^2/s$. This leads to $t = 30$ sec for $L = 2$ cm. Time t is shorter by a
494 factor $1/30$ when the pore fluid is gas owing to its lower viscosity (Gosman et al., 1969). This equation is for
495 constant k , but when k is a strong function of P_{eff} , decreasing perhaps 300 -fold at high effective pressures, up to 5
496 minutes may be required for small pore pressure transients to decay.

497 5.2 Simple model for pressure-dependence of permeability

498 The simplest approach to describing the influence of pore space geometry and connectivity on permeability is to
499 regard the pores as a bundle of circular capillary tubes, so that the equation for viscous Poiseuille flow can be
500 applied and permeability calculated as a function of capillary tube radius. The circular capillary tube is a special
501 case of flow through tubes of elliptical cross section. In this case the flow rate then becomes acutely sensitive to
502 the short radial dimension of the tube, and the more eccentric the tube cross-section the greater will be the
503 sensitivity of its shape to externally applied effective pressure (Seeburger and Nur, 1984). Ma et al. (2018) imaged
504 connected pores spaces in shales, including Haynesville shale, as thin, crack-like shapes lying parallel to bedding
505 and of nanometric widths. Such pores in shales are not identical to straight capillary tubes of elliptical cross
506 section, but we can explore the extent to which the pressure sensitivity of observed permeability can be modelled
507 as such (Mckernan et al., 2017).

508 For a single tube of long axis $2c$ and short axis $2b$ the volume flow rate q of a fluid of viscosity μ along a
509 hydraulic pressure gradient dP_p/dx is well known to be

$$510 \quad q = \frac{\pi}{4\mu} \left(\frac{b^3 c^3}{b^2 + c^2} \right) \left(\frac{dP_p}{dx} \right) \quad (13)$$

511 and for N parallel tubes embedded in an elastic matrix of volume V and intersecting a $1 m^2$ area normal to their
512 length the total flux $Q = Nq$. Separating out the viscosity and pressure gradient, the permeability k_o of the array is
513 $k_o = (N\pi/4) (b^3 c^3/(b^2 + c^2))$. Dimension c does not change with externally applied pressure for the elliptical
514 crack, whereas for the tapered crack it does, such as to keep the aspect ratio approximately constant (Mavko and
515 Nur, 1978), and Seeburger and Nur (1984) found that there is little difference in the effect of hydrostatic pressure
516 on flow rate when the tube cross section is elliptical or tapered. In terms of aspect ratio of an assumed elliptical
517 cross section $\alpha = b/c$, thus

$$518 \quad k = \frac{N\pi}{4} c^4 \left(\frac{\alpha^3}{1 - \alpha} \right) \quad (14)$$



519 The porosity $\phi = N\pi bc$. Parameters α , c and N that satisfy Eq. (4) are non-unique. N can be increased whilst pore
520 aperture is decreased, keeping k unchanged. A further constraint is therefore required, and this is provided by the
521 porosity ϕ , which is already known as a property of the material. Porosity is given by $\phi = Nc^2\alpha\pi$. Thus Eq. (14)
522 becomes

$$523 \quad k = \frac{\phi c^2}{4} \left(\frac{\alpha^2}{1 - \alpha^2} \right) \quad (15)$$

524 Applying a hydrostatic pressure P to a solid bearing elliptical cracks reduces the b dimensions of all pore spaces,
525 and hence reduces the hydraulic transmissivity. The spatial density of the ellipses is assumed to be sufficiently
526 small that the elastic strain fields of each do not interact significantly. From Seeburger and Nur (1984), following
527 Walsh (1965) and Mavko and Nur (1978) the bulk modulus K_{dry} of a solid of volume V containing N tubular
528 cracks of elliptical cross section and semi-major axis c is given by

$$529 \quad \frac{1}{K_{dry}} = \frac{1}{K_0} + \frac{1}{K_0} \left[2Nc^2d \frac{1 - \nu^2}{1 - 2\nu} \right]$$

530 Thus

$$531 \quad \frac{K_0}{K_{dry}} - 1 = 2Nc^2d \frac{1 - \nu^2}{1 - 2\nu} \quad (16)$$

532 d is the elliptical section tube length in the third dimension ($= V^{(1/3)}$).

533 Taking $m = (1 - K_{dry}/K_0)$, the left hand side is $m/(1 - m)$, and the expression can be rearranged with c^2 on the
534 left side:

$$535 \quad c^2 = \left(\frac{m}{1 - m} \right) \left(\frac{1 - 2\nu}{1 - \nu^2} \right) \frac{1}{2Nd} \quad (17)$$

536 This can replace c^2 in Eq. (15), to give :

$$537 \quad k = \left(\frac{\phi}{8Nd} \right) \left(\frac{\alpha^2}{1 + \alpha^2} \right) \left(\frac{m}{1 - m} \right) \left(\frac{1 - 2\nu}{1 - \nu^2} \right) \quad (18)$$

538 m is measured by pore volumetry as a function of Terzaghi effective pressure hence k is a function of effective
539 pressure. For $b \ll c$ it is primarily the reduction of the b dimension with increasing pressure that reduces
540 permeability. However, Mavko and Nur (1978) and Seeburger and Nur (1984) showed that the bulk modulus of a
541 porous solid of given porosity is not affected by the shape (eccentricity) of the pores. All pores change volume by
542 the same fractional amount. Only the distortion under pressure of the more eccentric ones is likely to affect the
543 permeability, although all pores will affect the storativity, according to how well connected they are. The
544 ‘connected’ porosity estimated from the log gain versus phase shift plot, that is much smaller than the total
545 porosity, is used in Eq. (18). Its small value implies that most of the porosity is not being inflated during the
546 passage of the pore pressure wave, hence during the time-scale of the pressure oscillation the greater part of the
547 porosity is closed off by the action of the effective pressure.

548 Eq. (18) can be fitted to the permeability data $\log k = f(P_{eff})$ measured for rock types studied using the non-linear
549 least-squares fitting routine Solver in MS Excel, to estimate the parameters N , ν and α . Via the inferred effective

550 porosity the conductive pore width can also be estimated. The results of the fitting exercise provide the parameters
551 for a bundle of capillary tubes that *behaves in the same way* as the measured rocks. This is not to say that the
552 geometric arrangement of a simple capillary tube bundle corresponds to the pore space configurations in these
553 rocks, nor that a solution can be found for all rocks. The pressure sensitivity lies in the function that describes m as
554 a function of pressure, obtained from pore volumetry, and incorporating the effective pressure coefficient n .
555 Figure 10 shows the fit to the data for the Pennant sandstone; fit parameters are in Table 3.

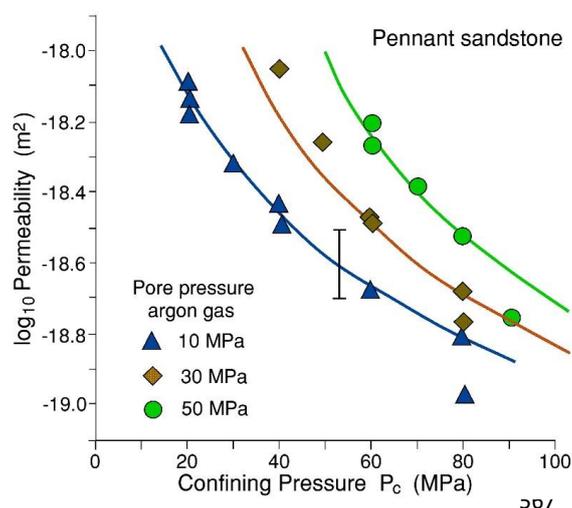


Figure 10. Gas permeability data for Pennant sandstone normal to layering, for three constant pore pressures. The three continuous curves are for Eq. (18) non-linear least-squares fits to the data.

568 down to the micron scale, in the Pennant sandstone the greater part of the rock volume is not porous, as it
569 comprises large quartz and feldspar grains. The 4.6% porosity is contained mostly in the spaces originally between
570 these grains that are now largely filled with phyllosilicate and oxide phases, i.e. about 26% of the total rock
571 volume, and is microstructurally in some ways comparable to a shale. Therefore in Table 3 the estimated
572 conductive channel dimensions are based on flow through this reduced volume fraction.

573 Figure 11 shows the fits to the permeability data for Haynesville shale. The cross-section shape of the elliptical
574 tubes is extremely eccentric and the shorter width of the tubes is measured in nanometres. This is consistent with
575 the observations of the dimensions of connected bedding-parallel porosity in the high-resolution tomography (CT)
576 observations of Ma et al. (2018) for Haynesville shale.

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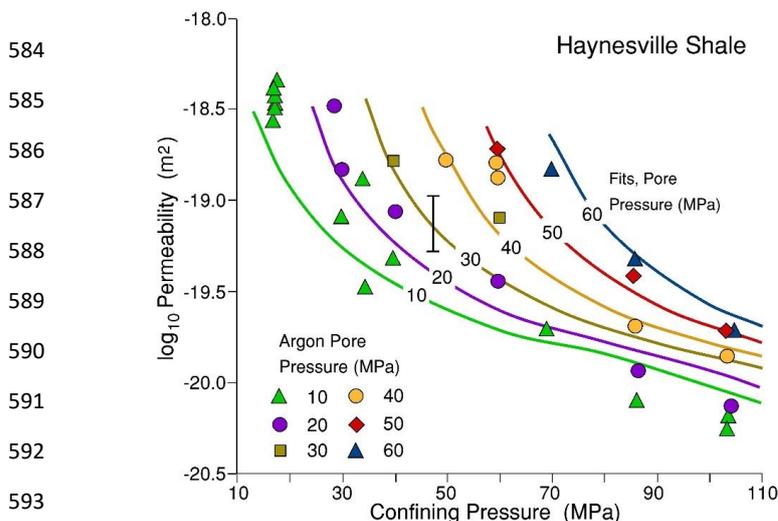
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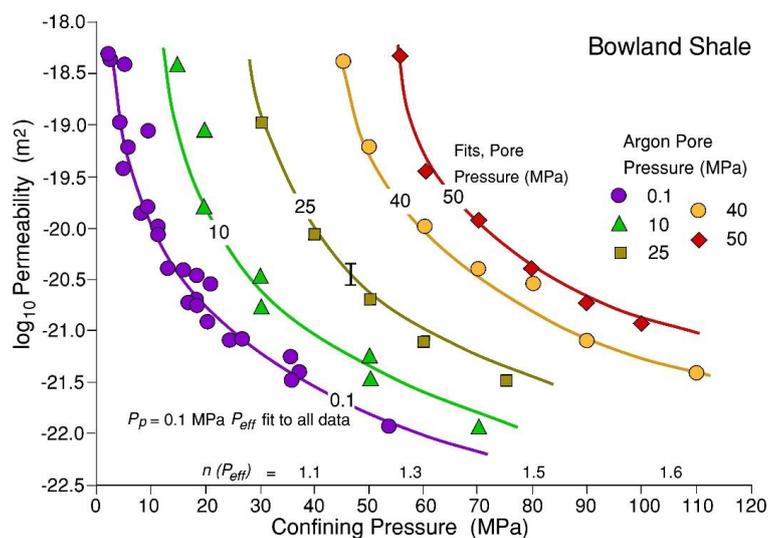
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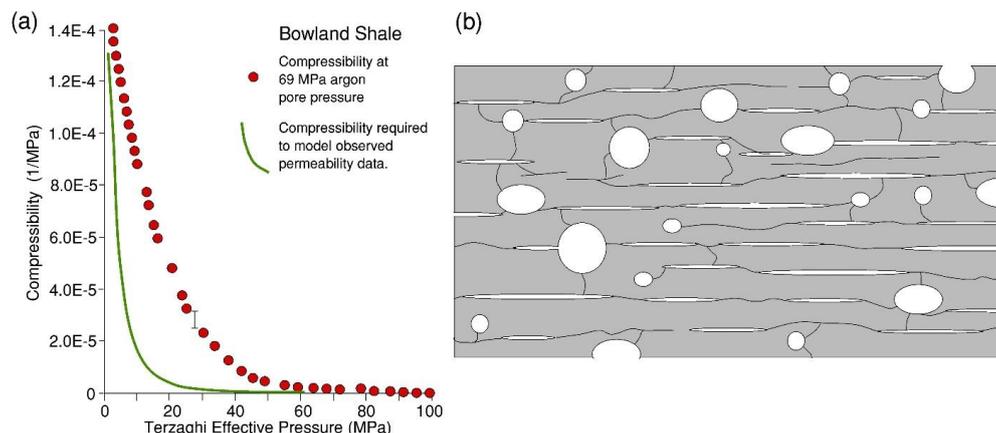


594 **Figure 11.** Permeability of Haynesville Shale versus total confining pressure for various values of constant
 595 gas pore pressure. The curves shown are the permeabilities calculated using the elliptical section pore
 596 channels model (Eq. (18)).

597



609 **Figure 12.** Permeability of Bowland Shale versus total confining pressure for various values of constant gas
 610 pore pressure. The curve and data shown for $P_p = 0.1$ is the effective pressure fit to all the data as shown in
 611 Fig. 3, collapsed onto a single least squares best-fit curve ($\log_{10} k = -0.503 \log_{10} P_{eff} - 17.26$) for a pore pressure
 612 coefficient made to vary linearly with Terzaghi effective pressure according to $n = (1 + P_{eff}(\text{MPa})/85)$.
 613 Measured data for the separate pore pressures are shown, with best-fit curves with the variable pore
 614 pressure coefficient. n -values are shown to indicate how they increase from left to right.



615

616 **Figure 13.**

617 (a) The experimentally observed variation of pore compressibility at 69 MPa pore pressure (filled circles) vs
618 Terzaghi effective pressure for Bowland shale, derived from the data in Fig. 9. The reciprocal of this
619 compressibility is $\phi/K\phi$. This rate of reduction of compressibility with effective pressure cannot predict the
620 observed pressure sensitivity of permeability that is observed experimentally. The continuous curve shows
621 what the trend would have to be like in order that the single capillary tube model can behave in the same
622 way as the rock.

623 (b) Schematic illustration of the porosity model best able to explain the permeability and bulk modulus
624 data in the shales. Highly eccentric pores and cracks lie parallel to layering but are well-connected,
625 accounting for easy gas transport yet using only a small fraction of the porosity. These narrow pores are
626 easily constricted by hydrostatic pressure. Most of the storage capacity resides in the larger, equant pores
627 of dimension about 1 micron that are poorly linked and not easily closed down by hydrostatic pressure.

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637 **Table 3. Fit parameters for the capillary tubes bundle model applied to describe the permeability of**
638 **Haynesville shale and Pennant sandstone at low effective pressures, when the permeabilities are not**
639 **strikingly different. n is the pore pressure multiplier for the permeability data, N is the number of pores**
640 **intersecting a 1 m^2 area normal to the flow path, a is the pore shape aspect ratio and ν is the Poisson ratio.**
641 **$2b$ is the mean short dimension (nm) of the elliptical cross section and s is the average pore spacing**
642 **(microns).**

643



	Haynesville Shale	Pennant sst
644		
645	n 0.99	0.86
646	N 1.03 E+11 m ⁻²	8.4 E+11 m ⁻²
647	α 0.0051	0.004
648	ν 0.17	0.10
649	$2b$ 13.5 nm	21 nm
650	s 3.1 μ m	1.5 μ m
651	Conductive porosity 0.3%	3.8%

652 -----

653

654 The form of the curve of K_{dry} vs Terzaghi effective pressure does not permit the simple elliptical section
 655 capillary tubes model to be fitted to Bowland shale, because the observed rate of decrease of m with effective
 656 pressure is insufficiently rapid to explain the three orders of magnitude decrease of permeability observed over
 657 this pressure range (see Fig. 12). Figure 13a compares the observed variation with effective pressure of pore
 658 compressibility factor m to the variation that would be required to be able to make such a fit. It is inferred that
 659 pressure must be able to act in this rock to close down pore connectivity in one or more additional ways to the
 660 elastic compression of elliptical channel cross-sections. These could involve development of increased tortuosity
 661 of channelways, or the existence of a more complex distribution of connected pores of different sizes and shapes.
 662 The simple model of a set of similarly-sized and shaped channels that can behave in a comparable way to a real
 663 pore network is clearly inapplicable to this rock.

664 **5.3 The effective pressure coefficients, m and n**

665 In the context of permeability, n is the multiplier of pore pressure in the definition of the modification of
 666 Terzaghi effective pressure that brings observed permeability data at different constant pore pressures onto a
 667 common curve (e.g. Fig. 12), thus $P_{eff} = P_c - nP_p$. n takes a value close to unity in the case of the experimental
 668 data on Haynesville shale and Whitby shale, and 0.86 in the case of Pennant sandstone. In other studies, observed
 669 departures from unity have been attributed to, for example, differences in the roles of elastically stiff and
 670 elastically soft mineral components surrounding the pore spaces in responses to changes in P_c relative to changes
 671 in P_p (e.g. Zoback and Byerlee, 1975; Kwon et al., 2001; Ma and Zoback, 2017), resulting in different rates of
 672 change of pore volume with P_c and P_p .

673 On the other hand, in Eq. (4), for a homogeneous, isotropic elastic matrix, it is the value of K_{dry} , the bulk
 674 modulus of the porous rock, that determines the change in geometry of pore spaces, and hence permeability, in
 675 response to effective pressure change. The theoretical expression for the effective pressure coefficient m for elastic
 676 deformations of a mechanically linear, homogeneous and isotropic rock is given by Eq. (7) and this parameter
 677 appears in the expression for the permeability according to the bundle of capillary tubes model (Eq. (18)). Using
 678 the pore fluid displacement method (Figs. 6, 8 and 9) we have found that in all cases m decreases from near unity
 679 with Terzaghi effective pressure according to the pressure dependence of K_{dry} , whereas for Pennant sandstone and



680 Haynesville shale, observed n remains close to unity for permeability data and exceeds unity for Bowland shale
681 over Terzaghi effective pressures from zero to *ca* 80 MPa, thus $m \neq n$. Nur and Byerlee (1971) took care to point
682 out that m as defined in Eq. (7) cannot generally be used as a predictor of effective pressure coefficient for
683 particular processes, like permeability, mechanical strength and elastic wave velocities, even though all involve
684 elastic distortions.

685 As was pointed out earlier, pressure sensitivity of permeability according to the simple capillary bundle model
686 cannot behave in the same way as was observed experimentally for Bowland shale. Also, a single value of n
687 cannot reconcile permeabilities at different pore pressures for this rock. Figure 12 shows the permeability data for
688 Bowland shale separated into measurements at different pore pressures. By extending the collective fit between
689 log permeability and effective pressure shown in Fig. 12 to the data at each pore pressure, the downward
690 divergence of the curves becomes apparent. This can be described empirically by fitting a linear variation of n
691 with Terzaghi effective pressure, such that $n = 1$ at low effective pressures, rising to $n = 1.6$ at the upper end of the
692 pressure range used. This is interpreted as a further manifestation of the pore structure complexities that mean that
693 this Bowland shale cannot be described by a simple capillary tube bundle model.

694 **5.4 Relationship between observed pressure-dependent permeability and mineralogy**

695 Several studies have reported the relationships between mineralogy of shales and related rocks and their
696 petrophysical properties (e.g. Kwon et al., 2004; Ma and Zoback, 2017). The rocks used in this study display a
697 spectrum of mineralogy that is reflected in their permeabilities, both in terms of absolute values and their
698 sensitivity to effective pressure.

699 Pennant sandstone is typical of tight gas sands in which the load bearing framework is of continuous quartz and
700 feldspar grains with what would otherwise be a large porosity that is mostly filled with some detrital muscovite
701 plus diagenetically-introduced clay and oxide phases (Wilson and Pittman, 1977; Howard, 1992). Prior to the pore
702 filling there was a degree of intergranular pressure solution and formation of quartz overgrowths around quartz
703 grains. The protective armour around the filled pore spaces afforded by the quartz framework is thought to have
704 limited degree of compaction of the pore filling, in which most of the present porosity resides. Relative to the
705 volume of the inter-quartz spaces, the porosity of the filling would be ~20%, and it is thought that this contributes
706 to the relatively high overall permeability and reduced pressure sensitivity of Pennant sandstone.

707 The Bowland and Haynesville shales are mineralogically and microstructurally strikingly different. It is
708 important to remember that these are particular samples taken from their respective sequences and may not be
709 especially representative of their host sequences at all. The Bowland shale sample is a phyllosilicate-rich,
710 carbonate-poor siliceous mudstone with sufficient phyllosilicate to form a contiguous matrix, and this is likely to
711 be responsible for the relatively low bulk modulus (53 GPa) of the rock and hence low permeability. The
712 Haynesville shale is a carbonate-rich (>50vol%), phyllosilicate-poor siliceous mudstone with a higher bulk
713 modulus (61 GPa). The carbonate grains (fossil fragments and diagenetic carbonate) provide a stiff framework of
714 contiguous grains, helping to maintain open porosity and to resist its elastic compaction. Despite these qualitative
715 observations that can be made about how mineralogy and microstructure impacts upon permeability, the present
716 results do not form a basis for making any quantitative correlations.

717



718 **5.5 Inference of key characteristics of pore space geometry in shales**

719 Much has been written on pore space geometry based on SEM, TEM and Xray CT imaging of shales, but
720 important characteristics can be inferred from observations of bulk petrophysical properties. Key points noted in
721 the present study are:

- 722 • The storativities for both shales are extremely small for flow paths lying parallel to the layering, such that over
723 90% of the available pore space is not participating in the flow.
- 724 • At low effective pressures, the permeabilities of all three rocks are similar, but with increasing effective
725 pressures they diverge at markedly different rates. Marked sensitivity of permeability to effective confining
726 pressure implies that conductive (well-connected) pores are flat and crack-like. This is supported by permeability
727 modelling, that suggests that for a bundle of elliptical-section capillary tubes of equivalent permeability behaviour,
728 their aspect ratios are extremely small and the narrow dimension is expected to be in the nanometric range (Table
729 3).
- 730 • For flow normal to layering, at least in Haynesville shale, storativity is much greater than for flow across the
731 layering, but still implies that over half of the pore space is not participating in the flow.
- 732 • Permeability in both shales is very low under elevated effective pressures compared to Pennant sandstone,
733 which is of similar overall porosity, implying that connected pore spaces are narrow and/or poorly
734 connected/tortuous.

735 The above observations suggest that the effective configuration of pores spaces corresponds to the sketch shown
736 in Fig. 13b, with a population of highly oriented, crack-like pores parallel to layering that account for only a small
737 fraction of the total porosity but dominate the hydraulic transmissivity through the rock mass parallel to the
738 layering and also account for the low storativity associated with flow along the layering. These are poorly
739 connected to larger, probably more equant pores by conduction channels trending across the layering, and which
740 contain most of the gas storage space in the rock. The equant pores are ‘seen’ more easily for flow across the
741 layering, so that this flow is characterised by higher storativity, as demonstrated for Haynesville shale. Such
742 storage pores are likely to be much slower to drain (or to fill) in response to an applied pore pressure gradient than
743 implicit in the laboratory-measured permeability data. This suggests that permeabilities measured by transient
744 flow methods in the laboratory may lead to an over-conservative estimate of the potential for drainage of a gas
745 reservoir in shale, and perhaps help partially to explain the long-term persistence of flows from some shale gas
746 reservoirs (e.g. Guo et al., 2017; Wang, 2017).

747 **6. Conclusions**

748 Permeabilities as functions of effective pressure were measured using the oscillating pore pressure method at 20
749 °C for three rocks (Haynesville and Bowland shales and Pennant sandstone) of low permeabilities and comparable
750 porosities. Tests were at effective pressures ranging up to 90 MPa with argon gas as permeant. From exhibiting
751 comparable permeabilities at low pressures they diverged markedly with increasing pressure. Pennant sandstone
752 showed permeability reduction with pressure of less than ten-fold, Haynesville shale became less permeable by
753 almost two orders of magnitude, whereas Bowland shale was reduced in permeability by more than 3 orders of
754 magnitude. The different pressure sensitivities of permeability correlated inversely with their (pressure sensitive)



755 bulk moduli and qualitatively with mineralogical differences, going from a continuous framework of stiff quartz
756 grains (sandstone) through a carbonate-rich framework (Haynesville shale) to a contiguous matrix of phyllosilicate
757 grains (Bowland shale).

758 High storativity of the sandstone implied that most of the available pore space was involved in the gas flow, but
759 in the shales, for flow parallel to the layering, less than 10% of the available pore space was involved in the flow.
760 For flow in the Haynesville shale across the layering a larger pore space fraction was involved, but still much less
761 than all the available pore space. Thus only a small fraction of the total pore space can be inferred to be well
762 connected in the shales. This implies that whilst the permeability we measure in the oscillating pore pressure
763 experiment is that associated with gas transport through the rock mass, a lower effective permeability applies to
764 the ability of the gas to flow into and out of the storage pores.

765 A simple model of permeability was developed based upon connected pore space behaving in a way similar to a
766 bundle of capillary tubes of highly eccentric cross section. By fitting the model to the experimental data, it was
767 possible to demonstrate that this model behaved in a similar way to the rocks for the case of Pennant sandstone
768 and Haynesville shale, but the model could not behave in a way compatible with the marked pressure sensitivity of
769 permeability for the Bowland shale. It was inferred that a more complex distribution of connected pore spaces of
770 varying dimension and tortuosity would be required to behave like the Bowland shale sample.

771 **Author contribution**

772 EHR was responsible for the conceptualization and methodology of the study, carrying out the bulk of the
773 experiments, compilation and analysis of data and writing the manuscript. JM was responsible for the acquisition
774 and management of financial support, carrying out the FEM analysis, contribution to experimental design and data
775 presentation, and preparation of the paper. YB carried out the experiments on Haynesville shale under the
776 supervision of JM and EHR as part of his doctoral research.

777 **Data availability**

778 All of the experimental data acquired in this research is freely accessible and collated in supporting datafile
779 DF1.csv. In correspondence with UK Research Council requirements is deposited in the UK National Geoscience
780 Data Centre, identified by the title of this paper. It is also downloadable from <https://zenodo.org/record/5675601>
The Authors declare that they have no conflict of interests.

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