se-2021-45 Reply to Reviewers: RC1

Rahul Prabhakaran, Giovanni Bertotti, Janos Urai, David Smeulders

28-7-2021

Dear Reviewer,

We would like to take this opportunity to thank you for such a detailed review. In the following paragraphs, we reply point-wise to the issues that were raised. We hope to have addressed the concerns sufficiently to warrant reconsideration for publication.

Kind Regards

Rahul Prabhakaran (On behalf of all Authors)

Anon 01

Reviewer Comment *Dr. Prabhakaran and co-authors here present a novel approach to spatial network analysis of fractures. This approach involves quantifying hierarchical clustering based on similarity of four statistics: fingerprint distance, D-measure, NetLSD, and portrait divergence. Hierarchical clusters are identified based on the similarity of areal sub-samples of a large fracture map (the example here being the Lilstock anticline, UK). The authors show maps of their results, noting apparent spatial autocorrelation in all analyses except for NetLSD.*

The manuscript is mostly well written and appears to me to be mathematically sound. However, despite using several highly sophisticated statistics, the method relies on at least two rather qualitative and/or arbitrary choices: (i) the sampling window size and overlap amount, which would seem to exert an effect on the degree of the aforementioned spatial autocorrelation, and (ii) the selection of the number of clusters present, which should sensitively affect the resulting map patterns.

Moreover, the utility of the approach remains elusive, despite some time spent on it in the introduction and discussion. These four statistics are novel, but their meaning is fairly obscure. The main output of the technique is a map that highlights spatial variation in these statistics, which indeed could be useful. But the same types of patterns show up on maps of more inuitive statistics, like intensity (Figure 9, 14, 19). What does the technique tell you about the spatial arrangement of fractures, fracture connectivity, fracture drivers, etc., that exceeds the explanatory power of more conventional approaches?

Use of the term "clusters" (Line 96-99) is non-standard. Marrett et al. (2018), which the present paper cites, defines a cluster as a place where fracture spacing is smaller than it is elsewhere; as such, a uniformly spaced fracture pattern would lack clusters entirely. This definition is consistent with the framework described earlier in the present study (Line 26-7). Why the change in usage? I suggest using a different term for the output of the present approach that is not already widely used in pattern-quantification literature.

Authors' Reply

Intensity maps versus graph similarity-based clustering maps

- Fracture intensity maps, despite being a reasonable indicator of spatial heterogeneity, misses out information on fracture network structure and its spatial variation. There is a need to incorporate network attributes in spatial heterogeneity maps and therefore, the attempt to use graph-based measures.

- We note that there are differences between spatial heterogeneity maps derived from HC and graph similarities and those produced by the conventional P_{20} and P_{21} computations. Since spatial P_{20} and P_{21} are often used to depict spatial variation in fracture network, our objective was to make a qualitative comparison. The main advantage of the proposed methodology as opposed to the ubiquitous spatial P_{20} and P_{21} is that a hierarchical structure of variation is derived that encapsulates network properties. Given that network spatial organization influences flow, transport, and geomechanics, the explanatory power of P_{20} and P_{21} has limitations.

Qualitative and arbitrary choices

The choice of the number of clusters was based on weight of sum of squares (wss) plots depicted in Figures 8, 13, and 18. This plot also referred to as "elbow plots" is often used to identify optimal number of clusters by considering the variation in slope. The choice of five clusters was based after considering the 12 wss plots corresponding to the four similarity metrics from each region. A second reason for the choice of 5 clusters is that we wanted to depicts results of spatial clustering across the considered similarity metrics. However, instead of making a single choice, we can easily depict the spatial variation for a range of cut-heights (clusters). When a diverging color scheme is chosen to depict sub-graphs under the various dendrogram branches, it can be observed that there is development of newer clusters and changing of cluster boundaries as one delves deeper into the dendrogram hierarchy. This is depicted in the figures below for the four similarity measures and corresponding to Region 2. In the revision, we extend this to all the Regions.

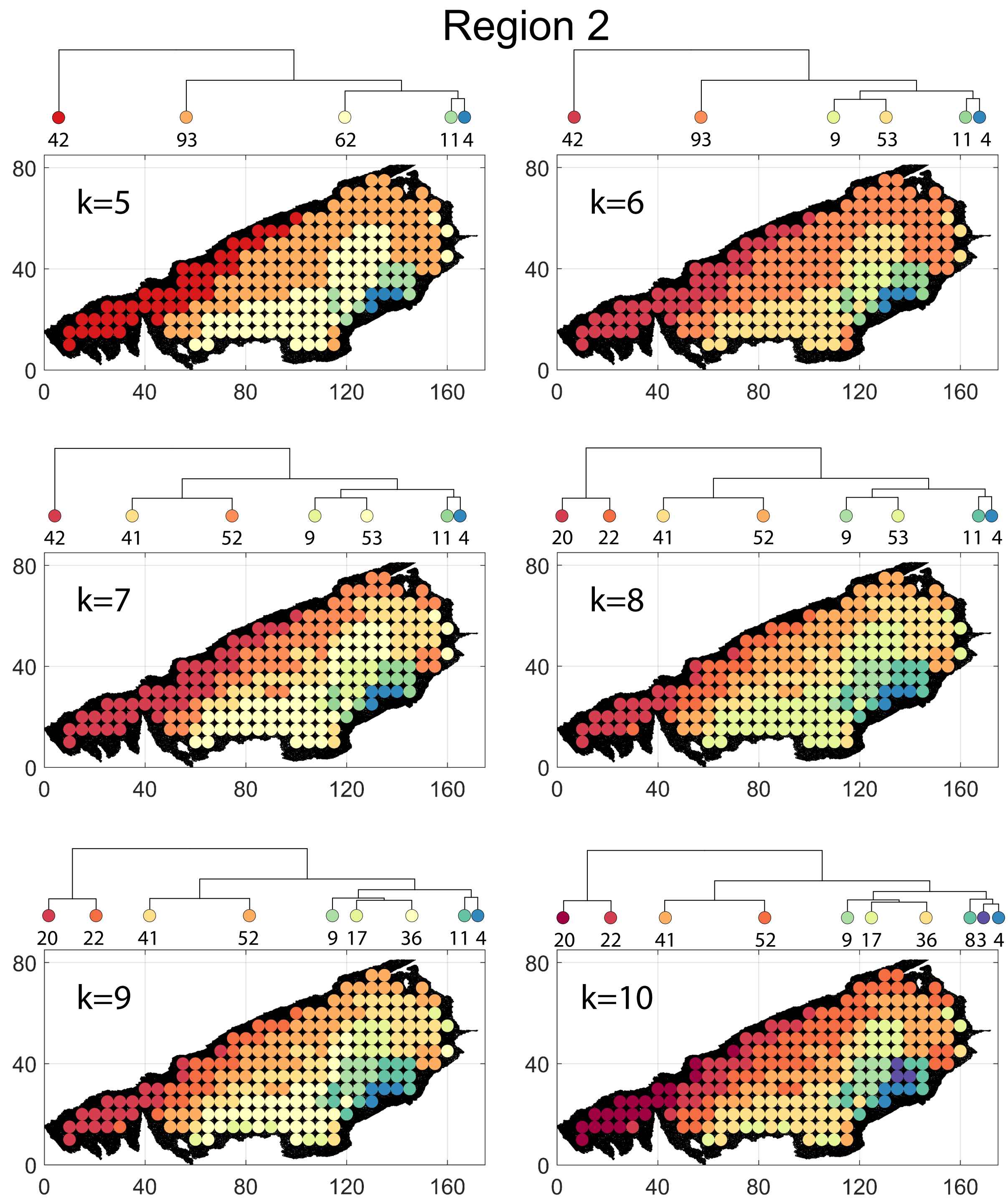

Figure 1: Region 2 Clustering using Fingerprint Distance at multiple dendrogram levels

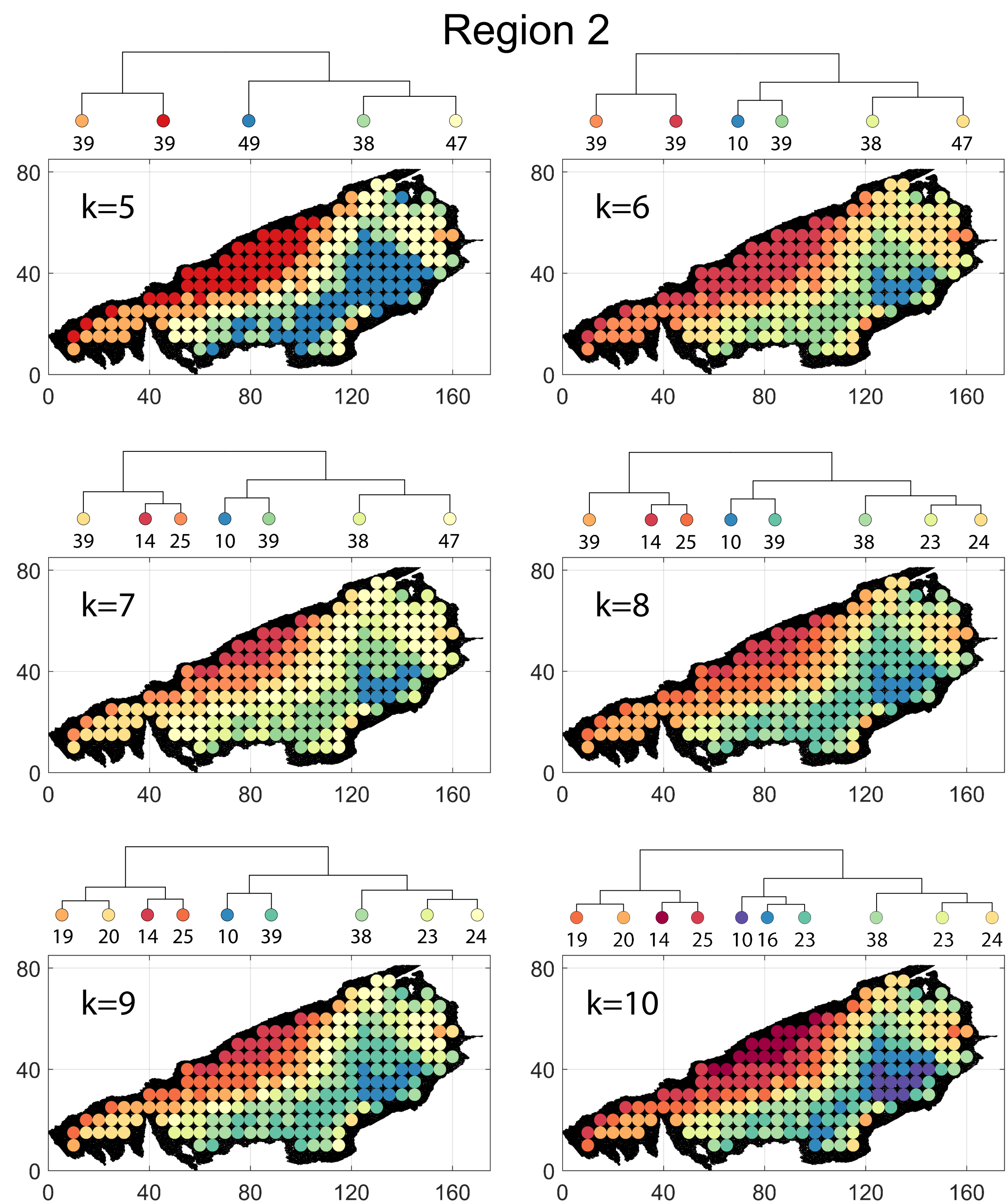

Figure 2: Region 2 Clustering using D-measure Distance at multiple dendrogram levels

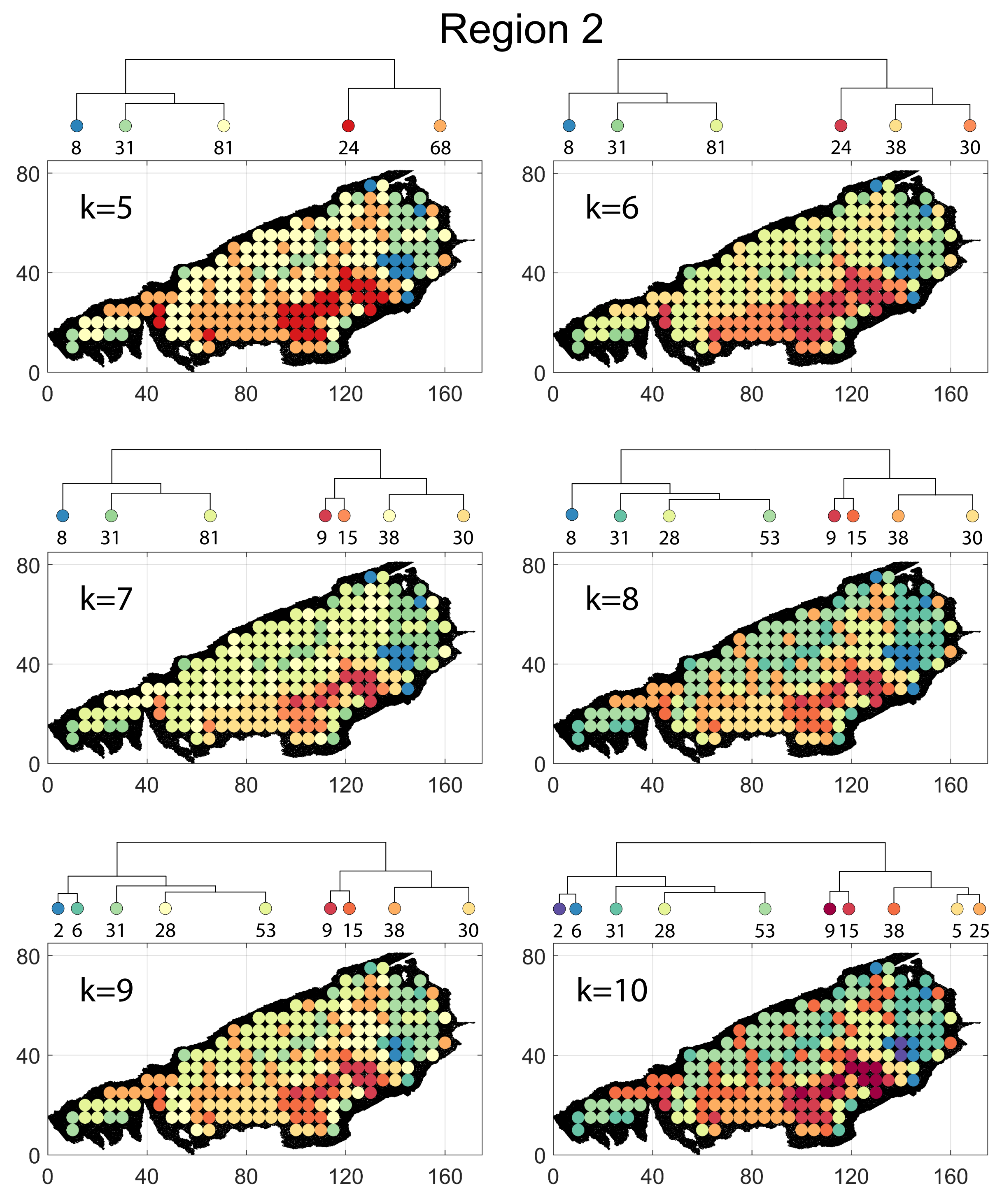

Figure 3: Region 2 Clustering using NetLSD at multiple dendrogram levels

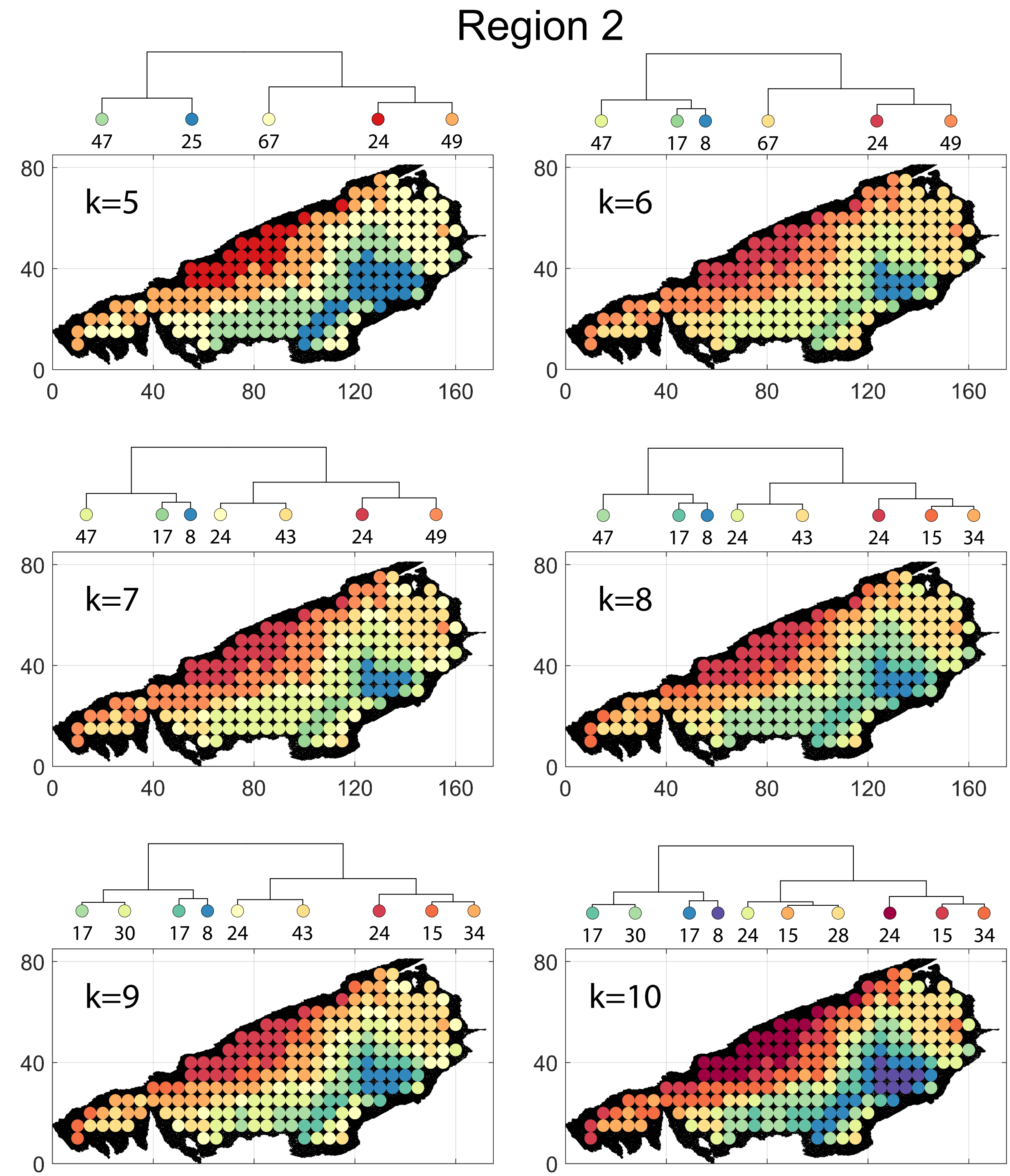

Figure 4: Region 2 Clustering using Portrait Distance at multiple dendrogram levels

Utility of the method in explaining connectivity, spatial arrangement, and drivers

- The method is essentially graph-based hence we can interrogate network properties of clusters in a more detailed manner since the dendrogram contains similarity information down to each sub-graph. In the revision, we add half-rose plots and topological summaries on the derived clusters to highlight the differences in network properties. An example from Region 3 using fingerprint distance, portrait divergence, and D-measure is depicted below. Sub-graphs pertaining to each cluster is extracted to depict fracture segment orientations. It can be observed that the orientations in each cluster visibly vary across dendrogram branches. The shape of the rose-plots pertaining are similar within clusters underneath a high-level branch in the dendrogram and different to those on other branches. Such a variation is identified without explicitly taking into account fracture edge orientations while weighting the primal graphs.

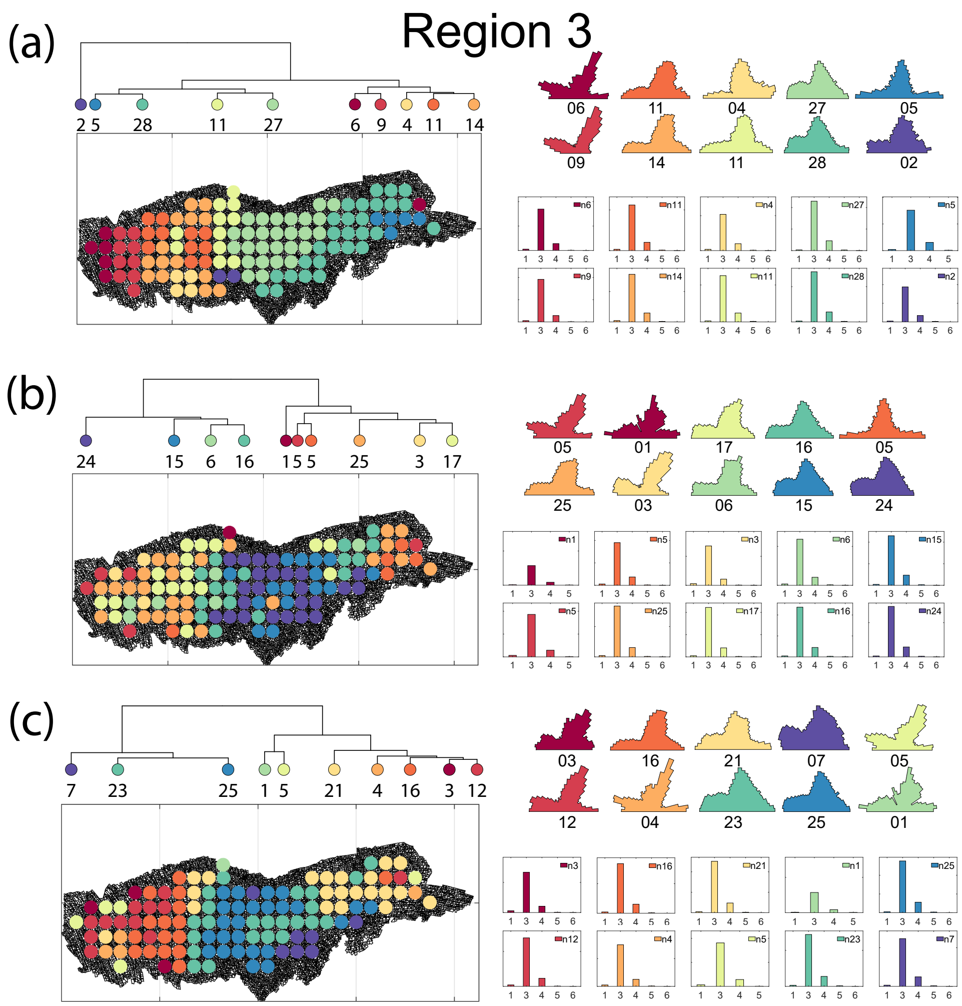

Figure 5: Fracture orientations and topological summaries for clusters

- Our aim in this contribution was not to use the technique to delve into details of the structural evolution of the region, but rather focus on a methodology for quantifiying spatial variation which is generally applicable to any fracture network. We chose the Lilstock dataset owing to the unprecedented spatial resolution, network size, and spatial extent. The reasons for the complexity and variation in fracturing styles in Lilstock, is still not fully explained despite several decades of work in the area. 2D fracture trace maps from which spatial variation is quantifiable, serves as one tool in a multidisciplinary set that probably requires more evidence, possibly incorporating data fusion from multiple remote sensing sources etc.

Usage of the term "cluster"

We agree that the context in which the term "cluster" was used by Marrett et al. (2018), is different from our usage. The clusters of Marrett et al. (2018) refer to regions of higher density of scanline intersections. In the revision, we make it clear as to the particular context where Marrett et al. (2018) derive their terminology. In the manuscript text which is essentially an application of a statistical clustering technique, we will again make it clear in Section 2.3, that we are referring to pattern similarity clusters.

Anon 02

Manuscript Text

Line 6: The results indicates presence of spatial clusters within fracture networks with that vary gradually over distances of tens of metres.

Line 8: Variation in fracture development in the other two regions are interpreted to be primarily influenced by proximity to faults that gradually gives way to background fracturing.

Line 11: . . . ; however, additional variations are highlighted that is not obvious from fracture intensity and density plots.

Line 289: . . . we can observe a cluster in red which closely correspond to the trend of high fracture persistence . . .

Line 344: A hierarchy of patterns are derived based on similarity scores and these can be examined at deeper levels.

Reviewer Comment *S/V agreement. Also 8, 10, 11, 289, 344.*

Authors' Reply We replace the sentences with corrected grammar in the revision.

- The results indicate the presence of spatial clusters within fracture networks varying gradually over distances of tens of metres.
- Variation in fracture development in the other two regions is interpreted to be primarily influenced by proximity to faults, gradually giving way to background fracturing.
- . . . however, additional variations, not immediately obvious from fracture intensity and density plots, are highlighted.
- Observing further branches of the dendrogram, a cluster in red, closely corresponding to the trend of high fracture persistence (compare with Fig.9(e) and Fig.9(f)), can be identified.
- A hierarchy of patterns are derived based on similarity scores. The hierarchy of patterns can be examined at deeper levels.

Anon 03

Manuscript Text Line 21: Systematically documenting near-surface fracture patterns is essential, for example, in mining applications where they often provide clues to ore deposit patterns (Jelsma et al., 2004), and in geotechnical engineering, where fractures influence stability in human-made structures such as tunnels (Lei et al., 2017).

Reviewer Comment *Line 21: ambiguous pronoun "they"*

Authors' Reply We replace the sentence by the following, "Systematically documenting near-surface fracture patterns is essential, for example, in mining applications where fracture patterns often provide clues to ore deposit patterns (Jelsma et al., 2004), and in geotechnical engineering, where fractures influence stability in human-made structures such as tunnels (Lei et al., 2017)."

Anon 04

Manuscript Text Line 24: An important property of natural fracture networks (NFRs) is that of spatial organization ...

Reviewer Comment *Line 24: I suggest abandoning this acronym. It is only used once more in the paper (Line 396) and furthermore it does not stand for its definition.*

Authors' Reply We remove the acronym in the revision.

Anon 05

Manuscript Text Line 26-27: Within such a framework, fracture objects are either clustered, periodically spaced, irregularly spaced, or regularly-spaced (Laubach et al., 2018).

Reviewer Comment *Line 27: See definition comment above; also, in this framework I believe that "irregularly spaced" is a synonym for "clustered" and "regularly spaced" is a synonym for "periodically spaced."*

Authors' Reply

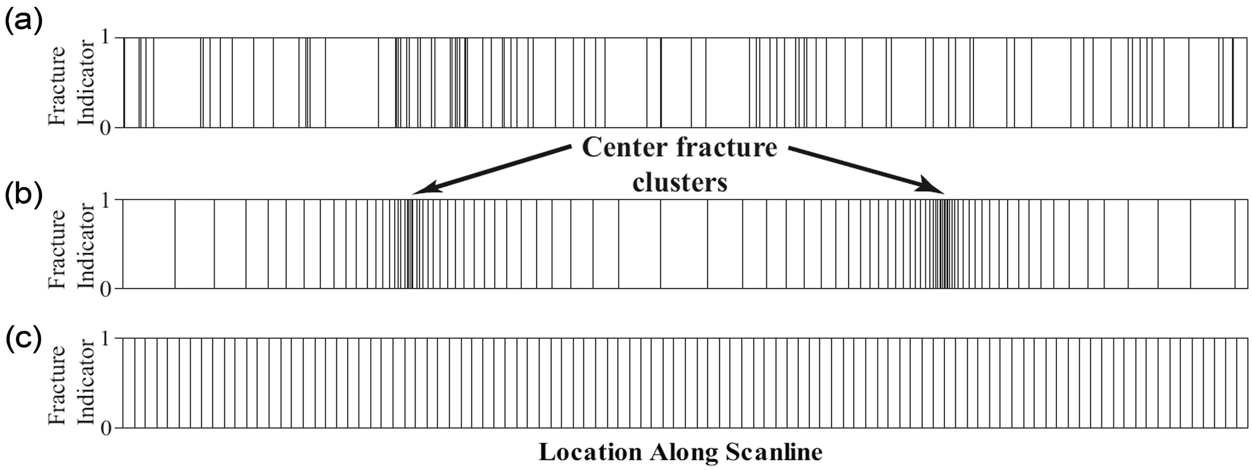

Figure 6: Clustering of scanline observations as described by Laubach et al. (2018)

Yes, Laubach et al. (2018) was referring to scanlines with statistically insignificant clustering in irregularly-spaced fractures, statistically significant clustering in irregularly-spaced fractures, and regularly-spaced fractures. We, therefore, replace the sentence to "Within such a framework, fracture objects are either regularly spaced, irregularly-spaced with statistically significant regions of close spacing, and irregularly-spaced with statistically insignificant regions of close spacing (Laubach et al., 2018)".

Anon 06

Manuscript Text Line 42: Other issues are associated with scanlines such as censoring and truncation effects, scale-dependence, and minimum sample size requirements.

Reviewer Comment *Line 42: None of these drawbacks is unique to 1D analysis.*

Authors' Reply Yes, these issues persist even in 2D trace maps. Hence, we remove this sentence and replace with the following "Scanlines do not provide information on properties such as fracture length, spatial arrangements, and relationships with other fractures."

Anon 07

Manuscript Text Line 56-57: However, given that any given spatial network (except regular lattices) is inherently non-stationary,

Reviewer Comment *Line 57: I am not an expert on stationarity, but I don't think this is true. Can't an irregular pattern nevertheless be stationary, meaning have attributes that do not change with position, above a certain size-scale?*

Authors' Reply The point we were making here is that conventional geostatistics, where transforming a spatial fracture network into a pixel/voxel property such as persistence measures, and then using variograms/semi-variograms to quantify spatial variation, is inadequate. A regular network pattern, that can be decomposed into a unit motif and then replicated over a region of interest, yielding the original pattern, can be considered to be stationary at the spatial scale of the motif. Since this sentence is quite ambiguous in its implication, we replace with the following, "However, given that natural fracture networks display spatial heterogeneity, the suitability of such REVs based on simplistic stationarity assumptions needs to be re-examined."

Anon 08

Manuscript Text Lines 65-67: By assigning positional information to the vertices (also nodes), fractures in the form of graphs encapsulate both topological and spatial information.

Reviewer Comment *Line 66: Also "called" nodes?*

Authors' Reply We rephrase the sentence as suggested in the revision.

Anon 09

Manuscript Text Line 92-93: In the case of spatial graphs derived from fracture networks, an undirected but weighted representation is sufficient.

Reviewer Comment *Line 92: What do undirected and weighted mean?*

Authors' Reply We add the following figure in the revision that depicts an example of an unweighted, weighted, and directed graph.

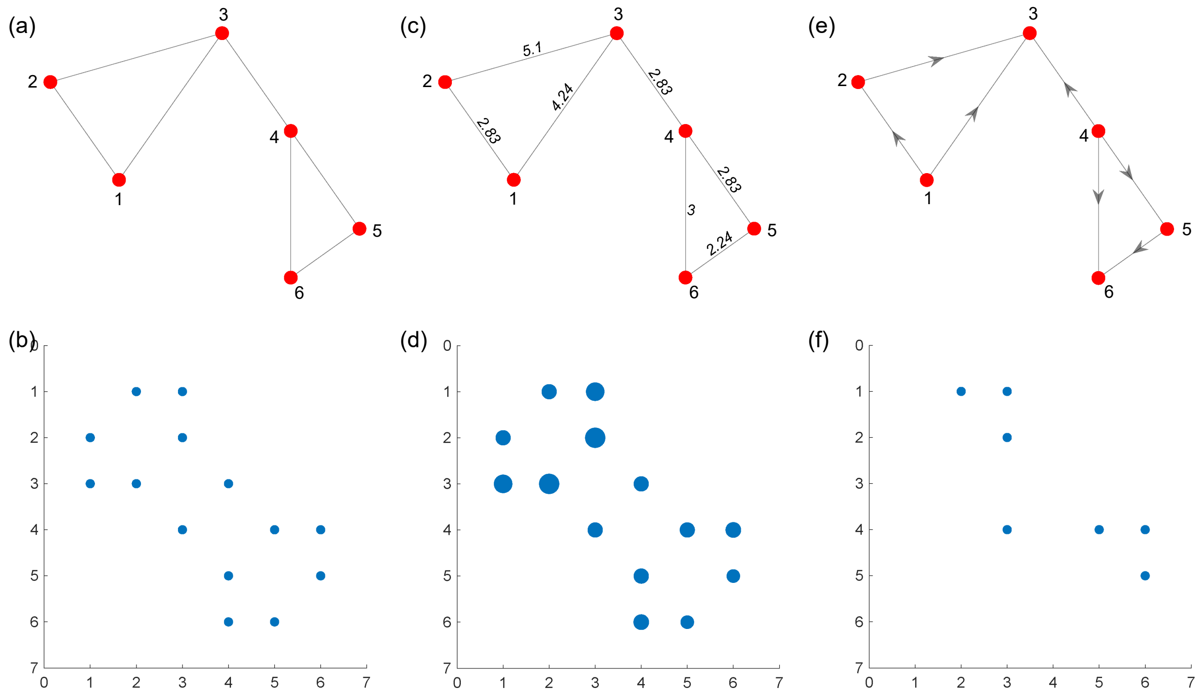

Figure 7: Examples of undirected, weighted, and directed graphs

Anon 10

Manuscript Image

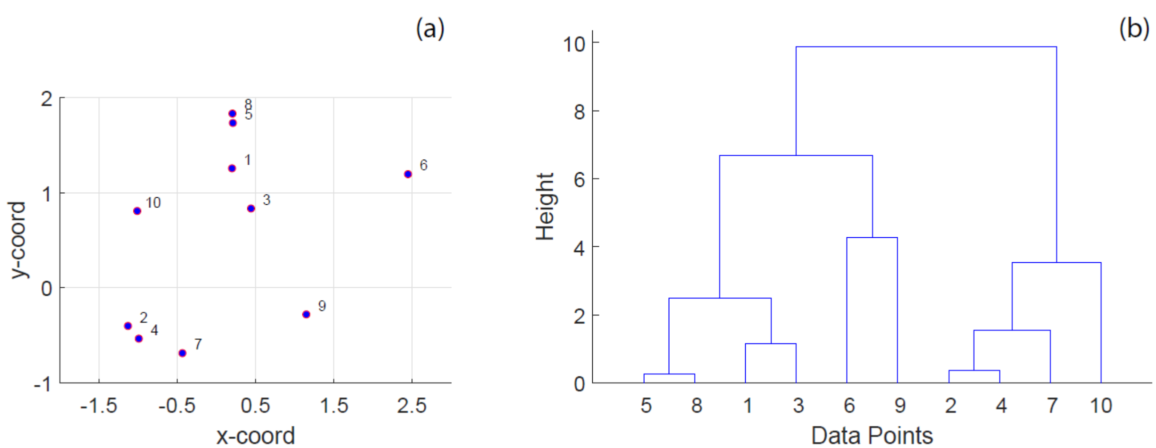

Figure 2. A simple example of hierarchical clustering using euclidean distance (a) 10 randomly positioned points (b) dendrogram computed from hierarchical clustering using the euclidean distance depicting clusters of the 10 individual points at different levels organized into a hierarchy

Figure 8: Fig.2 in first version of manuscript

Reviewer Comment *Figure 2: The construction of this figure is very difficult to understand until we read about Algorithm 1 below.*

Authors' Reply We modify the caption of the figure to refer to the Algorithm. We also briefly describe the figure in detail before the Algorithm is formally presented under Methods.

Anon 11

Manuscript Text Line 106: HC is an unsupervised statistical clustering method (Kaufman, 1990) ...

Reviewer Comment *Line 106: HC undefined acronym until Line 250, and even then it should be made explicit.*

Authors' Reply The acronym is expanded in Line 106.

Anon 12

Manuscript Text Lines 124 - 130:

– proximity and influence of faults explained by fluid-driven radial-jointing emanating from asperities within fault (Rawnsleyet al., 1998; Gillespie et al., 1993 etc)

– spatial variation of thicknesses of intercalated limestone and shale layers (Belayneh, 2004)

– proximity to high-deformation features such as folding (Belayneh and Cosgrove, 2004)

– interplay between regional and local stresses resulting in complex stress fields (Whitaker and Engelder, 2005)

– inheritance from spatial distribution of pre-existing vein / stylolite networks that influenced later joint network development (Wyller, 2019; Dart et al., 1995)

Reviewer Comment *Line 124-30: I would add synkinematic cementation (Hooker and Katz, 2015, Am. J. Sci.) to this list.*

Authors' Reply This reference is added in the revision. Another reviewer made a comment that our contribution is mainly methodological without seeking to explain the spatial variation particular to Lilstock. Hence, we add a sentence saying that we are simply reviewing some of the explanations for spatial variation in fracturing in Lilstock. The spatial variation maps that we generate using our approach can help in quantitatively identifying network variation, which needs to be used along side other approaches to satisfactorily explain the network variation.

Anon 13

Manuscript Text Line 124-125:

-proximity and influence of faults explained by fluid-driven radial-jointing emanating from asperities within fault (Rawnsley et al., 1998; Gillespie et al., 1993 etc)

Reviewer Comment *Line 125: "etc" inappropriate—if more studies have made the point and need to be listed, list them; else put "e.g." and then your chosen, representative references.*

Authors' Reply We remove "etc" from the sentence in the revised manuscript and add for e.g..

Anon 14

Manuscript Table

Table 1. Summary statistics for the three regions

Region	Approx. area (sq.m)	Fractures	Edges	Nodes
Region 1	6017	124006	364703	228661
Region 2	6749	141344	365333	235089
Region 3	1473	28892	78151	49771

Figure 9: Table 1 in Manuscript

Reviewer Comment *Table 1. Are these Edges and Nodes in the primal or dual graph sense?*

Authors' Reply The edges and nodes tabulated refer to the primal graph. The number of fractures in the table are equal to the nodes in the dual graph. We make this clear in the revised manuscript.

Anon 15

Manuscript Text Line 141: The long fractures in Regions 2 and 3 also exhibit large degrees of curvature, as can be seen when fractures are plotted based on logarithmic length bins.

Reviewer Comment *Line 141: I don't understand: how is curvature illustrated in these plots?*

Authors' Reply We were implying that Regions 2 and 3 have quite sinuous fracturing which is qualitatively observed from the networks and the photogrammetric images. To make a more quantitative comparison between the three regions, we introduce a metric that adds up the difference in orientation between each adjacent fracture edge in a tip-to-tip fracture. The slope of the summation of the strikes when plotted as a scatter against total fracture length is then an indicator of curvature and can be used to compare across regions. This plot depicted below is added to the revision. It may be observed that there is positive correlation for all regions. The slope in Region 1 is the least while that in Region 3 is highest. Region 2 containing observations with the most scatter indicating the presence of both curved and straight fractures.

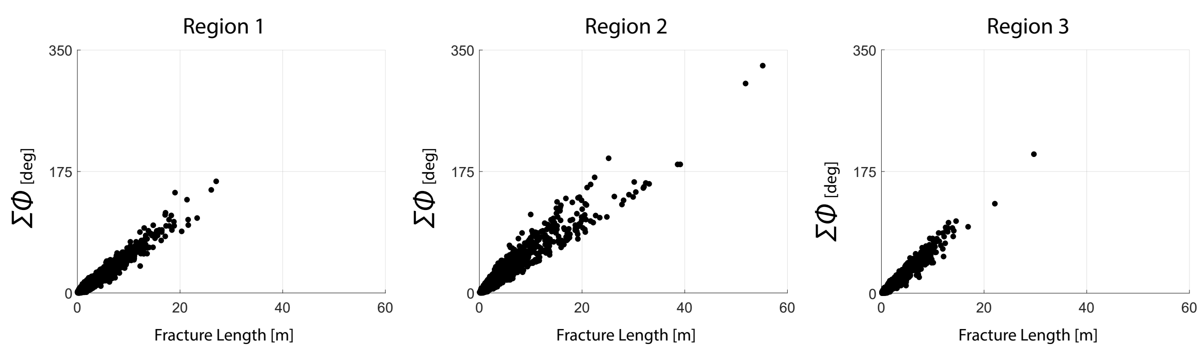

Figure 10: Comparing curvature across Regions

Anon 16

Manuscript Text Line 175 The value of ϕ is always smaller than 1, with larger values meaning greater regularity.

Reviewer Comment *Line 176: Greater regularity: can you justify this? A long, thin rectangular block would be highly "regular" and yet have a shape factor near zero.*

Authors' Reply The regularity that was implied here is the degree by which a block shape approaches a polygon with equal edge lengths. Such shapes would have the largest value of ϕ. We highlight this in the revision. We insert a figure to indicate the same.

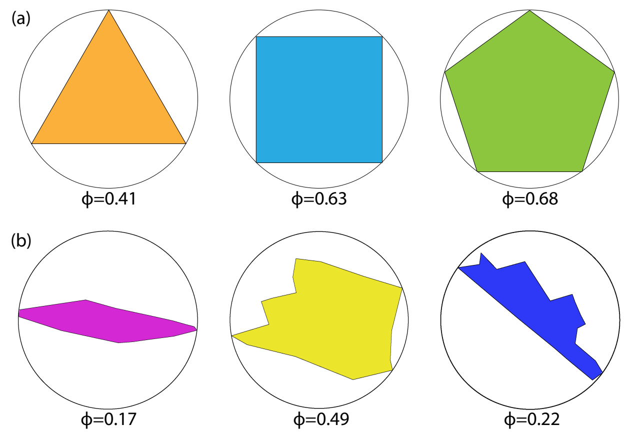

Figure 11: Illustration of shapefactors for a few regular shapes and block areas derived from the Lilstock data

Anon 17

Manuscript Text Line 179: The distribution of the block-face regions is binned logarithmically to integrate . . . .

Reviewer Comment *Line 179: "distribution of block-face regions"—you mean areas?*

Authors' Reply We change the wording in the revision to make clear that we are referring to block-face areas.

Anon 18

Manuscript Text Line 203-204: Schieber et al. (2017) defines NND, within the second term, as a measure of the heterogeneity of a graph w.r.t connectivity distances that capture global topological differences.

Line 424-425: The presence of these sub-regions can also serve as a guide to making decisions on stationarity w.r.t geostatistical modelling.

Reviewer Comment *Line 203: "w.r.t" seems needlessly curt; why not spell it out? Also 424.*

Authors' Reply This is expanded in the revision.

Anon 19

Manuscript Text Line 258: We visualize HC clusters using heatmaps of distance matrices, dendrograms, and spatial plots.

Reviewer Comment *Line 258: "HC clusters" redundant*

Authors' Reply The acronym HC is removed.

Anon 20

Manuscript Text Line 259-260: This plot along with weighted-sum of-squares plots enables picking of number of clusters and decisions on the height at which to cut the dendrogram.

Reviewer Comment *Line 260: "decisions on the height"—see major comment above about the arbitrariness of this decision. You put a tremendous amount of effort into quantifying aspects of the spatial arrangement, and then seem to make an arbitrary decision as to how to bin your clusters.*

Authors' Reply As explained previously under Anon 01, we now depict a range of dendrogram cuts that highlights how dendrogram branches bifurcate and the effects on sub-graph orientation. The "elbow plot" or "weighted-sum-of-squares plot" is generally used as an indicator for the cut-height based on the decreasing statistical significance as quantified by the similarity matrix. In our case, more granular division of clustering still depicts interesting variation and evolving development of cluster boundaries. It was not in the scope of this manuscript, but future work could involve more detailed comparison of sub-cluster with respect to effective permeabilities.

Anon 21

Manuscript Text Line 297: All four measures correctly identify the region of radial fracturing as described by Gillespie et al. (1993).

Reviewer Comment *Line 297: "correctly" a loaded term without more explanation.*

Authors' Reply This term is removed in the revision.

Anon 22

Manuscript Text Line 297-299: There are clear transition regions away from the influence of the fault located towards the SE of Region2, roughly following fracture persistence trend that progressively increases from SE to NW.

Line 315: For all the distance measures, there are clearly discernable clusters from the dendrograms.

Reviewer Comment *Line 298: I would omit "clear" and let the reader judge. Also "clearly" in 315.*

Authors' Reply This term is removed in the revision.

Anon 23

Reviewer Comment *Figure 18 caption: acronyms not used in figure?*

Authors' Reply The caption of the figure is modified in the revision.

Anon 24

Manuscript Text Line 342-343: The unsupervised statistical learning technique of HC was used along with graph distance metrics to extract spatial clusters.

Reviewer Comment *Line 342: "unsupervised"—see major comment above about arbitrary choices. The user has to make the call about what qualifies as a cluster here; I don't think this approach matches the spirit of unsupervised learning.*

Authors' Reply In the statistical literature, hierarchical clustering is "more unsupervised" than techniques such as k-means, since the latter requires an apriori number of clusters to be specified. In HC, the output is a hierarchy which the user can then make a call on the definition of a cluster. Some authors also refer to it as semi-supervised clustering since there is a known distance metric that is applied on the data. Since the usage may be confusing to the audience, we replace the phrasing of the sentence with "The statistical technique of HC was used . . . "

Anon 25

Manuscript Text Line 357: This is observed in Region 2, where clusters form roughly parallel to the E-N-E trending fault with . . .

Reviewer Comment *Line 357: "striking" for "trending"*

Authors' Reply "Trending" is replaced with "striking" in the revision.

Anon 26

Manuscript Text Line 359-360: The direction of cluster variation trends E-W. Region 1 is not affected by faulting and the network differences can be interpreted as background-variation.

Reviewer Comment *Line 360: "background-variation"—such an interpretive term, especially considering the only external clustering control you have eliminated, to some degree, is faulting.*

Authors' Reply Yes, we agree with the reviewer that the use of the term background-variation can be ambiguous. We remove this term in the revision.

Anon 27

Manuscript Text Line 385-386: Incorporating both length and strike into a single scalar can be done using a normalized dot product, and we will tackle this issue in future work.

Reviewer Comment *Line 386: "we will tackle"—this is inappropriate. Anyone who wants to extend your work is entirely free to do so. I am sure you are not intentionally "marking your territory," but we need to be especially cognizant of this kind of thing, especially in these days of pre-prints, self-archiving, predatory journals, and similar avenues whereby unscrupulous workers can "jump the gun" on an idea before proper peer review.*

Authors' Reply This point was also raised by another reviewer and we were trying to flag the audience to the way graphs are represented in this contribution. The primal graph representation using weighted graphs that we use for the graph similarity computation does not explictly encode orientation information. Hence, if we consider a square lattice and rotate it by 45 degrees, the graph similarity measure would not detect such a difference. Hence we proposed the idea of normalizing the edge weights using a combination of both length and orientation. Since we did not attempt this in the course of this work, we remove this sentence in the revision. As we show in the latest figures under Anon 01, the method is able to identify variations in orientations even when orientation is not explictly included as primal graph edge weights.

Anon 28

Manuscript Text Line 391: These methods are often unable to represent inherent non-stationarity in spatial variation (Thovert et al., 2017) ...

Reviewer Comment *Line 391: "inherent non-stationarity" see comment on Line 57*

Authors' Reply We feel that this context is different from our previous reference to non-stationarity in fracture networks in Line 57. In the previous case we were referring to ground observations that in the case of fractures, simple motifs or patterns do not replicate in a regular fashion forming a network. It is more common to see network spatial distribution vary quite irregularly.

In Line 391, we are making the point that in conventional geostatistics, spatial data are classified as continuously varying data, point data, and gridded data. Many spatial statistics procedures are well-developed for these three specific data types, but in the case of spatial network data it is not the case. The work of Thovert et al. (2017) use point process-based approaches to create DFNs while the work of Bruna et al. (2019) treat fractures as a facies types distributed as regularly-gridded pixels. Both the approaches try to recreate spatial variation without explicitly using a spatial network representation. Our contribution is directed to addressing the issue of spatial variation through the spatial graph representation and the reference to stationarity is for network data that does not fall into the three general classes of spatial data.

Anon 29

Manuscript Text Line 392: ... and work by Andresen et al. (2013) find that DFNs from nature exhibit disassortativity, which is not a property of generated networks.

Reviewer Comment *Line 392: define "disassortativity"*

Authors' Reply Since this is an important property of naturally occuring networks we briefly define dissortativity with a reference to Newman (2002) in Section 2.1 of the revision. Disassortative networks have a negative assortativiy coefficient. In the case of the three spatial networks we considered, the assortativity coefficients for the graphs in the dual form are negative. This is also reported by Andresen et al. (2013) and Vevatne et al. (2014) and this information is added to Section 2.1.

Anon 30

Manuscript Text Line 397: Regardless of the method used to extrapolate, stationariness decisions have to be made based on hard data, and this is where our approach is helpful.

Reviewer Comment *Line 397: "stationarity" for "stationariness"?*

Authors' Reply We replaced this term in the revision.

Anon 31

Manuscript Text Line 397: Regardless of the method used to extrapolate, stationariness decisions have to be made based on hard data, and this is where our approach is helpful.

Reviewer Comment *Line 397: "have to be made based on hard data"—see major comment on arbitrarily choosing the number of clusters, above. Have you made your decisions based on hard data?*

Authors' Reply In this work, we do not address the network extrapolation problem which in conventional geostatistics requires an assumption of stationarity. The hard data that we are referring to in this context are large-scale networks that are extensive and spatially continuous enough to be able to identify the spatial variation. We present our work as a graph-based method to identify variation in natural networks which can be then be used to guide decisions on stationarity to address the network extrapolation problem.

Anon 32

Manuscript Text Within the graph theory literature, there are other non-HC methods based on graph properties such as modularity (Blondel et al., 2008; Traag et al., 2019) or by graph spectral partitioning (Fiedler, 1973; Spielman and Teng, 2007).

Reviewer Comment *Line 408: define "modularity"*

Authors' Reply Modularity is a quantitative measure introduced by Newman and Girvan (2004) to identify possible clusterings within a graph. Modularity represents a fraction of graph edges connecting nodes within a cluster subtracted by an expected fraction if the edge distributions between nodes are random. Maximum possible modularity of 1, indicates networks with strong community structure. Blondel et al. (2008) and Traag et al. (2019) present fast computational procedures to compute graph clustering based on the modularity measure. Since the work of Blondel et al. (2008) and Traag et al. (2019) is based on Newman and Girvan (2004), we insert this reference as well in the revision so that the reader can easily refer to the original sources.

Anon 33

Manuscript Text Recent developments using graph neural networks and graph machine learning include modifications on the concept of modularity (Tsitsulin et al., 2020) and spectral methods (Bianchi et al., 2020) towards the goal of graph partitioning.

Reviewer Comment *Line 411: What is graph partitioning, and is that a worthy goal?*

Authors' Reply Graph partitioning is an interchangeable term for graph clustering. We make this clear in the revision.

Anon 34

Manuscript Text Line 418: . . . identify spatially-similar regions using the statistical technique of hierarchically clustering.

Reviewer Comment *Line 418: "hierarchical" for "hierarchically"?*

Authors' Reply We have corrected the spelling in the revision.

References

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