se-2021-45 Reply to Reviewers: RC2

Rahul Prabhakaran, Giovanni Bertotti, Janos Urai, David Smeulders

28-7-2021

Dear Prof.Dr.Sanderson,

We thank you for the detailed review of our submission. Please find below responses to the specific comments. We hope that we have sufficiently addressed all points that were raised.

Kind Regards

Rahul Prabhakaran (On behalf of all authors)

DJS 01

Reviewer Comment *This paper is an interesting contribution to an important topic – the analysis spatial variation in fracture networks. The paper introduces some methods, established in other areas but new to this area of earth science. The work is based on some interesting field data, and is both well written and illustrated. I recommend publication, but offer the following comments (see also comments on early sections in annotated pdf).*

Authors' Reply The authors would like to thank the reviewer for the detailed review of the manuscript.

DJS 02

Reviewer Comment

Section 1,2

The authors provide a concise, clear background to the treatment of fracture networks as graphs (Section 2). The level of explanation is appropriate for the rest of the paper, but I highlight, in an annotated version of the manuscript, a few areas that the authors might want to clarify or omit.

Authors' Reply We modify the Introduction section to address the issues raised by the reviewer in the annotated manuscript. The modifications that are made in the revised version are described in further detail below.

DJS 03

Manuscript Text Line 30: Regardless of how fractures' spatial arrangement is defined, quantitative analysis of spatial arrangements invariably leads to quantification of spatial variation.

Reviewer Comment *This is a somewhat convoluted sentence and add little - omit.*

Authors' Reply This sentence is omitted in the revision.

DJS 04

Manuscript Text Line 66-67: In fracture networks, the vertices are intersections between fractures and the edges represented by fracture segments connecting the vertices.

Reviewer Comment *This is only true for the node/branch models (e.g. Sanderson and Nixon 2015) but not for other representations (e.g. Andresen et al 2013).*

Authors' Reply We make clear in the revision that fracture intersections as graph nodes is the representation chosen by Sanderson and Nixon (2015) and the alternate representation of tip-to-tip fractures as graph nodes is preferred by others such as Andresen et al. (2013).

DJS 05

Manuscript Text Line 66-68: By assigning positional information to the vertices (also nodes), fractures in the form of graphs encapsulate both topological and spatial information.

Reviewer Comment *(e.g. Sanderson et al. 2019)*

Authors' Reply We add this reference to the sentence in the revised manuscript.

DJS 06

Manuscript Text Line 70-71: The degree of a graph node is simply the number of edges that intersect the particular node.

Reviewer Comment *incident at a*

Authors' Reply We modify this sentence using the suggested changes.

DJS 07

Manuscript Text Line 73-74: As can be seen in the case of the primal graph in Fig.1.c, the maximum node degree is 6, with the most common degree value being 3.

Reviewer Comment *In fracture networks the degree is almost ubiquitously 1, 3, or 4, with higher degrees almost always arising due to problems in resolving closely spaces nodes.*

Authors' Reply Yes, we agree. The uneven erosion in some locations in the Lilstock pavement make resolution of closely spacely nodes difficult. We insert the following two sentences in the revision, "It may be noted that node degrees in spatial graph representations of fracture network is most likely to be 1,3 or 4. For fracture networks interpreted from outcrop images, eroded fractures and enlarged apertures may lead to higher degrees due to issues in resolving closely spaced nodes."

DJS 08

Manuscript Text Lines 73-75: In the case of the dual graph, as depicted in Fig.1.d, the maximum degree can be much higher, and the longest fractures that have the highest number of intersections also have the highest degree. Andresen et al. (2013) and Vevatne et al. (2014) suggested that fracture networks are therefore disassortative in that shorter fractures preferentially attach on to the longer fractures.

Reviewer Comment *The use of the term "dual" is NOT that use in mathematics of Graph theory, where the dual involves interchange of nodes and regions, rather that nodes and branches/edges.*

Authors' Reply Yes, we agree. The use of the term "dual" was popularized by Marc Barthelemy's group in the context of spatial networks/graphs. Since the usage has a different meaning in the mathematics of graph theory we make this clear in the revision by modifying the sentence as "In the case of the alternate representation, referred to as dual graphs by Barthelemy (2018), and depicted in Fig.1.d, the maximum degree can be much higher, and the longest fractures that have the highest number of intersections also have the highest degree."

DJS 09

Manuscript Text Line 88-91: Graph similarity may be differentiated from graph isomorphism in that the latter comparison can only return a binary outcome. An isomorphism test on two graphs G1 and G2 can only yield two results, either isomorphic or not. Graph similarity on G1 and G2, on the other hand, should return a real-valued quantity that converges to zero when the two graphs approach isomorphism (or complete similarity).

Reviewer Comment *The comparison of "similarity" (as used in this paper) with "isomorphism" is unclear from this sentence, especially as the authors do not define "isomorphism".*

Authors' Reply In the revision, we add a simple definition of isomorphism and also a reference to the book of Van Steen (2010) for the interested audience.

DJS 10

Manuscript Text Line 108: HC is an unsupervised statistical clustering method. . . .

Reviewer Comment *Hierarchical clustering (HC)*

Authors' Reply The acronym is expanded in the revision.

DJS 11

Manuscript Text Lines 124 - 130:

– proximity and influence of faults explained by fluid-driven radial-jointing emanating from asperities within fault (Rawnsley et al., 1998; Gillespie et al., 1993 etc)

– spatial variation of thicknesses of intercalated limestone and shale layers (Belayneh, 2004)

– proximity to high-deformation features such as folding (Belayneh and Cosgrove, 2004)

– interplay between regional and local stresses resulting in complex stress fields (Whitaker and Engelder, 2005)

– inheritance from spatial distribution of pre-existing vein / stylolite networks that influenced later joint network development (Wyller, 2019; Dart et al., 1995)

Reviewer Comment *This list is a bit of a "straw man" and is very incomplete. There are many papers on these joints, and a discussion of how they relate to folds, faults, veins burial, uplift, etc. Do you really want to include this? How does your study contribute to the discussion.*

Authors' Reply The paragraph was an attempt to explain some hypotheses postulated by previous authors who have worked in the specific location. Since an in-depth explanation of the reasons behind the spatial heterogeneity deviates from the scope of this manuscript which is more methodological in its aims. Hence in the revised manuscript, we will convert this list into a simplified sentence so as to simply direct the reader to an unexhaustive set of works.

DJS 12

Manuscript Text Line 141-142: The distribution of joints within a particular length bin is also highly variable.

Reviewer Comment *This is the first mention that the fractures are "joints".*

Authors' Reply We make it clear in the start of the Section 3 Fracture Datasets, that the Lilstock data is a joint network system.

DJS 13

Manuscript Text Line 152-153: . . . we remove all edges from the sub-graphs emanating from degree-1 nodes that contact the periphery of the circular sample.

Reviewer Comment *Is this really what you did? The removal of these peripheral edges would convert some 3-nodes (Y) to 2-nodes, unless you also remove the nodes (i.e. produce an edge-induced sub-graph). This is unclear from Fig. 6.*

Authors' Reply Yes, the removal of edges create degree-2 nodes which was retained. The objective here was to prevent node degree-distributions to be dominated by I-nodes within the sub-graphs. We make this clear in the revision.

DJS 14

Manuscript Text

Reviewer Comment *Section 3*

The introduction to the field area is similarly clear and concise. There is no mention of a recent paper by Procter and Sanderson (2017) that discusses the spatial variability of fractures in the same geological units, just a few kilometres to the west. This paper not only uses graphs to represent the network, but also provides a statistical evaluation of the between layer and within layer variability of fracture intensity.

Authors' Reply We include this reference in the revision. The Kilve outcrops are similar to Lilstock in tectonic origin and the methodology is also of interest to the reader in the context of spatial heterogeneity in same geological units. We add the following sentence in the revision, "Recent work on fractures at the Kilve outcrop (Procter and Sanderson, 2018), exposing the same geological units as that of the regions of interest depicted in Fig.3, conclude that anomalous fracture intensity exists in fracturing at various locations and suggest that variability in fracture intensity cannot be fully explained by variations in thickness, compositional, or textural variations."

DJS 15

Reviewer Comment *Section 4: Methods*

Section 4.2 discusses a range of graph measures used in the subsequent hierarchical clustering. There is a lot of technical detail in the definition of these measures, which is difficult to follow but the sources are all clearly stated. What would be most helpful to the reader would be an evaluation of what each measure is contributing in terms of the geometry and topology of the network. For example:

- The "fingerprint measure" clearly defines a block "shape", both in terms of the number of sides and aspect ratio of the overall shape. Given that most block have 4-6 sides, an average aspect ratio would a little less than 2 and I would expect that this parameter would mainly be reflecting variation in aspect ratio.

- The D-measure is mainly based on the clustering of the node distribution. Given that the divergence and alpha centrality seem to vary little, I would think this measure mainly reflects variation in the node intensity, which seems to be supported by Fig. 14.

It would be good to have a similar evaluation of the other measures. In particular, it is not clear to me how the variation in fracture orientation is captured by these measures. Since the distribution of sets with differing orientation is a major feature of at least two of these regions (Passchier et al 2021), it is surprising that this aspect is omitted from description of the measures and appears to play little role in the clustering.

Authors' Reply

- We insert a new figure explaining the aspect ratio variation for a few regular shapes and for some fracture block areas in Section 4.2.1 of the revision.

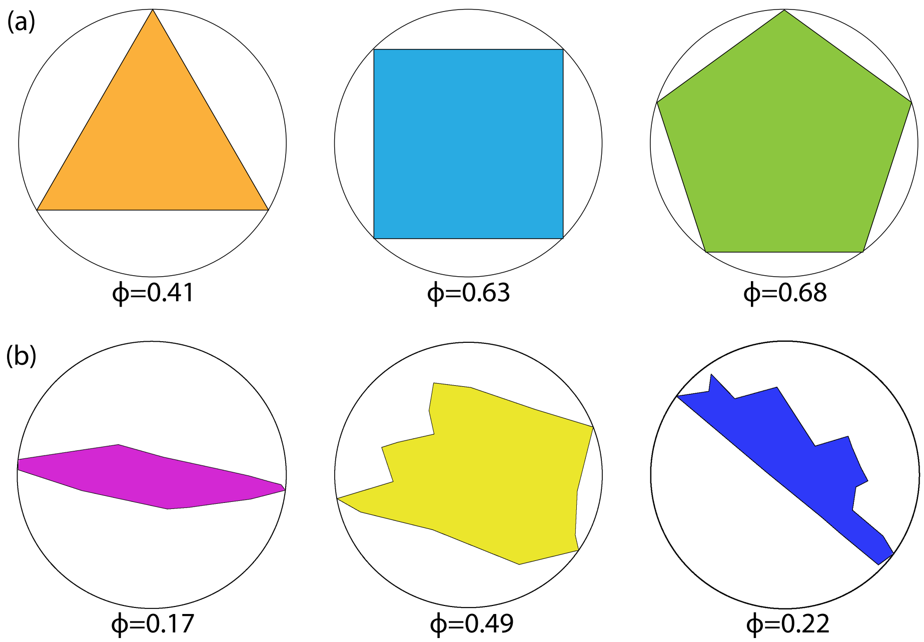

Figure 1: Illustration of shapefactors

- Within Section 4.2.2, we illustrate the distribution of NND, alpha centrality, and the network node dispersion for the same subgraph as depicted in Fig.7(c) using a new figure.

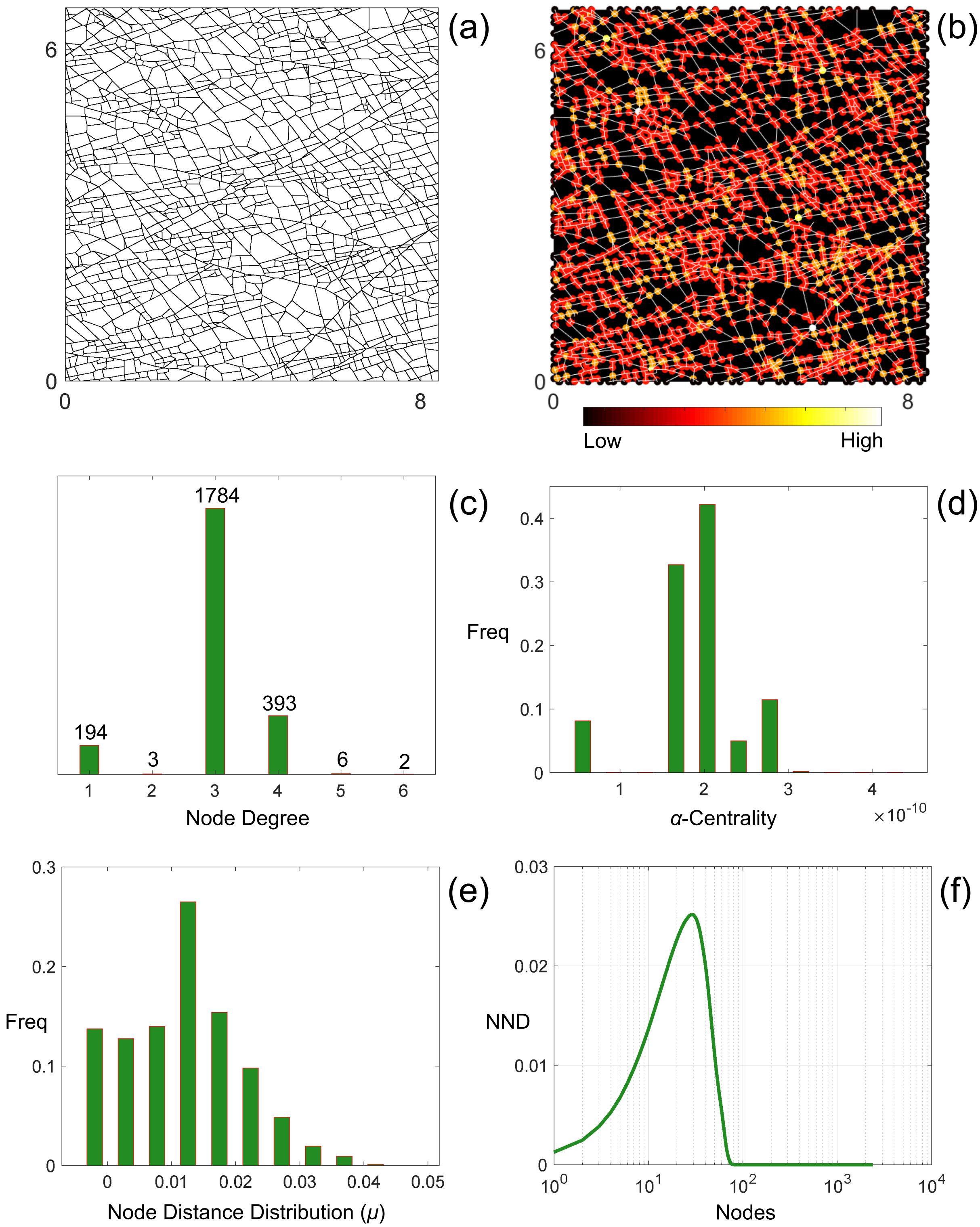

Figure 2: Network properties involved in computing D-measure applied to the example fracture graph

- For the portrait divergence and the NetLSD, we plot network portraits and heat traces for the same subgraph depicted in Fig.7(c). This is inserted in Sections 4.2.3 and 4.2.4 respectively.

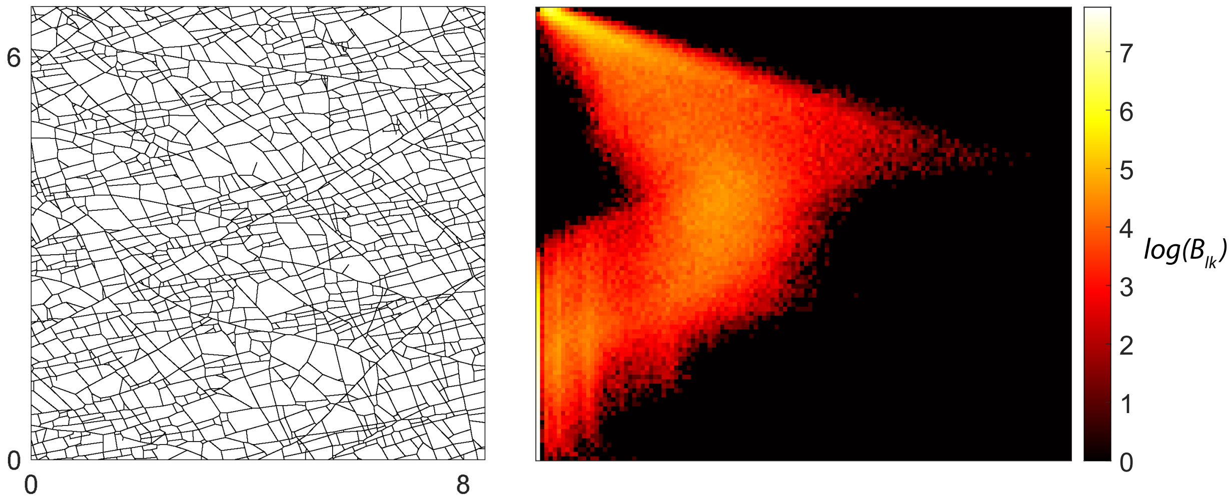

Figure 3: Network portrait of the example fracture network depicted as a heatmap

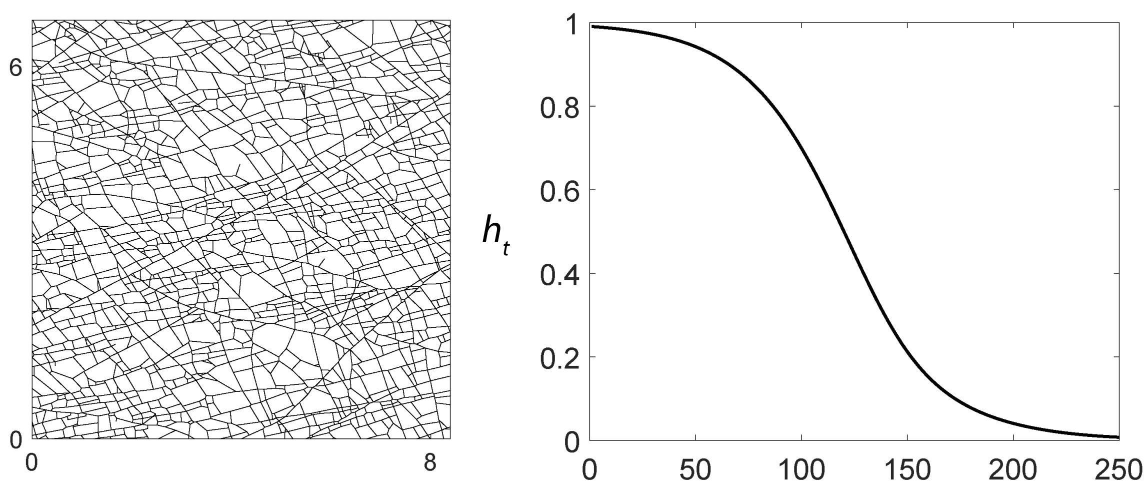

Figure 4: Heat trace signature vector for the example fracture graph

We do not incorporate fracture orientation directly in to the edge weights but only the segment lengths. The work of Passchier et al. (2021) did not consider fully traced networks but focussed on picking tip-to-tip sets manually. Despite not having considered orientation explicitly within the graph edge weights, the clusters are still able to identify differences in orientation. Examples of the variation of orientations pertaining to clusters for the similarity measures for the three considered regions are depicted below and also added to the revision. (Note: We exclude NetLSD for the lack of spatial autocorrelation in the results.)

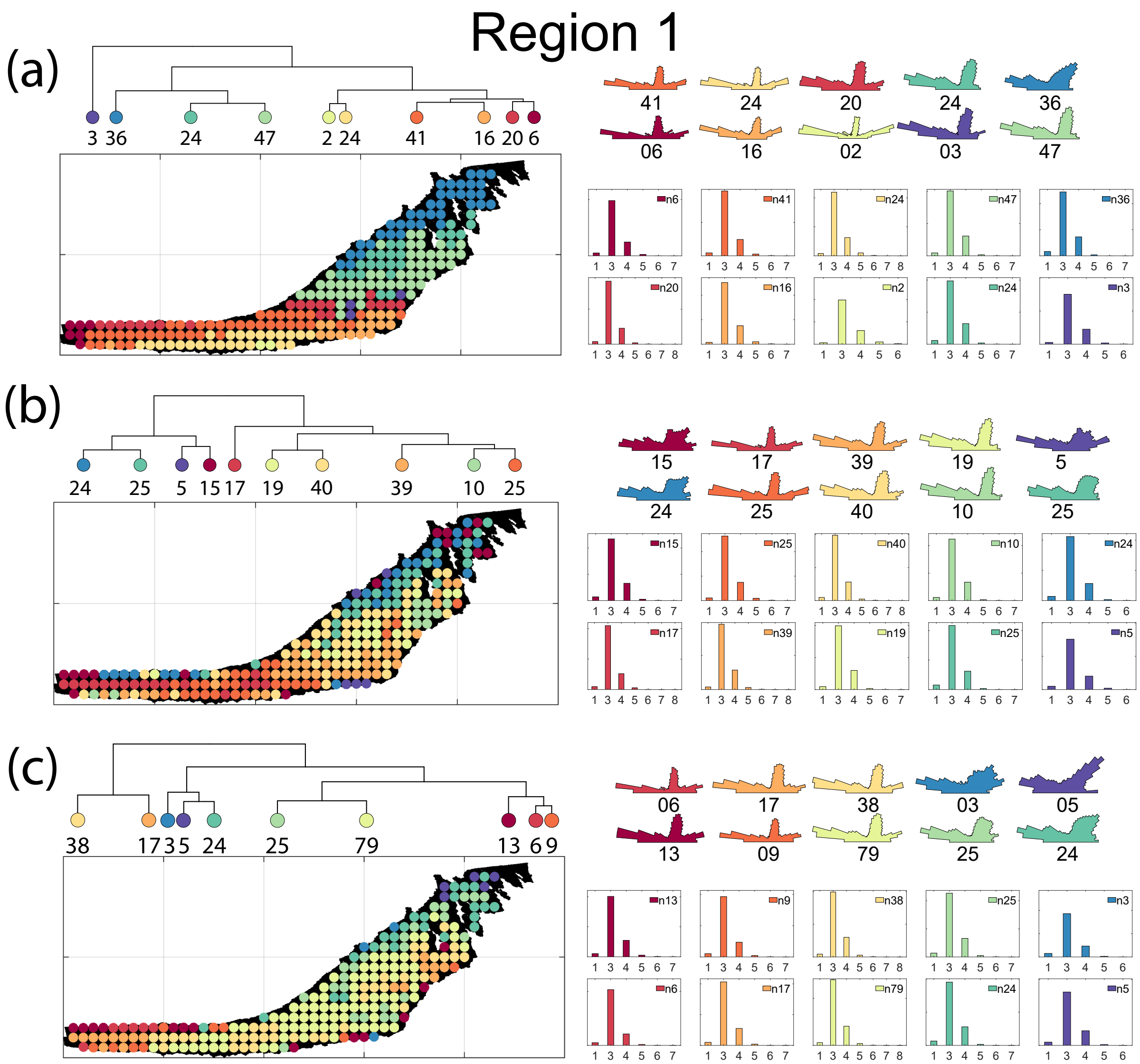

Figure 5: Fracture orientations and topological summaries for clusters Region 1 (a) Fingerprint (b) D-measure (c) Portrait Divergence

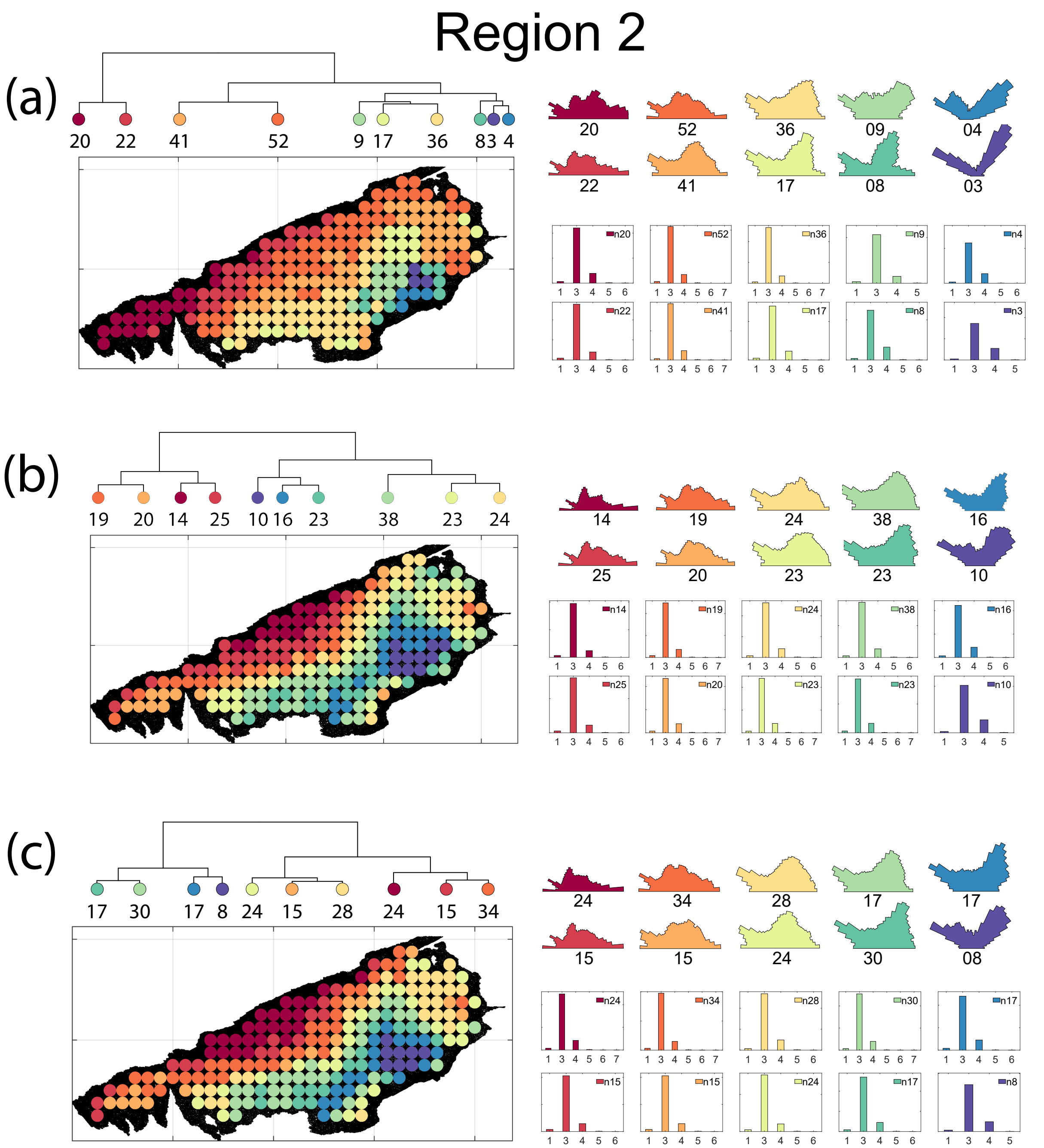

Figure 6: Fracture orientations and topological summaries for clusters Region 2 (a) Fingerprint (b) D-measure (c) Portrait Divergence

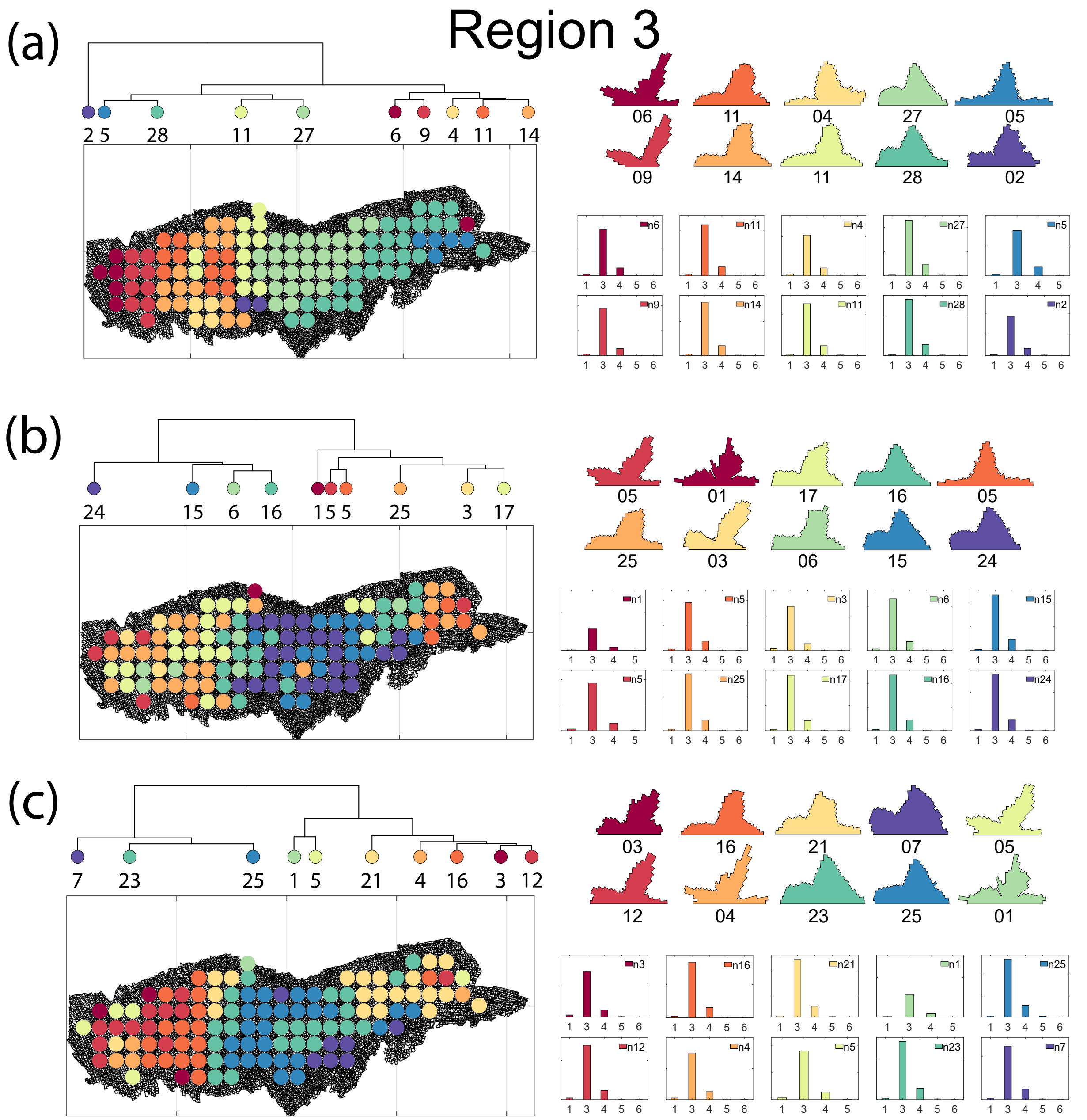

Figure 7: Fracture orientations and topological summaries for clusters Region 3 (a) Fingerprint (b) D-measure (c) Portrait Divergence

DJS 16

Reviewer Comment

Section 5 Results

This provides a detailed analysis of three regions and presents the results of the mapping of spatial variability in terms of the HC of the measures used. The results are presented through sets of five similar diagrams for *each of the three regions. Some of the material in these diagrams could be transferred to "supplementary material" (e.g. heat maps and dendrograms of the clustering).*

It is probably worth comparing the characteristics of each region. From Table 1, the number of fractures/branches/nodes, with similar ranges of fracture intensity (P21) in Figs 9, 14 and 19. The average degrees are also very similar (~3.15) indicating a close approximation to a 3-regular graph (or mesh). Apart from the "fingerprint" of block shape (Fig. 7e), there is no information on the distributions of the other measures used in the clustering.

It seems to me, that Region 2 illustrates the strengths and weaknesses of the methodology, so starting with a complete analysis of this area would make sense. The other two regions could then be treated more concisely, with emphasis on what they contribute to the study.

Authors' Reply

The plots depicting the distance matrix heatmaps, dendrograms and w.s.s plots are moved to an Appendix A in the revision. Based on the suggestions of another reviewer, we depict spatial clustering variations for different levels of dendrogram cuts rather than just 5 clusters to highlight the development of new clusters and evolving cluster boundaries as one goes deeper into the dendrogram. An example for such a result for the portrait divergence similarity corresponding to Region 2 is depicted below.

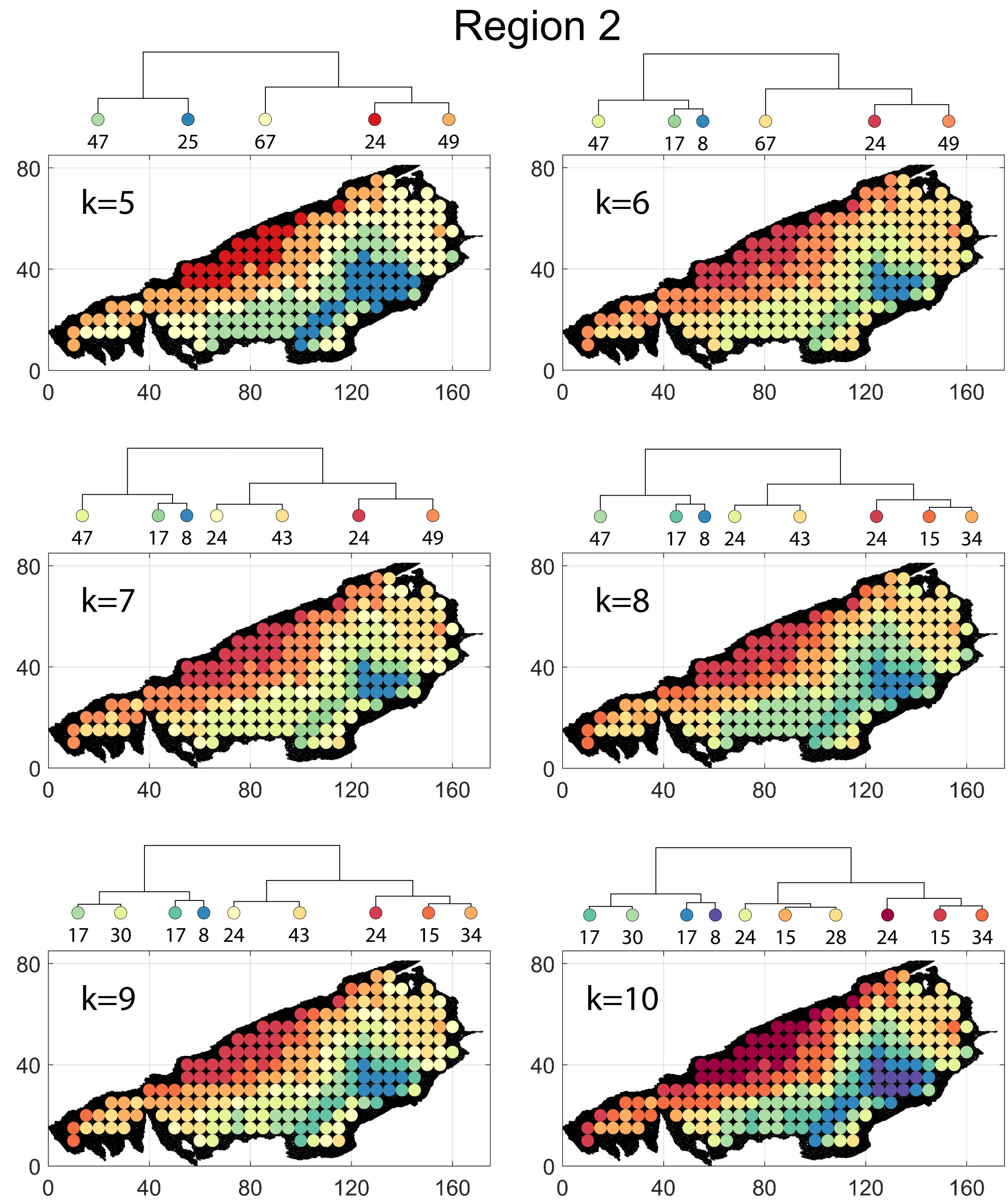

Figure 8: Region 2 Clustering using Portrait Distance at multiple dendrogram levels

Similar plots for the other regions and similarity measures are added to an Appendix B. The zoomed-in section plots depicting variation in fracturing pertaining to sub-graphs from the identified clusters are placed in Appendix C. We think this arrangement both reduces the number of figures in the main text of the Results and also makes it convenient for the readers who can refer to Appendices rather than a Supplement.

Full region properties are easily computed for the fingerprint measure as depicted below and is also inserted in the revision.

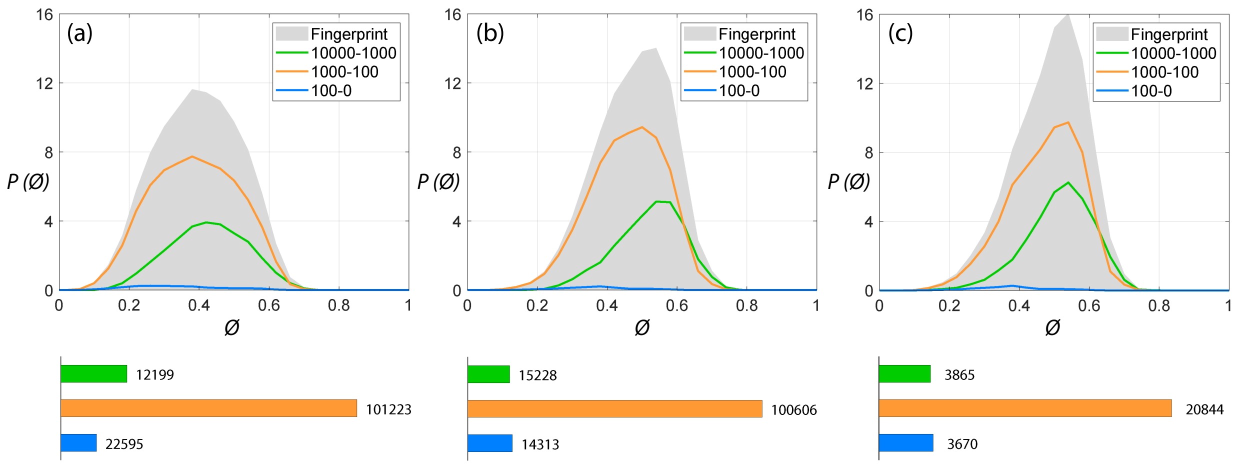

Figure 9: Fingerprint plots for the three regions (a) Region 1 (b) Region 2 (c) Region 3

However, for the other three similarity metrics, full region comparison is computationally difficult as the graphs are too large. We could perhaps insert ensembles of sub-graph properties as is depicted below for heat traces computed using NetLSD. In the revision, we can extend this to D-measure properties and network portraits.

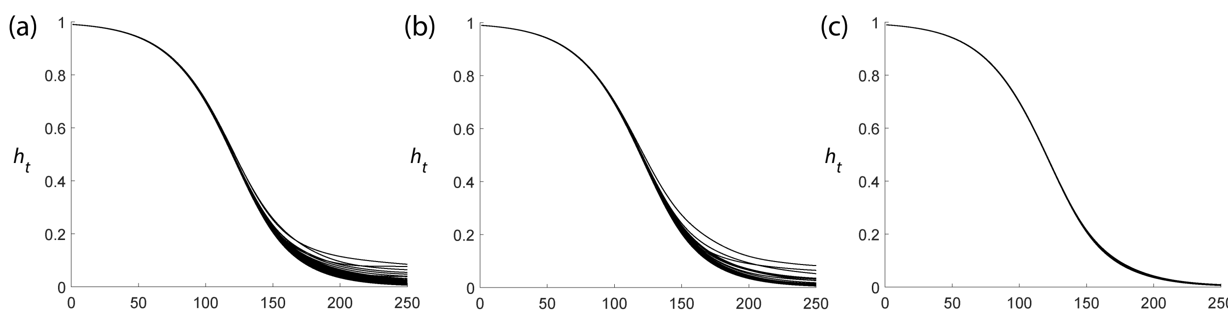

Figure 10: Heat trace vectors for all constituent sub-graphs corresponding to (a) Region 1 (b) Region 2 (c) Region 3

DJS 17

Manuscript Text Line 378-386: As may be observed from our results, the metrics highlight certain aspects of the fracture network while not considering others. For instance, the fingerprint distance only considers block area and shape factor distributions of the blocks and neglects orientations. The other three distances use graph properties directly, and hence orientation information (or the lack of it) is a consequence of how the spatial graph is defined. We used weighted graphs that incorporate euclidean distance between nodes as edge weights for the similarity computations. However, each edge also has a striking attribute to completely describe its position in 2D space (in the case of 3D, it needs a dip). Ideally, the edge weight should then be a vector, $w = [l,\theta]$ incorporating both lengths, 'l' and orientation, 'θ', but the distance metrics we use do not allow the use of non-scalar weights. Incorporating both length and strike into a single scalar can be done using a normalized dot product, and we will tackle this issue in future work.

Reviewer Comment *In the section on choice of graph metrics, the paragraph from lines 378-386 is key to this paper. An unweighted graph cannot contain information on geometry, since a graph is invariant to change in shape and size. Thus, geometry must be represented through the embedding of the graph in a (geographical) space or by including geometrical measures in the weights. The former allows specification of orientation and length, and measure of the frequency and intensity of elements to unit area. The way this paragraph is written implies that length and orientation could be incorporated but were not in the present study – this is "shooting one's self in the foot".*

Authors' Reply In our work, the graph edges are weighted using geometric lengths of the fracture segments. We have not incorporated orientation as a weighting parameter for the edges. As pointed out earlier under DJS 15, despite not using orientation explicitly in the edge weights, the similarity measures are still able to identify variation in orientations. A simple dot-product as a scalar weight is not straight-forward as the weights would not have a clear minimum or maximum, as the weight would combine circularly distributed data (orientations) as well as data that follow a negative power-law (segment lengths). The last sentence was intended to convey this difficulty, but since the wording is confusing we remove it in the revision.

DJS 18

Reviewer Comment *Conclusions and Abstract*

Both these sections are written in a vague way, expressing the aims and aspirations of the study, instead of focusing on the main findings.

Authors' Reply We rewrite these sections to better focus on the key findings.

References

Andresen, C., Hansen, A., Le Goc, R., Davy, P., and Hope, S.: Topology of fracture networks, 1, 7, https://doi.org/10.3389/fphy.2013.00007, 2013.

Barthelemy, M.: Morphogenesis of spatial networks, 2018th ed., Springer International Publishing, https://doi.org/10.1007/978-3-319-20565-6, 2018.

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Procter, A. and Sanderson, D. J.: Spatial and layer-controlled variability in fracture networks, 108, 52–65, https://doi.org/10.1016/j.jsg.2017.07.008, 2018.

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Van Steen, M.: Graph theory and complex networks: An introduction, 1st ed., 2010.