Thermal non-equilibrium of porous flow in a resting matrix applicable to melt migration: a parametric study

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The manuscript describes thermal non-equilibrium in two-phase systems using a simplified one-dimensional model where the solid is stationary, fluid velocity is constant and prescribed, and material parameters are constant. The authors take the additional step of seeking analytical solutions near the peak of disequilibrium by imposing a thermal gradient in the solid phase and assuming a steady state. This allows them to examine the limits of the equations and examine the behavior in several important regimes. This is done quite rigorously with significant discussion of several scenarios.

Overall the manuscript is rigorous and compelling but I agree with John Rudge's review and think it could be significantly streamlined. This would make the impressive body of work much more digestible and useful to the wider community.

Scaling

As John pointed out the number of parameters could be reduced with a redefinition of the length-scale L_0. This would have a huge impact on all the subsequent sections, making the conclusions much more tractable. I would encourage the authors to take a small extra step and redefine the length-scale such that:

$$L_0^2 = \frac{\delta}{S}, \tag{1}$$

where I have adopted John's notation and used $\delta = dm$ and have dropped the φ dependence. The equations then become:

$$\frac{\partial T_f}{\partial t} + Pe\boldsymbol{v} \cdot \nabla T_f = \nabla^2 T_f - \frac{1}{\varphi}(T_f - T_s) \tag{2}$$

$$\frac{\partial T_s}{\partial t} = \nabla^2 T_s + \frac{1}{1-\varphi}(T_f - T_s) \tag{3}$$

$$\tag{4}$$

with controlling parameters:

$$Pe = \frac{L_0 v_0}{\kappa_0} = \sqrt{\frac{\delta}{S}}\frac{v_0}{\kappa_0} \quad \text{and} \quad \varphi \tag{5}$$

This has the small additional appeal of removing the φ dependence from Pe (or A) and having independent controlling parameters.

Boundary Conditions

The top boundary condition is frequently referred to as a constant flux condition but it appears to only set the diffusive part of the condition and ignores any discussion of the advective component. This has large implications for the behavior at the top of the domain, where most of the discussion is focused because it is where the largest (or most persistent) non-equilibrium conditions are seen (at least in the case with a linear initial condition). The persistence of the disequilibrium is a product of the boundary conditions however, with the solid temperature being prevented from further equilibrating with the fluid by its flux being fixed. I was a little confused by the physical motivation of this boundary condition where fluid is (presumably) allowed to escape but the solid temperature gradient is fixed and would like to have it discussed or justified more at its introduction. The authors do later vary the boundary condition in the discussion and supplement but again don't offer physical motivations for the options and again don't discuss the advective flux component.

Analytical Justification

In section 4.1 the authors simplify their model to seek the temperature difference around a fixed solid temperature profile. The description of this simplification could be improved. For a start I believe the solid temperature gradient has a sign error on line 210 $\partial T_s/\partial z = -\Delta T/H$ (the subsequent equations are correct) and, as far as I could see, ΔT has not been defined at this point. ΔT_0 was previously used for the temperature drop across the domain (and ΔT_{max} for the peak non-equilibrium temperature difference). Beyond that more clearly stating that this fixes the solid temperature gradient and seeks a solution in this state would make reading this section easier (the authors do try to clarify that the steady state behavior they are seeking isn't the same as that in the "full" model).

Discussion

The discussion section rigorously describes a number of different scenarios and variations on the previous results. It covers an impressive range of topics though section 5.1.5 seems redundant and could be removed. It would be hugely enhanced by reducing the complexity and only considering a two (with presumably only one important) parameter model. Currently a full understanding of the section depends on several figures that are only available in the supplementary material, which are not particularly well captioned. If it were possible to improve the labeling and move them or a subset of them to the main text that would enhance the discussion significantly. In particular, though the color coded time evolution works well when lines are close together, it fails to clearly show the order of the solutions once significant jumps occur between them. It would also be good to know what (non-dimensional) time-ranges are being plotted in all figures.

Minor Points

There are quite a few small typos throughout the paper (such as extra or missing spaces) that should be corrected. Below I just list a few things that caught my eye:

- line 72: "allowing easily to decide" → "that allow the easy determination of"?
- start of section 2.1: a constant porosity and fluid velocity are already stated here but aren't used in the following derivation, instead being repeated at the end of the section (where they are used)
- line 86 and throughout where appropriate: "resumes to" → "becomes"
- equation 9a: κ_0 is introduced but isn't defined until equation 11 and even then is related to κ_f and κ_s, which also haven't been introduced before. I think a statement that $\lambda_f = \lambda_s = \lambda_{eff}$ might be necessary first.
- line 134: "is to emphasized" → "is to be emphasized"
- start of section 2.3: the domain height, H is introduced before it is used, perhaps consider adding to the first sentence: "are solved in a 1D domain of height H"

- line 148: sign error in “flux” boundary condition
- line 152: extra space in “R2018b)”
- lines 169-171: the sentence “As the fluid temperature increases” could do with rewriting for clarity.
- line 177: “boundary conditions are applied” → “boundary conditions at $z = H$ are applied”
- line 210: sign error in solid temperature gradient and ΔT introduced without definition
- line 224: missing space between “for” and “$(T_f - T_s)$”
- line 231: “For 91% of the models” - does this include those models with a different initial condition or those with varying material properties?
- line 245: I think “complex” could be confusing here, maybe try “complicated”
- line 273: “L’Hospital” → “L’Hôpital”
- line 287: extra space after “dykes”
- line 315: “onm” → “on”
- line 340: “As initial condition” → “As an initial condition”
- section 5.1.4: redefining A for this section is potentially a little confusing. Hopefully A is no longer necessary anyway but, if not, perhaps in earlier sections the constant version could be A_0 so that the variable wouldn’t have to be re-used.
- section 5.1.5: this seems redundant and could be removed
- line 428: it might be nice here to clarify that this is the steady state of the full model, not the analytic model
- supplementary material line 23: this looks like it’s the solution of Eq. (19) from the main text not the homogeneous Eq. (S2)
- line 46: “into Taylor series” → “into a Taylor series”
- Figure S4 caption: spacing and punctuation errors