Reply to Anonymous Referee # 2 comments

Dear reviewer,

Thank you for the detailed and thoughtful comments. We took all your comments into consideration. We have copied and numbered your comments and addressed them point-by-point below.

Sincerely,
Mahtab Rashidifard, Jeremie Giraud, Mark Lindsay, Mark Jessell, Vitaliy Ogarko

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RC2_1: This paper describes an application of a level set method to the problem of reconstruction density from gravity data in 3D with additional constraints obtained from 2D seismic survey. The constraints are implemented as a regularization penalizing the evolution of the level set functions where the model is deemed to reliably constrain by from prior information, in this case an inverted 2D seismic section. The method is applied in synthetic examples and to a field data set from Yilgarn Craton in Western Australia.

I think the main contribution of this paper is the adaptation of the methodology to the specifics of the area studied in the field data example and the specific kinds of data available there. I recommend accepting the paper with some revisions.

Reply: We would like to commence by thanking the referee for providing a constructive and exact review of the manuscript, as shown by this precise summary.

RC2_2: Comments on the substance of the paper:

I think the theory section is adequate, considering that a previous paper by the authors where the methodology is explained in more details is referenced. Examples section is adequate in scope, however some results were a bit difficult to interpret. I believer that several figures might benefit from editing and some additional comments would help.

Reply: Thank you for your feedback about the substance of the manuscript. We have made some modifications to the example section and made the results clearer based on your comments.

RC2_3: Specifically, I have the following suggestions:

RC2_4: Lines 106-108: The meaning of the sentence is hard to understand, I suggest reformulation for clarity. Gravity level set inversion actually is quite well-posed when gravity values are known and does not need much regularization, in my experience. When minimum length/area regularisation is used, usually it does not lead to overly simplified shapes when the data constrain the problem well.

Reply: Thanks for the hint. The mentioned sentence has now been reformulated in the text (line 117-119:) “Minimizing the length of the geometries, generates shapes with the smallest area and regularizing these inversion problems can be limited to the specific shape of units that can introduce a bias towards unrealistically simple geometries.”

This is a fair point. In the referenced level-set inversion methods (Li et al. 2016, Li et al. 2017, Zheglova et al. 2018) using known constant physical property values has already reduced the non-uniqueness of the inversion problem however, non-uniqueness of problems are even reduced more using prior information in additional regularization terms. In the introduced approach we follow a different strategy for the application of regularizations. In this manuscript, we use area-specific regularizations, which require minimum updates of the model (not minimum structure as per the works referenced above). What we enforce as constraints in this study comes from a higher resolution method and it might, on occasions, violate the minimum area constraint in cases where seismic reflectors show a non-smooth surface while being perfectly valid geophysically.

RC2_5: Lines 125-126: It would be nice to reformulate the sentence, because its meaning is hard to understand.

Reply: The mentioned sentence is now re-written as (line 134-135:), “We show that by updating global and local regularization terms with low-uncertainty information from seismic datasets, they can act as global and local constraints.”

RC2_6: Line 129: What exactly does “uniform” mean here?

Reply: Thank you for noting this detail. It was a mistake to use the term “uniform” in the text. The word is deleted in the revised manuscript.

RC2_7: Line 135: Could you please elaborate a little bit on the difference of the effect of Ws and Wp?

Reply: We have updated and modified the theory section based on Referee_1 comments to elaborate more on the effects of the constraints. We have emphasized that W_s is a constraining term that includes all rock units at once while W_p considers each rock unit separately. We have added this to section 2.3 based on your suggestion (from line 140 to 142:) “We use global regularization term to encapsulate the information about all lithologies in one vector while local terms are defined to include different lithologies separately in the inversion problem.”

RC2_8: Lines 141-142: Could you please make it clearer, what is meant by "arrays", also what the "sample section" means here.

Reply: Thank you for pointing that we have not used understandable terms in this section. The mentioned sentence is explaining part of Fig. 1. In the revised manuscript we have updated Fig. 1 so we have updated the corresponding explanation in the text (from line 153- 154 :) “All parts of the 3D model that lie within the constraining section are weighted accordingly”. In this figure, it

is assumed that within a 3D model, there is a section (colored section) that is going to be used for constraining. So in this sentence, the message is that parts of the 3D model that overlays the 2D section are given different weights for the constraining purpose.

RC2_9: Lines 148-150: The meaning of the sentence is not clear.

Reply: The mentioned sentence is reformulated in the new text after updating Fig. 1 (line 161-162:), "As shown in Fig. 1b, in a 3D volume with the same size as the model, the extracted section along the seismic profile is weighted accordingly for lithology 2."

RC2_10: Line 165, Figure 1: The top two images on this figure need labels, captions and more explanation. How do the bottom figures, especially figure 1b fit into the right top image? Why do images (a) and (b) show two different plots of the same matrix Ws2?

Reply: We agree that Fig. 1 needs modification. This figure has been updated and the corresponding explanations have been reformulated in a clearer fashion in the revised manuscript. In the manuscript in the method section (lines 166 -171 of the revised manuscript) it has been already explained that why two kinds of W_{s2} can be defined. This should be clearer in the revised text given that Fig. 1 has been updated.

RC2_11: Section 2.4. As far as I understand, this regularization prevents small pieces of one lithology to be isolated inside another lithology, reducing fragmentation of the model, but it's not quite clear why such a constraint needs to be applied. What is meant by "nucleation"?

Reply: Thank you for the hint. We have updated the sentence (at line 201-204:) "We take advantage of a certain type of the morphological rules of image processing techniques to prevent the nucleation of a given unit into another and for the model to obey topological rules. This becomes important for retaining the integrity of the predefined unit boundaries during the inversion and ensuring geological plausibility of the inverted model (age and deformation history)"
The exact meaning of Nucleation in geophysical inversion is the inclusion of one lithology into the other. We have added Fig. B1 to Appendix B in the revised manuscript to show the effect of applying this constraint on the second example.

RC2_12: Line 218: I wouldn't call the starting disc model random. Maybe it is better to use a different word to describe the choice of the initial model.

Reply: Thank you for noticing this detail. The starting model is not random and we corrected the term. We replaced the word "random" with "simplistic" as in line 233.

RC2_13: Lines 254-255: The sentence seems to contradict later sentences: it says that the seismic section is only applied in the construction of the initial model. However, around line 260 it is said that the reflectivity from the seismic section is also used as a constraint during inversion.

Reply: After re-examining the text we agree that the sentence is misleading. We have reformulated the sentence to prevent further misunderstanding. What is meant by mentioned sentences is: for generating the starting model, we have assumed that only the seismic section is available (meaning that the starting model follows the seismic section only and not the gravity datasets). The sentence is replaced (at line 270:) "…we generate the starting model using only information from the seismic section".

RC2_14: Line 286: "The difference between the structural similarities" and "an indication of the applicability of the approach to spatially distributed constraints in the level-set inversion" -- these two phrases are hard to take in and could be simplified for clarity.

Reply: Thanks for the suggestion. We simplified the sentence as (line 303-304:) "This implies that the method can be applied to real case scenarios where gravity and seismic datasets with different coverages are available."

RC2_15: Line 295, Figure 5: It might be nice to show the true and inverted models from the same angle.

Reply: Thanks for the suggestion which we agree with. We have changed the view of Fig. 4a to the same slices and same angle as Fig. 5 which represent the true and inverted model respectively. Now, Fig. 4 and Fig. 5 are in the same direction for viewing and the same slices are being used.

RC2_16: Line 440, Figure 12: Compared to the starting model, Figure 10b, it appears that the green, blue and brown units have switched placed and moved away from their original locations quite a lot. It is usually hard to recover the shape of a unit if there isn't some overlap between the initial and true unit location, so such a result doesn't look plausible. Considering that also the evolution of the level set function was suppressed at the seismic section, this final reconfiguration of the facies is very unlikely.

Reply: Thank you very much for noting this detail. As you mentioned, there was a mistake in colorbars of Fig. 10 and 15 and also some mistakes in density contrast values in the table which had led to different colorbars. We have updated the density contrasts in Table 1 and color-map of the Figure 10 and plotted different colorbar for (a) and (b) so that the color of the units matches with Figure 12. Having the color bar corrected, the problem about switching units is resolved now.

RC2_17: Are you sure that the units are plotted in the correct color? The color bar from Figure 15 would make the models on Figure 12 much more plausible and consistent with Figure 10 and the discussion. This needs to be fixed or explained. It would be helpful if Figures 10, 12 and 15 used the same color scheme, so I suggest replotting Figures 12 and 15 using the color scheme of Figure 10.

Reply: This is a very correct point which we are really appreciated for pointing it. As was suggested from the previous comment, we updated the color-scheme of Figures 10 and 15 to be compatible with Fig 12. Fig 10 is replotted with new colorbar and for Fig 15 we have updated the seismic section background color

to be more comparable with Fig 10 and 12. The seismic section in Fig. 15 now is in grayscale so when overlain with the model, slight changes in colors are inevitable.

RC2_18: Also, it would help to plot the final models on the same set of axes as the initial model in Figure 10a, to better visualize the shape changes of the bodies. It would also help to plot the initial and both final models along the seismic section overlain on the seismic image as in Figure 9d.

Reply: We agree that plotting the results in 3D might be a good visualisation. However, plotting all of the units in one frame in 3D for this case-study section will be a bit messy and we believe that the final model if plotted in the same way as Figure 10, won't be informative enough for the conclusion. The main focus is to compare the resulted constrained model along the seismic section, which we have included in the manuscript as Fig. 15. For showing that the resulted 3D model can be messy we have provided 2 figures (Fig. 2_1 and Fig. 2_2) that show 3D visualization of the Yamarna Greenstone belt as a sample in this document.

RC2_19: Lines 443-444: Again, it's a bit difficult to compare the initial and final models and also see how well the final models fit the constraints. If Figures 12 (a) and (b) were plotted on the same kind of axes as Figure 10, perhaps this would make understanding the changes in the model after the inversions easier.

Reply: In the original manuscript, there were some mistakes and contradictions between colorbars. We have corrected Fig. 10 colorbars based on previous comments. Now the two images can be compared easier given that the colorbars are fixed.

RC2_20: Lines 448-449: The fact that the models differ in constrained parts and do not much differ elsewhere seems to indicate that the information in the constraints does not quite agree with the information in the gravity data. I wonder if it might indicate that the constrained inversion result is incorrect elsewhere, or that the constraints themselves are incorrect? Could the authors comment on this? Have you tried a synthetic, in which the constraints were assumed correct, while they weren't, to see how robust the inversion is to such errors? There is a relevant comment on uncertainty in the conclusion section, but it is a bit far below and hard to tie to this particular place, so it would be nice to make a comment here.

Reply: By definition, the seismic section is uncertain and the model proposed is one among the possible ones that gravitate around the causative model. As a consequence, such seismic constraints might stir the gravity in the right direction, and the information from the gravity data is used to adjust that model, not completely wildly but somehow in accordance with the seismic. On the other hand, this area as explained in the manuscript is poorly known and unfortunately, the petrophysical constraints are not available in the studied area. The seismic section also being in a crooked 2D line and with a low signal-to-noise ratio (hard-rock) results in high imaging and interpretation uncertainties.

About the second part of the comment, yes it has been tested and can be well referenced later in the revised manuscript once the abstract is online. We have

provided this experiment in a conference abstract to AEGC which showed that in case of the wrong constraint the inverted model is still plausible compared to the true model. Also, the synthetic case of the SEG salt dome shows an example where the constraining information is not complete.

RC2_21: Line 465, Figure 15: The numbers and colors on the color bar are out of order and inconsistent with Figures 10 and 12. Could this figure be replotted in the color scheme of Figure 10, for easier understanding? How does the unconstrained inverted model compare with the prior interpretations? Could you perhaps show an example?

Reply: Thank you very much for the suggestion, we have re-plotted Figure 15 with the grayscale seismic image that has the least effect on the color scheme. We have corrected and adjusted the colorbar based on consistency with figures 10 and 12 too.

Also, we have added a new figure (Fig. 2_3) in this document that shows the suggested comparison along the seismic line. We think that this figure is not necessary to be included in the manuscript as a comparison between the constrained inversions overlain with existing seismic interpretation is already provided in Figure 15. The shape of the recovered models in Fig. 2_3a and Fig. 2_3b can be seen in Fig 10b and Fig. 12a in the manuscript and the overlain with seismic image might not be informative for the conclusion.

RC2_22: Line 505: Could Yamarna Greenstone unit be marked on Figure 12b? It would probably help to better appreciate the shape changes, if this body were plotted in 3D.

Reply: It could be a great idea to plot the Yamarna unit in 3D if it wasn't messy as what is shown in Fig. 2_1 and Fig. 2_2. We think is messy and doesn't add much information to the manuscript.

RC2_23: Line 509-510. Again, this is a bit difficult to see from the plots on Figure 12, I think a 3D plot of this body would help.

Reply: We have addressed this issue by replying to comments No. 18, 19, and 22.

RC2_24: Technical comments:

Lines 111-112: "in the same fashion as that smallness terms regularize inversion problems (Calvetti et al., 2000)", remove "that".

Line 118: I suggest using small "w" if the sentence is continued, or start a new sentence with "Here" with the capital "H".

Line 145: "as follow:" -> "as follows:"

Lines 161-162: Word "plausible" seems to be out of place here. It's not quite clear what this sentence grammatically means.

Line 165, Figure 1: In the caption, second line remove "of" from "Distribution of constraint matrix of from lithology 2".

Line 256: Replace “area. In” by “area, in”, otherwise the second sentence is grammatically incomplete.

Lines 480-482: The first sentence grammatically needs improvement. The next small sentence needs to be reformulated for style.

Reply: Thanks for all of the technical comments which we found really useful to improve the text passage. The technical comments were all applied and the text was modified in the revised manuscript.

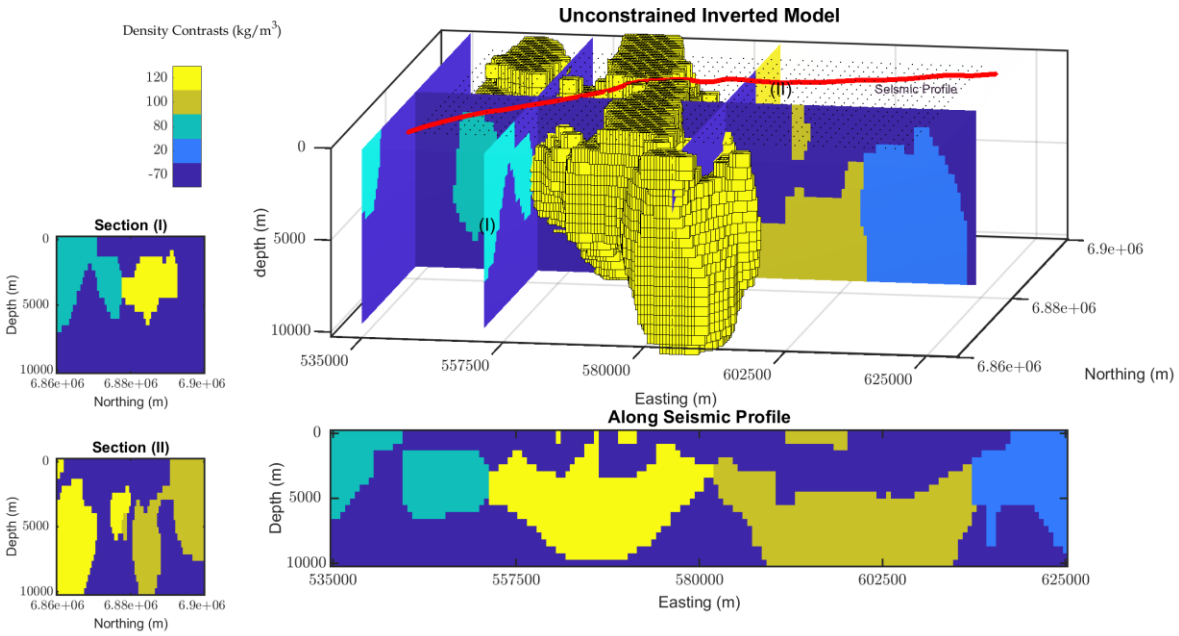

Fig 2_1: 3D visualisation of Yamarna Greenstone belt within the unconstrained inverted model

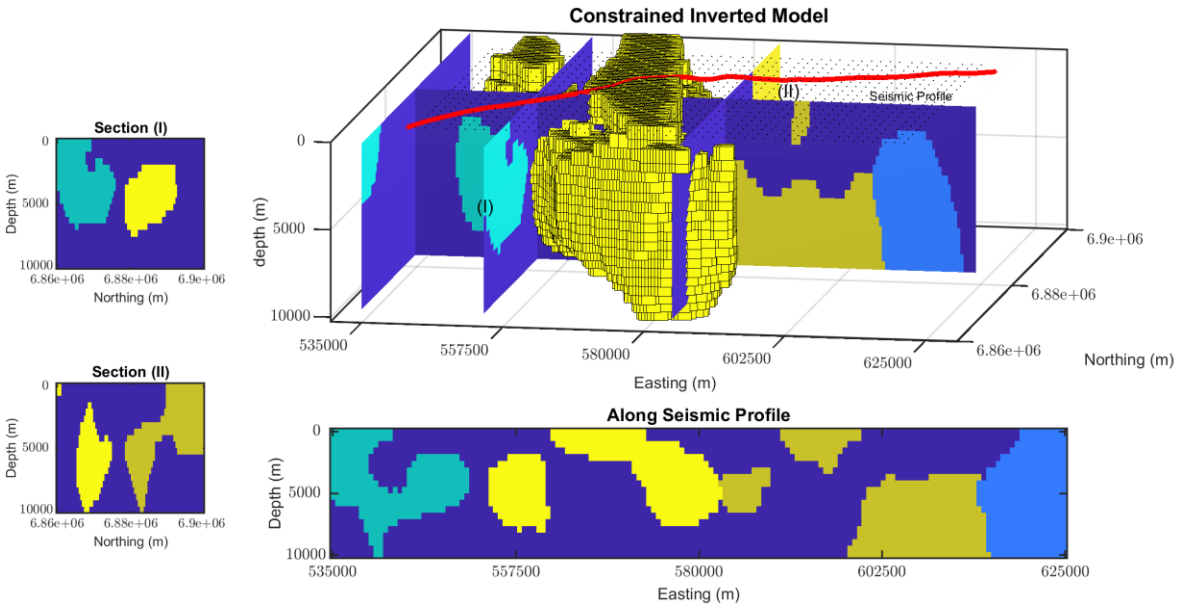

Fig 2_2: 3D visualisation of Yamarna Greenstone belt within the constrained inverted model

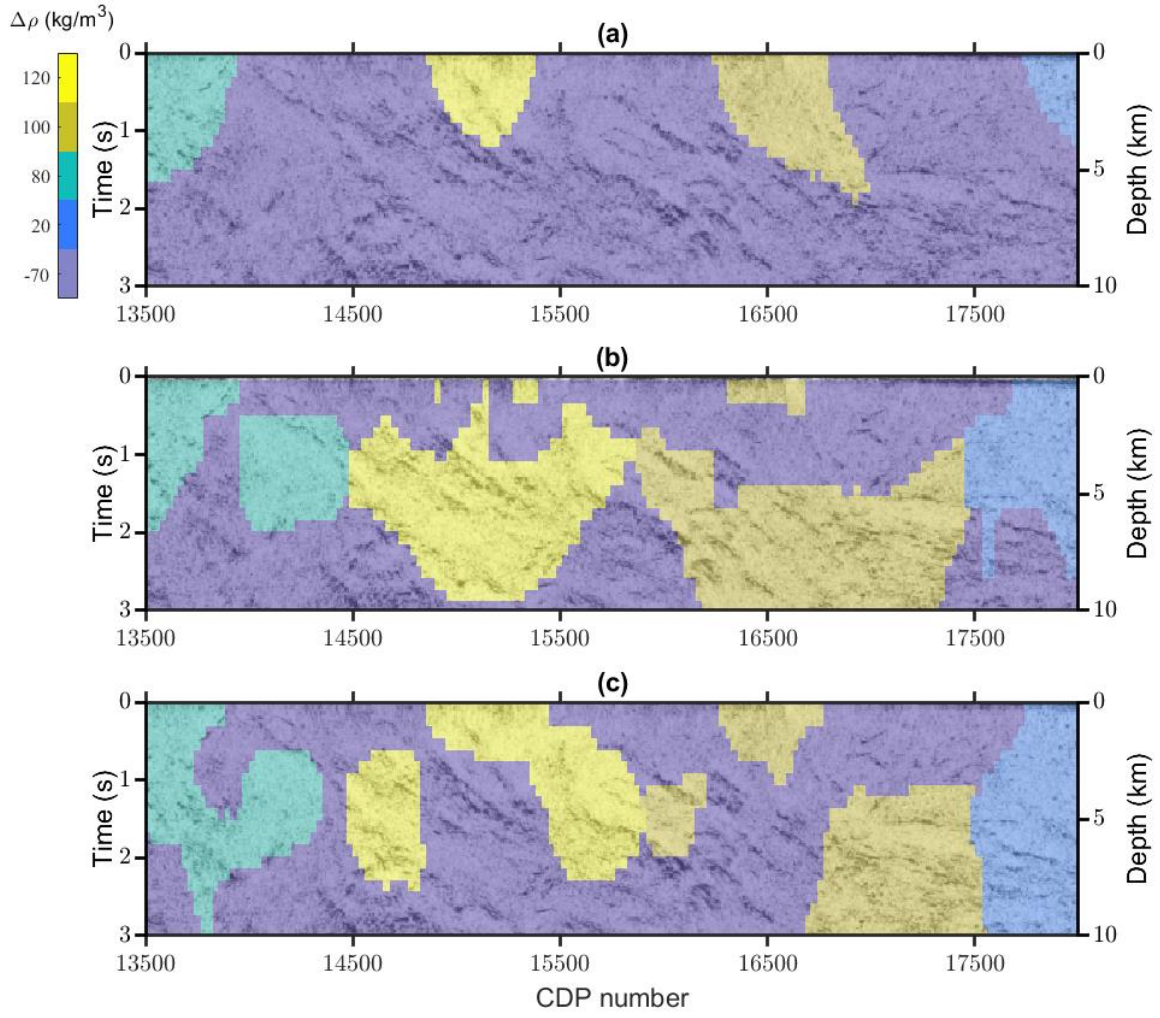

Fig 2_3: Comparison of the models (starting, unconstrained, constrained) overlain with the seismic image.